AN EXAMPLE TO ANALIZE DISCRETE VECTOR FIELDS ON THE SPHERE USING QUANTITATIVE AND QUALITATIVE METHODS

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ABSTRACT. Our aim is to compatibilize the correction of massive ground- based catalogs and the study of properties that are missed in DR2. Two advantages justify these studies. On the one hand, it is usual to suppose that in the correction process the signal and the noise are accurately detected, but this is not necessary true if the statement "with respect to me adjustment model and a precision order" is not added to the assertion. The improvement of two ground-based catalogs and their comparison may explain a common part in the residuals depending on certain physical properties. On the other hand a question arises: the observation from the Earth involves some intrinsic errors but is it possible to align them to the ICRF while conserving the above-mentioned intrinsically terrestrial properties? This is not possible, but we can seek for quantitative improvements that eliminate bias and determine qualitative properties of the residual vector field on the celestial sphere with radius r considering magnitudes and spectral types. This is applied to assign a proper motion vector field in the domain of work.

1. INITIAL STEPS

Choice of a set of stars common to the Hipparcos and the massive catalog 2MASS. Denote this set as Ω_0. Now, we consider different properties such as the spectral type (splitting the data into a KM set and a no-KM set), H-magnitudes (splitting the data into $m_4 = [5.750, 7.153)$, $m_5 = [7.153, 8.556)$, $m_6 = [8.556, 9.959)$ and $m_7 = [9.959, 11.363)$) and also the distances assigning the data to sets, following Astraatmadja and Bailer-Jones (2016) in DR1. We can built different subsets of stars as, for instance, Ω_{KM,m_4} for KM stars with m_4 magnitude; or $\Omega_{m_5}|_{r=200}$, containing m_5 stars in the slice $[100pc, 300pc]$, for example. The intervals for r $[25pc, 200pc], [100pc, 300pc], [200pc, 400pc]$, are named after their "center" $r = 100, 200, 300, ..$pc. We will denote as Ω any of these possible data work sets.

2. STEP 2: OBTENTION OF THE VECTOR FIELDS

From each set of data points Ω, we supose a relation $Y_i = m(X_i)$ where $X_i \in \Omega$ is a random vector and we assume that:

$$m(\mathbf{X}_i) \simeq m(\mathbf{x}) + (D^1 m)_x(\mathbf{X}_i - \mathbf{x}) \tag{1}$$

with x near some X_i. We obtain the estimator for the vector field and the estimators for the first derivatives of the vector field: $\widehat{m}(x)$, $\widehat{m}_1(x)$, $\widehat{m}_2(x)$,... as the solution of the problem described in:

$$\{\widehat{m}(x), \widehat{m}_1(x), \widehat{m}_2(x)\} = \left\{\widehat{b_{00}}(\mathbf{x}), \widehat{b_{11}}(\mathbf{x}), \widehat{b_{12}}(\mathbf{x})\right\} =$$
$$= \min_{\left\{\widehat{b_k}(x)\right\}} \{Y_i - b_0(\mathbf{x}) - b_{11}(\mathbf{x})(X_{i,1} - x_1) - b_{12}(x)(X_{i,2} - x_2)\}^2 K_{h,ix}$$

$$K_{\mathrm{h},ix} = \frac{1}{h_1} K(\frac{X_{i,1} - x_1}{h_1}) \frac{1}{h_2} K(\frac{X_{i,2} - x_2}{h_2}) \tag{2}$$

3. STEP 3: USING VECTOR FIELDS TO OBTAIN QUANTITATIVE DATA

Suppose that the vector of the residuals is developed in vector spherical harmonics depending on r, α, δ by means of:

$$\mathbf{X}(r, \alpha, \delta) = \sum_{n,|m| \leq n} [r_{nm} \mathbf{R}_{nm} + s_{nm} \mathbf{S}_{nm} + t_{nm} \mathbf{T}_{nm}] \tag{3}$$

where $\mathbf{R}_{nm}, \mathbf{S}_{nm}, \mathbf{T}_{nm}$ are the vectors (orthogonal and complete system of the functions with integrable square in the sphere of radius r) whose coefficients represent the radial, spheroidal and toroidal parts, respectively of the field $\boldsymbol{X}$. These vectors are given by the expressions:

$$\mathbf{R}_{nm} = Y_{nm} \mathbf{r}, \mathbf{S}_{nm} = r \nabla Y_{nm}, \mathbf{T}_{nm} = -\mathbf{r} \times \nabla Y_{nm} \tag{4}$$

Using inner product $\langle , \rangle$ in the Hilbert Space of the spherical S^2-vector, the coefficient for the normalized basis vector ϕ_k, are $\alpha_k = \langle \widehat{m}, \phi_k \rangle$. In Marco et al (2019) one can see an exhaustive set of coefficients of developments for each set Ω. There, it can be observed that the values of the obtained parameters and their evolution (in the distance) depend on both magnitudes and spectral types, so that the corrections must be more specific than what is usually considered. This step, in itself, was qualitatively and quantitatively finer and more precise than other more usual procedures.

4. STEP 4: USING VECTOR FIELDS TO OBTAIN QUALITATIVE DATA

Before applying any correction (which is in J2000), we study stars where (or very close where) the residual field is singular and, in addition, the rotational component of the field is irrelevant. These points have the particularity of being maximum or minimum of the function of magnitude VT of Tycho, used in the reduction of the 2MASS. Note that from Helmholtz decomposition, a vector field is split into two components (rotational and irrotational) by means of $\overrightarrow{X} = \overrightarrow{\nabla \phi} + \overrightarrow{\nabla} \times \overrightarrow{u}$ where X is an spherical vector field. Taking divergence operator, we deduce the relation $div \overrightarrow{X} = \Delta \phi$ and assuming $\phi(\alpha, \delta) = \sum_{n,m} a_{nm} Y_{nm}(\alpha, \delta)$, from the application of properties of the Laplace-Beltrami operator we deduce:

$$\Delta \phi = \sum_{n,m} a_{nm} \Delta Y_{nm} = \sum_{n,m} [-n(n+1) a_{nm}] Y_{nm} \tag{5}$$

where

$$a_{nm} = -\frac{1}{n(n+1)} \frac{\langle \Delta \phi, Y_{nm} \rangle}{\langle Y_{nm}, Y_{nm} \rangle} \tag{6}$$

We show here only two examples of singular points of the vector field near singular points of the potential (see Figures 1 and 2). For more examples, see the above mentioned paper.

5. CONCLUSION

Returning to the text of the Abstract, after the correction process, we obtain the possibility of exploring the aforementioned "second advantage". On the other hand, regarding the "first advantage: both "ground based" catalogs will be improved using Hipparcos2 so that by comparing two improved catalogs with each other, we can launch the hypothesis that the Hipparcos effect has only affected the improvement and not the residual component related to the "Earth-observed"

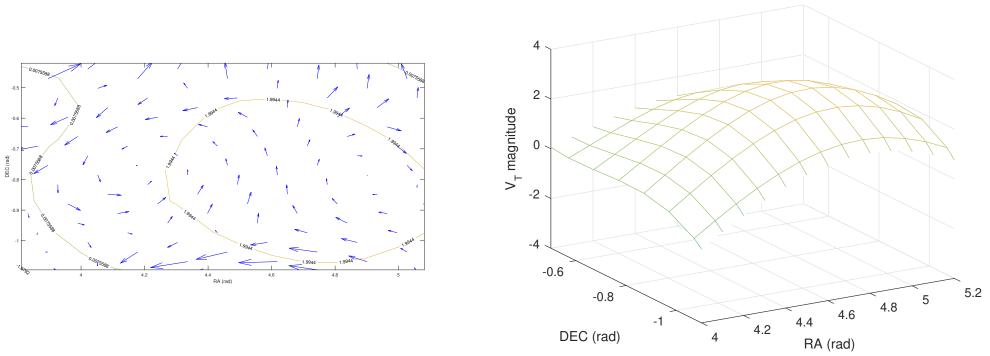

Figure 1: Slice 25-200. Area III, $KM - m_5$, Shrink (1.6,1); On the left, the potential vector field. On the right, the corresponding V_T surfaces.

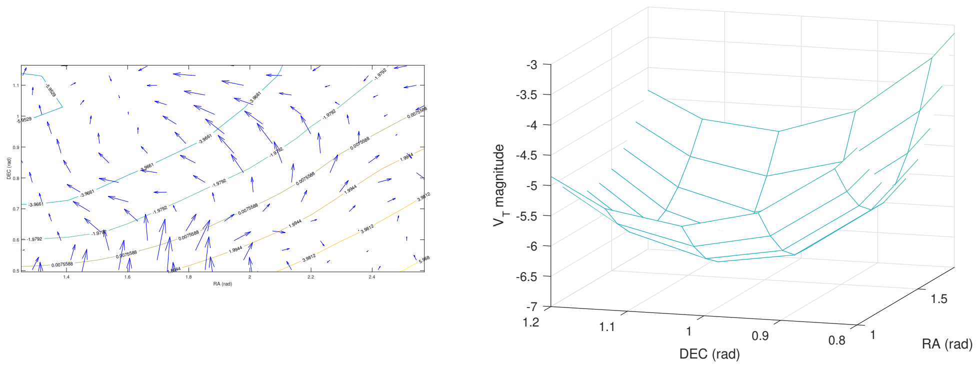

Figure 2: Slice 25-200. Up, Area I, $KM - m_4$, Source (4.9,-0.8); On the left, the potential vector field. On the right, the corresponding V_T surfaces.

Table 1: Comparison between the proper motions from DR1, PMA and our obtained results for some stars in the neighborhood of some singular points of their vector field. Hip stands for the number of the star in the Hipparcos catalogue, DR1 for the DR1 identifier. Subindex (1) means $DR1$ and (2) means PMA. The proper motions are given in mas, being the last two colums $\Delta\mu^*_\alpha$ & $\Delta\mu_\delta$ our obtained results.

Hip	$DR1$	$\alpha_{(1)}$	$\delta_{(1)}$	$\Delta\mu^*_{\alpha(1)}$	$\Delta\mu_{\delta(1)}$	$\Delta\mu^*_{\alpha(2)}$	$\Delta\mu_{\delta(2)}$	$\Delta\mu^*_\alpha$	$\Delta\mu_\delta$
28951	1008018207212849024	91.6628127275	63.4538738263	−35.558	8.594	−34.73	21.84	−36.35	21.33
30031	998050069154382592	94.7912065760	56.5264619828	−37.640	−31.332	−	−	−39.10	−31.05
24771	188796557490084480	79.6695937289	−32.3228506874	4.963	−13.891	3.08	−20.75	4.47	−23.68
27047	4756115082715073920	86.0379453784	−65.1018594755	−2.259	21.541	−4.01	26.0	−2.24	27.01
23865	279443525899748352	76.9426318743	55.7590984639	26.115	−14.343	22.01	−16.58	29.28	−21.70
89345	6721441368029854976	273.4528707722	−43.2043919512	−9.007	−13.989	−8.55	−17.27	−9.91	−18.69
44931	3841861165533628672	137.2857737710	−1.5880525106	−27.981	−35.349	−28.80	−45.08	−23.34	−43.16
23863	279443525899748352	76.9426318743	55.7590984639	26.115	−14.3434	22.01	−16.58	28.47	−22.17

character of both catalogs that may remain. Extension to Tycho-2 stars must be performed preserving the Hipparcos-2 corrections.

4. REFERENCES

The 2MASS All-Sky Catalog of Point Sources: `http://cdsarc.u-strasbg.fr/viz-bin/cat/II/246`.

Astraatmadja, Tri I. and Bailer-Jones, A.L., 2016."Estimating distances from paralaxes. III. Distances of two million stars in the Gaia DR1 Catalogue". The Astrophysical Journal 833(1), p. 119.

Gaia Data Release 1: `https://www.cosmos.esa.int/web/Gaia/dr1`.

Hipparcos-2 Catalogue: `https://www.cosmos.esa.int/web/Hipparcos/Hipparcos-2`.

F.J. Marco Castillo, M.J. Martínez Usó, J.A. López Ortí, 2019, "Global and Local Three-dimensional Studies of The Residual Vector Field from 2MASS and Hipparcos-2 Catalog", Publications of the Astronomical Society of the Pacific.

PMA Catalogue : I/341 : `http://cdsarc.u-strasbg.fr/viz-bin/cat/I/341`.