SOLID BODY TIDES
IN THE DYNAMICAL MODEL OF THE MOON

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ABSTRACT. The dynamical model of the orbital-rotational motion of the Moon, used in numerically integrated lunar ephemeris, accounts for tidal dissipation. However, it does not explain some long-term signatures that are detected by lunar laser ranging (LLR) observations. One cause of those signatures may be that the lunar crust experiences tidal deformations at different bands, and tidal Q factor depends on band; however, the dynamical model has a single tidal delay and does not have a concept of tidal band. An attempt to improve the dynamical model, introducing two delays instead of one, is made in this work. One delay is applied to the diagonal elements of the inertia tensor, while the other delay is applied to the non-diagonal elements. It turns out that the two delays are well separated and some of the residual long-term signatures are significantly different from those obtained with one-delay model.

1. INTRODUCTION

Modern study of lunar interior is mainly based on three kinds of data:

- Gravimetry measurements done by the GRAIL mission and its predecessors;
- Earth-Moon lunar laser ranging (LLR) measurements which allow to validate hypotheses about lunar structure via the indirect effect on lunar orbital and rotational motion;
- Photometry, laser altimetry and other measurements of the lunar surface performed by the LRO spacecraft and its predecessors. Those measurements allow to explore the history of the formation of the Moon via the present state of its exterior.

While there are similarities between methods of study of Earth's and Moon's structure, the latter has two distinct features:

- The rotational model of the Moon, unlike that of the Earth, is modeled with a set of dynamical equations directly representing lunar figure, core, tidal and rotational dissipation.
- The observations of lunar orbiters, most importantly GRAIL, are not tied directly to the lunar surface. There is an indirect tie via Earth–spacecraft measurements and a geocentric lunar ephemeris which is built using the aforementioned dynamical model.

The most precise lunar reference frame to date contains five retroreflectors points on lunar surface whose positions are determined from LLR. The accuracy of this frame is presently estimated at 0.2 m (optimistic) or 2 m (pessimistic) (Pavlov 2020). While there are obvious technical ways to improve the accuracy (more LLR measurements, more lunar retroreflectors), it is no less important that further improvement of lunar reference frame should go together with the improvement of the lunar dynamical model.

LLR has existed for 50 years. A continuous orbit of the Moon is fit to 50 years of LLR observations to form lunar ephemeris. LLR has been proven sensitive not only to the initial conditions of the Earth–Moon dynamical system and positions of retroreflectors, but also to parameters of the lunar core (Williams et al. 2001), low degree gravitational harmonics of the Moon, degree 2 tides on the Earth (Williams and Boggs 2016), and Earth's orbit plane.

However, the lunar dynamical model in its present state does not explain all signatures that are visible in LLR residuals. One major direction of research to explain the signatures is the improvement of the model of the lunar core. Another part that could be improved, which this work is devoted to, is the model of the solid body tides on the Moon.

2. USAGE OF GRAIL'S RESULTS IN THE DYNAMICAL MODEL OF THE MOON

A particular inconsistency in the lunar gravitational field solutions obtained from GRAIL data indicates a problem in the lunar dynamical model. Selenocentric orbits of GRAIL spacecraft (Ebb and Flow) were determined by Doppler tracking from Earth and tied to a specific lunar ephemeris.

The lunar ephemeris is built in such a way that the coordinate axes are supposed to be the principal axes (PA) of the whole Moon. Under that assumption, the gravity field coefficients C21, S21, and S22 are supposed to be zero. However, different GRAIL solutions, which include the said coefficients, all have S21 equal approximately 1×10^{-9} (see Table **??**).

Solution	Team	Ephemeris	C21	S21	S22
GL0660b	JPL	DE421	0.123×10^{-9}	0.101×10^{-8}	-0.249×10^{-9}
GRGM900c	GSFC	DE421	0.223×10^{-9}	0.101×10^{-8}	-0.105×10^{-9}
GRGM1200a	GSFC	DE430	0.147×10^{-10}	0.117×10^{-8}	0.908×10^{-9}
GL1500e	JPL	DE430	0.173×10^{-9}	0.104×10^{-8}	-0.102×10^{-9}

Table 1: Values of C21, S21, and S22 coefficients in different solutions obtained from GRAIL data

The nonzero GRAIL's S21 value does not depend much of the used ephemeris, and does not differ much in solutions done by different teams. Most probably, the nonzero S21 indicates an inconsistency in the lunar dynamical model rather than an artifact in GRAIL data or processing.

Another fact that further supports that point: nonzero S21 can be detected from LLR itself. While normally S21 is fixed to zero in the lunar dynamical equations (together with C21 and S22), it is possible to add S21 into the set of determined parameters in the solution. Such an experiment has been done in a special test solution of EPM lunar ephemeris, with the result of

$$\mathrm{S21} = (0.74 \pm 0.05^{(1\sigma)}) \times 10^{-9}$$

which is reasonably close to the GRAIL's value.

So, most probably the present lunar model (Folkner et al. 2014, Pavlov et al. 2016) has an inconsistency: a mass misalignment that manifests itself in the external torque but not in the mantle's inertia tensor. Once supposed cause of that was the shape of the lunar core (Williams et al. 2014). A tilted core would make the mantle's inertia tensor asymmetric, while not changing the external torque in the PA frame. Unfortunately, that hypothesis did not work out, because the supposed tilt would affect not only the inertia tensor, but the pressure torque, too. The effects nearly cancel each other out and the overall result of the titled core on lunar rotational dynamics would be very small (Pavlov 2018, Viswanathan et al. 2019).

It is also important that GRAIL solutions can have systematic biases that come from imperfections of the lunar rotational model. Also, fixing low-degree gravity coefficients to GRAIL values in the dynamical model can cause artificial "drag" of the resulting ephemeris towards the ephemeris consistent with the used GRAIL values. An attempt has been made (not shown here) to use the GRAIL gravity model in the DE430 frame while keeping the dynamical equations in their own (determined) frame; the resulting orbit turned out to be very bad in terms of LLR fits. A further attempt, to not only use the GRAIL values in DE430 frame, but also use kinematic model of lunar solid body tides (Konopliv et al. 2013) instead of the dynamical equations of tidal variations, did not give an acceptable result, either.

At this point it seems reasonable to avoid using GRAIL values for low-degree harmonics in the lunar dynamical model until a more elaborate dynamical model appears. First, at least, the nonzero S21 problem should be resolved; afterward, it is recommended to re-obtain GRAIL solution using the ephemeris obtained with the improved dynamical model.

3. JPL DE430 MODEL OF (DELAYED) TIDAL DISSIPATION

The DE430 lunar model (Folkner et al. 2014) is presently accepted, with minor modifications, also in EPM (Pavlov et al. 2016) and INPOP (Viswanathan et al. 2018) lunar ephemeris.

Let $I^* = I + I_c$ be the inertia tensor of the whole Moon, consisting of the mantle and the core. The inertia tensor of the whole Moon, according to the model, obeys the following equation:

$$\begin{aligned} \frac{I^*}{m} = \frac{I_0}{m} - k_2 \frac{\mu_{\mathrm{E}}}{\mu_{\mathrm{M}}} \left(\frac{R_{\mathrm{M}}}{r}\right)^5 \begin{bmatrix} x^2 - \frac{1}{3}r^2 & xy & xz \\ xy & y^2 - \frac{1}{3}r^2 & yz \\ xz & yz & z^2 - \frac{1}{3}r^2 \end{bmatrix} + \\ + k_2 \frac{R_{\mathrm{M}}^5}{3\mu_{\mathrm{M}}} \begin{bmatrix} \omega_x^2 - \frac{1}{3}(\omega^2 - n^2) & \omega_x\omega_y & \omega_x\omega_z \\ \omega_x\omega_y & \omega_y^2 - \frac{1}{3}(\omega^2 - n^2) & \omega_y\omega_z \\ \omega_x\omega_z & \omega_y\omega_z & \omega_z^2 - \frac{1}{3}(\omega^2 + 2n^2) \end{bmatrix}, \end{aligned} \tag{1}$$

where I_0 is the (diagonal) inertia tensor of the whole undistorted Moon; μ_{E} and μ_{M} are the gravitational parameters of the Earth and the Moon, respectively; R_{M} is the equatorial radius of the Moon; k_2 is the Love number; $\mathbf{r} = (x, y, z)^T$ is the geocentric position of the Moon; $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)^T$ is the angular velocity of the lunar mantle; n is the lunar mean motion.

The second and the third terms of Eq. (**??**) account for tidal and spin distortion, respectively. They come with a time delay: $I^*(t)$ depends not on $\boldsymbol{r}(t)$ and $\boldsymbol{\omega}(t)$, but on $\boldsymbol{r}(t-\tau)$ and $\boldsymbol{\omega}(t-\tau)$. τ is the time delay, a determined parameter.

The second-order gravitational potential varies over time accordingly with the inertia tensor:

$$\begin{aligned} C_{20} &= \frac{1}{R_M^2}\left[\frac{1}{2}\left(\frac{I_{11}^*}{m} + \frac{I_{22}^*}{m}\right) - \frac{I_{33}^*}{m}\right] \\ C_{22} &= \frac{1}{4R_M^2}\left[\frac{I_{22}^*}{m} - \frac{I_{11}^*}{m}\right] \\ C_{21} &= -\frac{1}{R_M^2}\frac{I_{13}^*}{m} \\ S_{21} &= -\frac{1}{R_M^2}\frac{I_{32}^*}{m} \\ S_{22} &= -\frac{1}{2R_M^2}\frac{I_{21}^*}{m} \end{aligned} \tag{2}$$

It is well known (Williams et al. 2013) that the described single-delay model is a compromise. In reality, the tidal delay strongly depends on the tidal band. The concept of band is not natural to the dynamical model. As a result, few-mas periodic longitudal libration terms appear in the LLR residuals. Some of them are: $\cos l'$ term (1 year), $\cos(2l - 2D)$ (206 days), and $\cos(2F - 2l)$ (3 years). Another, not presently included into ephemeris solutions, is a 6-year term $\sin(F - l)$, relatively recently noted by James Williams from NASA JPL.

One approach to allow tidal delay to vary with frequency is using an orbital delay in addition to the rotational delay (Williams and Boggs 2016); however, that approach was developed for Earth tides and is not applicable to the Moon.

Irrespective to the tidal delay problem, the currently developed EPM lunar ephemeris (Pavlov 2020) accounts for the tides not only from the Earth, but from the Sun, too. The single delay τ is applied to tides from both bodies.

4. TWO-DELAY MODEL

First of all, the spin part of Eq. (**??**), while originally proposed to have a delay, is mathematically almost non-sensitive to the delay. So for further calculations $\boldsymbol{\omega}(t)$ can be taken instead of $\boldsymbol{\omega}(t-\tau)$ with neglibible impact on the outcome.

As for the tidal distortions: (Williams and Boggs 2015, Table 1) lists an overview of variations of degree 2 spherical harmonic coefficients at different bands. There are monthly bands (F, l) and semi-monthly bands ($2D$, $2l$, $F+l$). There is also $2D-l$ band (31.812 days) and the aforementioned $F-l$ band (2190.350 days). All of them cause variations of second-degree coefficients; the set of affected coefficients depends on band. There are also other bands that cause smaller variations than the ones listed here.

Ideally, one would want to separate the bands entirely and use a dedicated delay for each band (and also separate the effect of tides of different bands on the inertia tensor). It does not seem to be easy, or possible at all. One small step towards the reduction of the disorder, however, is made in this work.

The idea is the following: use two delays τ_{02} and τ_1 instead of a single τ. Apply τ_{02} to the diagonal elements of the tidal part of I^* in Eq. (**??**), and apply τ_1 to the off-diagonal elements. Consequently, in Eq. (**??**), variations of C_{20} and C_{22} will depend on τ_{02}, while variations of C_{21}, S_{21}, and S_{22} will depend on τ_1.

The ABMD numerical integrator (Aksim and Pavlov 2019), developed at the IAA RAS, can handle differential equations with arbitrary number of delays, and can integrate both forward and backwards in time. The source code of the integrator is available at `https://gitlab.iaaras.ru/ iaaras/abmd`.

5. RESULTS

Two lunar solutions were obtained for comparison. The LLR observations used in the solutions, as well as the statistics of the residuals, are presented in (Pavlov 2020). Table **??** lists the determined values of the delays in the two solutions, and also the determined amplitudes of the kinematic longitudinal libration terms. Separate solutions were obtained where S_{21} coefficient was determined while $\sin(F-l)$ amplitude was not considered. The given errors are 1σ.

$\sin(F-l)$ amplitude and S_{21} are strongly correlated. If the lunar dynamical model is improved so that it naturally absorbs the $(F-l)$ longitudinal libration, the S_{21} frame misalignment problem will most likely be gone.

6. CONCLUSION

- In the lunar dynamical model, solid body lunar tides are of no less importance than the core.
- GRAIL data should be later reanalyzed with an improved underlying lunar dynamical model.
- 6-year period in longitude (discovered by James Williams) is strongly connected to the frame misalignment that results in nonzero S_{21}.

Fitted value	Single delay	Two delays
τ, days	0.098 ± 0.001	—
τ_{02}, days	—	0.068 ± 0.002
τ_1, days	—	0.144 ± 0.002
$\cos l'$ amplitude, mas	4.2 ± 0.1	4.2 ± 0.1
$\cos(2l - 2D)$ amplitude, mas	1.6 ± 0.1	0.8 ± 0.1
$\cos(2F - 2l)$ amplitude, mas	-0.8 ± 0.3	-3.4 ± 0.3
$\sin(F - l)$ amplitude, mas	-9.0 ± 0.6	-7.2 ± 0.6
$S_{21} \times 10^9$ (separate solution)	0.74 ± 0.5	0.58 ± 0.5

Table 2: Comparison of solutions obtained with single-delay and two-delay models

- Two different delays can be detected from LLR data. Separation significantly affects 206 days and 3 year amplitudes, slightly affects S_{21} / 6-year amplitude.

Regardless of the tidal model, work should continue on improving the model of the lunar core. One particular hypothesis worth checking there is the shifted core, see e. g. (Wieczorek et al. 2019) where shifted core is derived from joint analysis of GRAIL and LOLA data.

7. REFERENCES

Aksim, D., Pavlov, D., 2019, "On the extension of AdamsBashforthMoulton methods for numerical integration of delay differential equations and application to the Moon's orbit", Math Comput Sci., doi: 10.1007/s11786-019-00447-y.

Folkner, W., Williams, J., Boggs, D., Park, R., Kuchynka, P., 2014, "The Planetary and Lunar Ephemerides DE430 and DE431", IPN Progress Report 42-196, NASA JPL.

Konopliv, A.S., Park, R.S., Yuan, D.N., Asmar, S.W., Watkins, M.M., Williams, J.G., Fahnestock, E., Kruizinga, G., Paik, M., Strekalov, D., Harvey, N., Smith, D.E., Zuber, M.T., 2013, "The JPL lunar gravity field to spherical harmonic degree 660 from the GRAIL Primary Mission", J. Geophys. Res. (Planets) 118(7), pp. 1415–1434, doi:10.1002/jgre.20097.

Pavlov, D.A., Williams, J.G., Suvorkin, V.V., 2016, "Determining parameters of Moon's orbital and rotational motion from LLR observations using GRAIL and IERS-recommended models", Celest. Mech. Dyn. Astr. 126, pp. 61–88, doi: 10.1007/s10569-016-9712-1.

Pavlov, D., 2018, "Results of processing of the lunar laser ranging observations of 1970–2017", Izvestia GAO 225 (in Russian).

Pavlov, D., 2020, "Role of lunar laser ranging in realization of terrestrial, lunar, and ephemeris reference frames", J Geod 94(5), doi: 10.1007/s00190-019-01333-y.

Williams, J.G., Boggs, D.H., Yoder, C.F., Ratcliff, J.T., Dickey, J.O., 2001, "Lunar rotational dissipation in solid body and molten core", J. Geophys. Res. (Planets) 106(E11), pp. 27933–27968, doi: 10.1029/2000JE001396

Williams, J., Boggs, D., 2015, "Tides on the Moon: Theory and determination of dissipation". J. Geophys. Res. : Planets 120, pp. 689–724, doi: 10.1002/2014JE004755.

Williams, J., Boggs, D., 2016, "Secular tidal changes in lunar orbit and Earth rotation". Celest. Mech. Dyn. Astr. 126:89, doi: 10.1007/s10569-016-9702-3

Williams, J. G., Konopliv, A. S., Lemoine, F. G., Goossens, S., Asmar, S. W., Park, R. S., Yuan, D. -N., Boggs, D. H., Mazarico, E., Kiefer, W. S., Wieczorek, M. A., Watkins, M. M., Smith, D. E., Zuber, M. T., 2014, "A Glimpse of Lunar Core Shape and Deep Gravity Field", 45th Lunar and Planetary Science Conference.

Williams, J.G., Boggs, D.H., Folkner, W.M., 2013, "DE430 Lunar Orbit, Physical Librations, and Surface Coordinates", JPL IOM 335, California Institute of Technology.

Viswanathan, V., Fienga, A., Minazzoli, O., Bernus, L., Laskar, J., Gastineau, M., 2018, "The new lunar ephemeris INPOP17a and its application to fundamental physics", Monthly Not R Astron Soc 476(2), pp. 1877–1888, doi: 10.1093/mnras/sty096.

Viswanathan, V., Rambaux, N., Fienga, A., Laskar, J., Gastineau, M., 2019, "Observational constraint on the radius and oblateness of the lunar core-mantle boundary", Geophys Res Let 46, pp. 7295–7303, doi: 10.1029/2019GL082677.