PROGRESS IN UNDERSTANDING NUTATIONS

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ABSTRACT. We present an overview of the recent activities within the project RotaNut (Rotation and Nutation of a Wobbly Earth), an ERC Advanced Grant funding from the European Research Council. We have recomputed the Basic Earth Parameters (BEP) from recent VLBI series and we interpret them in terms of physics of the Earth's deep interior. This includes updates of the nutational constraints on Earth's internal magnetic field as well as of the coupling constants at the core-mantle boundary (CMB) and inner core boundary (ICB). We have explored on simplified Earth models the interactions between rotational and inertial modes. With the help of numerical simulations, we have also addressed the coupling between the global rotation and the inertial waves in the fluid core, including ohmic and viscous dissipation effects. Special interests have been given to the influence of the magnetic field onto the large scale flow in the liquid core and its influence on the different dissipation mechanisms. The role of precession and nutation forcing for the liquid core is characterized as well as the interaction between the Free Core Nutation (known as the spin-over mode in the fluid core community) and inertial waves.

1. VLBI OBSERVATION

We first analyzed the VLBI observation for obtaining the nutations. We used several nutation series among which, the C04 IERS nutation series. The Basic Earth Parameters (see Mathews et al., 2002, Koot et al., 2010, Zhu et al., 2017) are shown in Table 1.

BEPs	Mathews et al 2002	Koot et al 2010	Zhu et al 2017
$10^3\ (e_f + \Re K_{CMB})$	2.6681 ± 20	2.6752 ± 15	2.6760 ± 8
$10^3\ \Im K_{CMB}$	-0.0185 ± 14	-0.0186 ± 5	-0.0188 ± 5
$10^3\ \Re K_{ICB}$	1.11 ± 10	0.98 ± 6	1.01 ± 4
$10^3\ \Im K_{ICB}$	-0.78 ± 13	-0.87 ± 22	-1.11 ± 4

Table 1: Values of some of the Basic Earth Parameters (BEPs) for different nutation series.

The coefficient $(e_f + \Re K_{CMB})$ is in the range $[2.674\,10^{-3}, 2.677\,10^{-3}]$ and $\Im K_{CMB}$ in the range $[-0.019\,10^{-3},\ -0.018\,10^{-3}]$ in the last two studies. They did not change too much. So it remains to interpret these values as previously. Concerning the ICB parameters, their ranges differ quite a lot between the studies. For the last studies of Zhu et al. (2017) and Koot et al. (2010), $\Re K_{ICB}$ is in the range $[0.92\,10^{-3},\ 1.05\,10^{-3}]$ and $\Im K_{ICB}$ in the range $[-1.15\,10^{-3},\ -0.65\,10^{-3}]$ respectively.

2. INTERPRETATION OF THE CMB BEPS

Buffett et al. (2002) have interpreted the CMB BEP by considering that the differences with respect to the hydrostatic value of the Free Core Nutation (FCN) are due to an increase of the flattening of the core and the presence of a dipolar and uniform magnetic field at the CMB.

However, the value of the uniform field that is necessary to explain the data is at the level of 0.6 mT, twice the value of the observed field (e.g. Olsen et al., 2009).

We consider that it is indeed necessary to include the existence of a magnetic field inside the core to explain the observed nutations but that, while previously thought to have a very minor influence, it is also necessary to incorporate the presence of viscosity within the core as well as its influence on the core flow to explain the value of the BEP K_{CMB}.

3. INTERPRETATION OF THE ICB BEPS

The values of the ICB coupling constants have tentatively be explained by Koot and Dumberry (2011) by considering the inner core viscosity. They computed the effects on the K_{ICB} as a function of the inner core viscosity. The observed values of the real and imaginary parts provide lower bound on the inner core viscosity but as seen by the observed values, there is too much energy dissipated to be explained by the inner core viscosity only.

Just as for the case for the CMB, we consider as well that it is necessary to include viscosity in the outer core as well as the existence of a magnetic field to explain the observed K_{ICB}.

4. FLUID CORE BEHAVIOUR FOR EXPLAINING THE BEPS

Usually, the scientists studying the core in the frame of geodynamo ignore variations of the mantle's spin axis. Scientists studying the nutations usually consider that the core motion is prescribed as a uniform vorticity (or Poincar) flow. Within the RotaNut project, Rekier et al. (2019) propose a numerical method to compute the inertial modes of a container with near spherical geometry based on the fully spectral discretization of the angular and radial directions using spherical harmonics and Gegenbauer polynomial expansion, respectively. This permits to solve simultaneously the Poincaré equation and the no penetration condition as an algebraic polynomial eigenvalue problem. This allows them to recover the inertial modes of an exact oblate spheroid undergoing uniform rotation.

On the other hand, no core flow motion beyond the simple Poincar flow model is considered in traditional geodetic studies. In reality, in a viscous rotating fluid, there are Coriolis-restored (or inertial) oscillations, and, in a flattened core, there are rotational modes (related to nutations). These motions interact. In the vicinity of the FCN, as parameters such as the mantle's moment of inertia vary, the FCN frequency can increase at the same time that it becomes more damped. Under certain conditions, the FCN can interact with nearby inertial modes and can even exchange personality with them. In this situation, the FCN ceases to be the least damped mode. This has been shown in the paper of Triana et al. (2019). It shows the importance of considering the inertial modes in the core.

For the Earth, we are not in the parameter range where the FCN interacts with other modes, at least not through viscous interactions. However, the presence of other inertial modes have consequences on the FCN itself. For a viscosity value in the core that is close to the real Earth, Triana et al. (2020, in preparation) have considered the presence of a magnetic field, which complicates the matter.

Buffett (2010) has studied the core flow and ohmic dissipation associated with the FICN by 'mimicking' the radial flow that the FICN mode would produce at the inner core surface. Thus the model is based on a boundary flow forcing at the inner core at the theoretical FICN frequency ($\omega = 0.9975\,\mathrm{day}^{-1}$). Assuming a uniform background magnetic field, he showed that the ohmic dissipation $\mathcal{D}_{\mathrm{ohm}}$ increased as the viscosity decreased, following a $\mathcal{D}_{\mathrm{ohm}} \propto E^{-2/3}$ scaling law. However, he only reached an Ekman number near $E \sim 10^{-7}$ while the low viscosity for the real Earth's fluid core corresponds to an Ekman number $E \sim 10^{-15}$.

Triana et al. (2020, in preparation) are able to go further down in the Ekman number at the level of 10^{-10}. Surprisingly, the behaviour of the viscous and ohmic dissipations are not as expected by Buffet (2010). Instead of a continuous increase of the ohmic dissipation and a continuous decrease of the viscous dissipation, the Ohmic dissipation saturates! And the viscous dissipation can even start increasing in some cases. What we are seeing is that, at that particular forcing frequency, we are hitting a resonance of the inertial modes, which completely perturb the smooth behaviour shown by Buffett (2010). This behaviour is found to be different for different magnetic field amplitudes. At a moderate field like in the Earth's core (corresponding to a Lehnert number $Le \sim 10^{-4}$), inertial mode resonances still persist in the curve showing the ohmic dissipation as a function of the frequencies around the FICN (see figure below) and the total energy dissipation will depend on how close the forcing frequency is to an inertial mode resonance. We emphasize that, in this model, the FICN is represented as a forced boundary flow coming radially from the inner core surface. This effectively 'mimicks' a wobbling inner core but with the notable exception that no torques or any back-reaction from the fluid core on the solid inner core are allowed. In other words, the FICN eigenmode itself is missing in the eigenmode spectrum. This indicates that a more suitable model is needed, i.e. a model including a spheroidal inner core responding dynamically to torques (pressure, viscous and magnetic) exerted by the fluid core and gravitational torques by the mantle, thus incorporating the FICN eigenmode along with the rest of inertial modes.

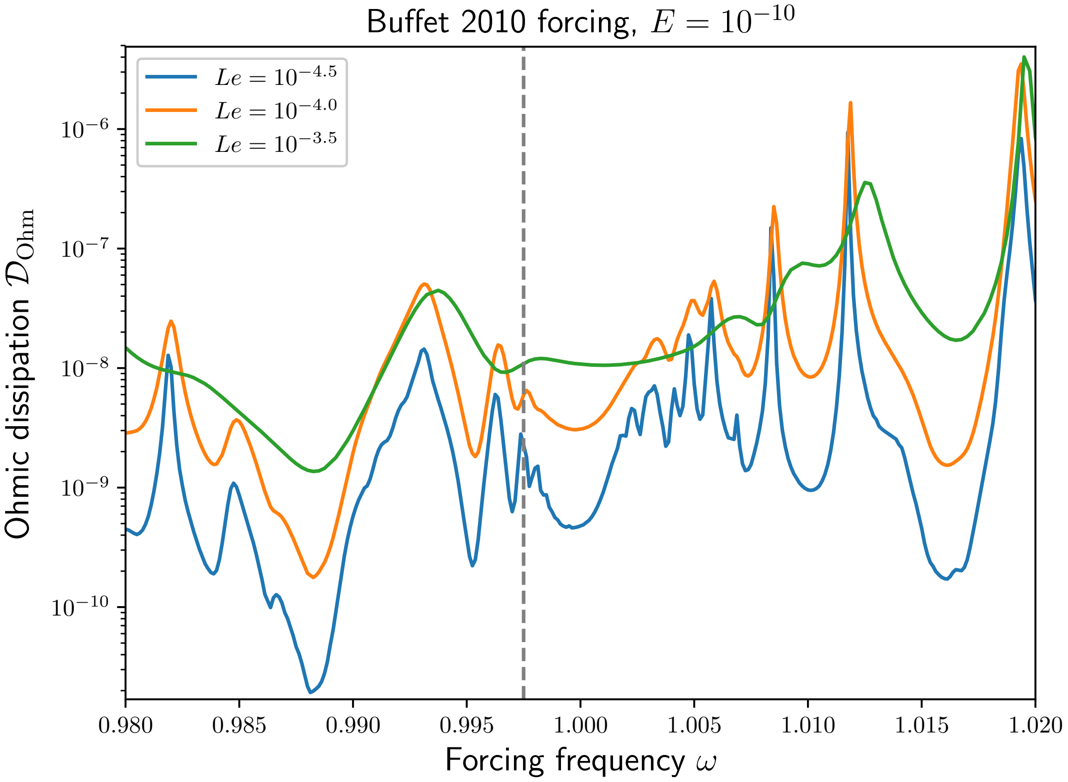

Figure 1: The ohmic dissipation in the model proposed by Buffett (2010) depends on the frequency used for the forcing. At the Ekman number in this figure ($E = 10^{-10}$), a small change in the frequency can lead to a more than an order of magnitude change in the dissipation, or even more if the Ekman number is smaller (note that $E \sim 10^{-15}$ for the Earth's fluid core). Increasing the background magnetic field (here represented adimensionally by the Lehnert number Le) reduces the resonances' quality factor.

5. CONCLUSION

After having presenting the Basic Earth Parameters (BEPs) that are fitted on VLBI observations of the nutation, we have discussed how to explain them. We have considered a fully coupled system of fluid flow obeying the classical Navier-Stokes equation in the core and the mantle rotational excitation (tidal excitation). In such a system, inertial waves and global nutation in the fluid core interact. The frequencies and damping of the inertial modes and global rotation mode change with the core flattening, the viscosity, and the mantle moment of inertia. The FCN in particular can be very much influenced. Both ohmic and viscous dissipations are important to consider. While a simple model as the one from Buffett (2010) provided Ekman number scalings for the ohmic and viscous dissipations, the real Earth case seems to be much more complicated due to the likely interaction between the inertial modes and global rotation inside the fluid core. We are in the process of deducing effects on nutation of this mechanism.

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6. REFERENCES

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