TESTING THE IMPROVEMENT OF THE IAU PRECESSION USING DIFFERENT J_2 VARIATION WITH TIME

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ABSTRACT. At its 2006 General Assembly, the International Astronomical Union (IAU) has adopted a new precession theory, called the "IAU 2006 precession", that is dynamically consistent and compliant with the IAU 2000 nutation. The time variation of the Earth's dynamical flattening J_2 is one of the contributions to the IAU 2006 model for the precession rate in longitude. However, the uncertainty in the J_2 model is one of the greatest sources of uncertainty in this precession theory. In this paper, we use the latest observational data from the satellite laser ranging to investigate the effect of different J_2 long time variations in solving the precession of the equator. The polynomial expressions for precession quantities are developed with a method similar to the IAU 2006 approach and are checked using the latest VLBI series of celestial pole offsets. However, a longer time span of VLBI data is required to reveal which J_2 model is more realistic. A serious study should be carried out before introducing a new J_2 variation in the IAU precession model.

1. INTRODUCTION

The current precession model recommended by the IAU and IERS is named the IAU 2006 precession (Capitaine et al. 2003). It is compatible with the IAU 2000 nutation and provides polynomial formulas for a number of quantities for the CRS-to-TRS transformation paradigms.

As one part of the IAU 2006 model, the precession of the ecliptic was derived by fitting the analytical ephemerides VSOP87 to the long term numerical ephemerides DE406 over the time span J1000.0 to J3000.0. For the precession of the equator, the basic quantities ψ_A and ω_A were derived by solving the dynamical differential equations using the improved ecliptic precession, updated integration constants provided by the IAU 2000 model, and the best non-rigid Earth model available at that time (Capitaine et al. 2003). One important feature of the IAU 2006 precession of the equator is the inclusion of a negative J_2 rate (J_2 is known as the Earth's form factor or the second-degree zonal harmonic of the Earth's gravitational field) that is generally attributed to the postglacial rebound of the Earth's mantle. The value for the J_2 rate adopted in the IAU 2006 model, according to Williams (1994), is such that

$$\dot{J}_2 \quad = \quad -3.001 \times 10^{-9}\ \mathrm{cy}^{-1}. \tag{1}$$

However the relative uncertainty of the J_2 rate reaches about 20% (Williams 1994), which is therefore one of the main limiting factors of the accuracy of the precession in longitude.

More recently, Liu & Capitaine (2017, denoted LC17) tried to construct an improved precession model by taking into account various progresses in Earth rotation theories, solar system ephemerides, as well as the best available celestial pole offsets monitored by the very long baseline interferometry (VLBI). In the LC17 work, new ephemerides INPOP10, DE406, and VSOP2013 were used to build the precession of the ecliptic. Several progresses in theoretical precession rates including contributions from revised non-linear terms, tidal Poisson terms, second-order torque, Galactic aberration, and more importantly, new determinations of the J_2 variation, were applied to

Table 1: Theoretical contributions of J_2 to precession rates in X component. Unit: $\mathrm{mas\,cy^{-2}}$ and $\mathrm{mas\,cy^{-3}}$.

		t^2	t^3
IAU 2006	$r_\psi(J_2)$	-14.0 ± 3.0	0
LC17	$r_\psi(J_2)$	-2.5 ± 1.2	$+50.6 \pm 9.2$
LC19	$r_\psi(J_2)$	-1.4 ± 1.1	$+87.0 \pm 9.1$
J_2 data (2002-2019)	$r_\psi(J_2)$	$+17.7 \pm 1.0$	0
No J_2	$r_\psi(J_2)$	0	0

integrate the dynamical differential equations for the precession of the equator:

$$\sin\omega_A \frac{d\psi_A}{dt} = (r_\psi \sin\epsilon_A)\cos\chi_A - r_\epsilon \sin\chi_A, \qquad \frac{d\omega_A}{dt} = r_\epsilon \cos\chi_A + (r_\psi \sin\epsilon_A)\sin\chi_A. \tag{2}$$

The solution in Liu & Capitaine (2017) has significant difference for the quadratic and cubic term in the polynomials of ψ_A. It shows certain improvement with respect to the IAU 2006 precession as indicated by VLBI residuals. However, due to large uncertainty in the J_2 empirical models and limited time span of the VLBI observations, the authors recommended to retain the current IAU model. In this short paper (denoted LC19), we report our new effort of improving the IAU precession model. The main updates of the present study are consideration of new J_2 data provided by NASA GSFC SLR observations and the use of a longer VLBI series of celestial pole offsets.

2. LONG TERM VARIATION OF THE EARTH'S J_2

Generally the long-term trend in J_2 has been approximated by a negative linear drift attributed to postglacial rebound of the Earth's mantle or the ongoing global isostatic adjustment, therefore a constant J_2 rate was adopted in the IAU 2006 precession. The observations from SLR data used in LC17 (up to 2011, denoted by gray dots in Figure 1) demonstrates that the deceleration in J_2 variation is significant. One important conclusion is that the long-term variation of the Earth's dynamical form-factor J_2 appears, from SLR observations up to 2011, to be more quadratic than linear in nature (Cheng et al. 2013).

According to newly added SLR data from 2002 to 2018 plotted as green dots in Figure 1, the J_2 rate seems positive in recent years (Loomis et al. 2019), which is opposite to the value adopted by the IAU model . This fact gives us adequate reason to believe that J_2 variation adopted in LC17 is still appropriate and motivates us to study more carefully this effect on the Earth's precession. In all, we have 43-year data from 1976 to 2019 as illustrated in Figure 1, and they are fitted to a parabola again in the whole interval. Based on the theoretical contribution of the Earth's oblateness to the precession rate in longitude (Capitaine et al. 2003), we calculated the numerical values of r_ψ corresponding to different J_2 empirical models. In Table 1 the first and second lines give the theoretical contribution of J_2 used in the IAU 2006 and the LC17 paper. The third line corresponds to the updated coefficients when new SLR data are added in the present analysis, which have been brought into the integration of the precession equation. We call the new solution L19 in this work. The last line has been obtained from only new data between 2002 and 2018 such that the trend of J_2 is positive.

3. UPDATED SOLUTION

The upgraded precession of the equator is obtained by solving the differential Eq. 2 with using (i) as in LC17, the updated ecliptic precession expressions derived from VSOP2013 and DE422 as given in LC17, (ii) the theoretical contributions to the precession rates including the $\dot{J}_2$ as listed in

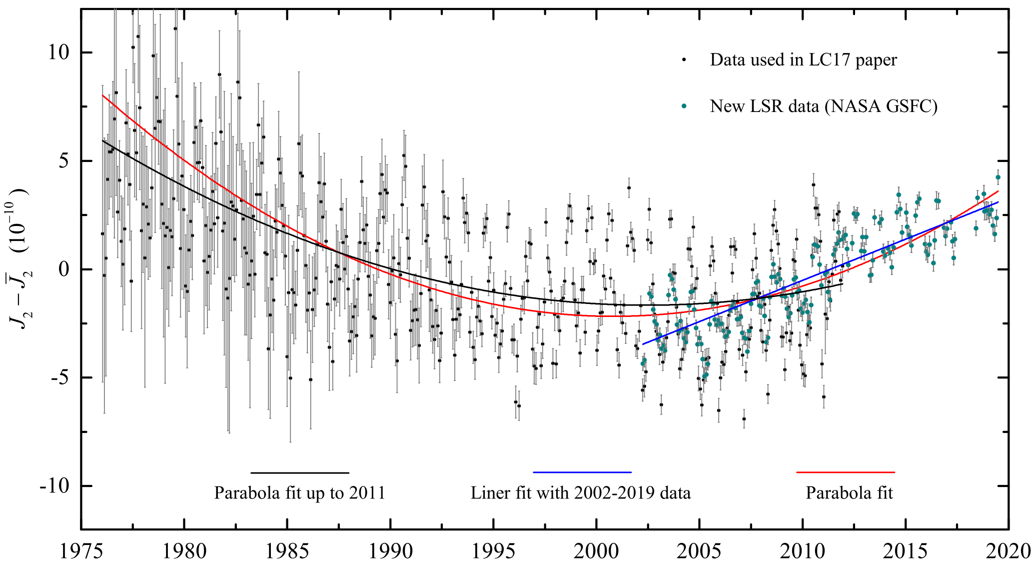

Figure 1: The Earth's J_2 values evaluated from SLR and its long term variation. The constant $\bar{J}_2$ is the mean value for J_2, which equals 0.0010826359797. The original data are provided by Cheng et al. (2013) and Loomis et al. (2019). The error bars are shown in grey on the plot.

Table 2: Difference to IAU 2006 of the coefficients of the polynomial expressions up to the third degree for the quantity ψ_A from various precession solutions. Unit: μas cy^{-1}, μas cy^{-2}, μas cy^{-3}.

$\Delta\psi_A$	t^1	t^2	t^3
IAU 2006	0	0	0
LC17	532	5 767	16 847
LC19	534	6320	28 995
Positive J_2 rate	534	15 862	−5
No J_2	534	7 012	−2

line 3 (LC19) of Table 1. The expressions corresponding to the updated solution for the primary precession quantities of the equator, ψ_A and ω_A, are such that

$$\begin{aligned}
\psi_A &= 5\,038''.482\,041\, t - 1''.072\,687\, t^2 + 0''.0278\,555\, t^3 + 0''.000\,123\,42\, t^4 - 0''.000\,000\,109\,6\, t^5 \\
\omega_A &= \epsilon_0 - 0''.025\,754\, t + 0''.0512\,626\, t^2 - 0''.0077\,249\, t^3 - 0''.000\,000\,086\, t^4 + 0''.000\,000\,221\, t^5
\end{aligned} \quad (3)$$

with $\epsilon_0 = 84\,381''.406$ being the obliquity of epoch. The secondary precession quantities (p_A, ϵ_A, and χ_A) are not listed here.

The comparison of the different solutions for ψ_A (mainly different in J_2 contributions) by taking the IAU 2006 as a reference is shown in Table **??**. The largest differences in the quadratic and cubic terms for ψ_A and p_A are attributed to the use of updated empirical model for J_2 variation. The signs for the t^3 terms of ψ_A are now positive while it was negative in IAU 2006. The precession in obliquity ω_A is identical to the IAU 2006 value because the integration constant for both cases are the same: the only difference for ω_A, which is at an order smaller than 1 μas cy^{-1}, originates from the ϵ-dependence of these new theoretical contributions. The largest uncertainties in our solution for the precession in longitude are still attributed to the imperfection modeling of J_2 variation. Based on the numerical values in Table 1, the relative error in t^2 and t^3 terms in r_ψ are as high as 48% and 18% for the LC17 solution, but the they are even higher for the the new solution (78% for the quadratic term).

4. COMPARISON OF PRECESSION EXPRESSIONS WITH VLBI CELESTIAL POLE OFFSETS

The geodetic/astrometric VLBI technique plays a crucial role in understanding the Earth's rotation. It monitors the celestial coordinates of the CIP and the Universal Time (UT1), which are known as the Earth orientation parameters (EOP). The current accuracy of VLBI observation is unprecedented high, namely at microarcsecond level, thus it provides the best observational material for studying the behavior of the precession-nutation models. The VLBI observations have shown that there are deficiencies in the IAU 2006/2000 model of the order of 0.2 mas, mainly due to the fact that the free core nutation (FCN) is not part of the model. The differences between the CIP positions estimated by VLBI observations and the CIP positions predicted with the IAU 2000/2006 model are reported as "celestial pole offsets" (CPO) $\mathrm{d}X_{\mathrm{IAU}}$ and $\mathrm{d}Y_{\mathrm{IAU}}$:

$$\mathrm{d}X_{\mathrm{IAU}} = X_{\mathrm{obs}} - X_{\mathrm{IAU}}, \quad \mathrm{d}Y_{\mathrm{IAU}} = Y_{\mathrm{obs}} - Y_{\mathrm{IAU}}, \tag{4}$$

the subscript "IAU" meaning that the reference model is the standard IAU 2006/2000A$_{\mathrm{R06}}$ precession-nutation model.

To interpret more deeply the residuals between the observations and different precession solutions, we have used (i) straight line plus 18.6-year nutation, and (ii) a parabola plus the 18.6-year nutation as in Capitaine et al. (2009) for the least squares fit. The 18.6-year nutation is the largest nutation term and is expected to be sensitive to the errors of the secular precession model. The equations used for the fit of celestial pole offsets are such that:

$$\mathrm{d}X = \begin{cases} A_0 + A_1 t + A_{\mathrm{s}} \sin\Omega + A_{\mathrm{c}} \cos\Omega & \text{(i)} \\ A_0 + A_1 t + A_2 t^2 + A_{\mathrm{s}} \sin\Omega + A_{\mathrm{c}} \cos\Omega & \text{(ii)} \end{cases}, \tag{5}$$

where Ω (polynomial function of t) is the mean longitude of the ascending node of the Moon with a period of 6798.38 days (approximately equals 18.6 years).

The coefficients ($A_0,\ A_1,\ A_2,\ A_{\mathrm{s}},\ A_{\mathrm{c}}$) in the two functions of Eq. (**??**), as well as the pre- and post-fit weighted root means squares of the residuals are estimated (see Table 3). Note that only the results for the dX component is presented in this report.

Table 3: Weighted fits of the CIP coordinates X to VLBI residuals for different precession models. Unit: Unit: μas cy^{-1}, μas cy^{-2}, μas cy^{-3}.

Model	A_0	A_1	A_2	A_s	A_c	WRMS$_{\mathrm{pre}}$	WRMS$_{\mathrm{post}}$
IAU 2006	13 ± 1	382 ± 12	$--$	36 ± 1	-18 ± 1	126	116
	2 ± 1	-90 ± 23	4436 ± 184	54 ± 1	-31 ± 1	126	115
LC17	9 ± 1	-210 ± 12	$--$	48 ± 1	-31 ± 1	128	115
	6 ± 1	-353 ± 23	1355 ± 184	54 ± 1	-35 ± 1	128	115
LC19	9 ± 1	-330 ± 12	$--$	52 ± 1	-36 ± 1	135	115
	8 ± 1	-391 ± 23	579 ± 184	54 ± 1	-38 ± 1	135	115
Positive J_2 rate	-2 ± 1	-503 ± 12	$--$	61 ± 1	-37 ± 1	152	115
	3 ± 1	-301 ± 23	-1884 ± 184	54 ± 1	-31 ± 1	152	115
No J_2	6 ± 1	82 ± 12	$--$	47 ± 1	-26 ± 1	152	115
	3 ± 1	-92 ± 23	1635 ± 184	54 ± 1	-31 ± 1	152	115

From Table 3, one can see that the pre-fit WRMS for the first three precession expressions, i.e. IAU 2006, LC17, and LC19, are of the same level, but that for forth and fifth experiments with positive and no-J_2 effects, respectively, the pre-fit WRMS are higher by about 20% with respect to the previous solutions. Note that the pre-fit WRMS can be used to indicate the global consistency between the observed CIP location and the theoretical CIP positions predicted by the corresponding precession-nutation models, one can conclude that a positive or zero value for J_2 rate are flawed, although the post-fit WRMS are of the same level as the others.

From this table, we found that the coefficient of t^2 term decreased significantly when the empirical model adopted for J_2 includes a quadratic term. Since the most important changes in the LC17 or LC19 precession is the introduction of an updated J_2 variation, which mainly modified the quadratic term and cubic terms of the precession in longitude, we have shown that the use of J_2 quadratic variation eliminated most of the residual quadratic curvature in the celestial pole offsets. Furthermore, the smallest coefficient A_2 appears in the LC19 precession model which is constructed using the whole SLR observational data. This indicates that the updated model for the J_2 variation dervied from longest time span is appropriate even though the duration of SLR observations is not long enough.

Table 4: Correlation coefficients of the VLBI fitting results with two empirical models. The precession model is LC19.

Term	A_0	A_1	A_2	$\sin\Omega$
A_1	-0.7			
A_s	$+0.0$	$+0.4$		
A_c	$+0.2$	-0.3		-0.0
A_1	-0.1			
A_2	-0.3	-0.8		
A_S	-0.2	-0.3	$+0.5$	
A_c	$+0.2$	$+0.3$	-0.4	-0.2

Table 4 presents the correlation matrix of the coefficients in different empirical models for the precession solution LC19. In the model comprising of a linear plus a 18.6 year periodic term, the correlation coefficients ρ_{A_0,A_1} and ρ_{A_1,A_s} are significant. For the parabola plus a 18.6-yr model, the correlation coefficients ρ_{A_1,A_2} are also unexpectedly high. This should be attributed to the relative short time interval of the VLBI CPO series, which may be not sufficient to separate the secular and periodic signal in the residuals.

5. CONCLUSION

In this study, we have investigated the possibility of improving the IAU 2006 precession model with updating our work in 2017. We have introduced an empirical expression for J_2 based on the most recent and accurate determination by the SLR observations over 43 years before integrating the precession equations for the equator. The quadratic and cubic terms in the precession quantity ψ_A have difference at the order of 6 mas cy^{-2} and 29 mas cy^{-3} with respect to the IAU 2006 (see Table **??**). With the help of additional two years of VLBI data, we tried to check the precession models against observations. The newly developed precession have shown some advantages with respect to the IAU 2006 model, but due to the limited time span of VLBI data and relatively large uncertainties in the J_2 variation, this effect should be studied more carefully in the future before introducing such a different Earth model in precession-nutation. Moreover the model for the J_2 variations should be consistent with a dynamical model. Therefore it is still preferable to retain the IAU 2006 as the standard model before the corrections such as J_2 variation are significant and robust enough.

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6. REFERENCES

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