

# ANALYSES OF CELESTIAL POLE OFFSETS WITH VLBI, LLR, AND OPTICAL OBSERVATIONS

Y.-T. Cheng, J.-C. Liu, Zi Zhu

School of Astronomy and Space Science, Key Laboratory of Modern Astronomy and Astrophysics, Nanjing University - China - jcliu@nju.edu.cn

## ABSTRACT.

**Aim.** This work aims to explore the possibilities of determining the long-periodic part of the Earth's precession-nutation with techniques besides VLBI. Lunar laser ranging (LLR) is chosen for its relatively high accuracy and long period. Results of previous studies could be updated using latest data with generally higher quality, as well as adding ten years to the total time span. Historical optical data are also analyzed for its rather long time coverage to look into its possibility to improve the current Earth's precession-nutation model.

**Methods.** Celestial pole offsets (CPO) series are obtained from LLR and optical observations, and analyzed separately by weighted least square fits of three empirical models. A joint analysis of VLBI and LLR data is also presented for further discussion.

**Results.** Determination of the nutation terms with both VLBI and LLR data have been improved. LLR shows its potential to determine celestial pole offsets with a comparably high accuracy with VLBI in the future and to serve as an independent check for the VLBI results. CPO series from historic optical observations have a typical standard error of about two hundred times larger than that of the VLBI series, hence is hardly able to make any contribution to the contemporary precession-nutation theory.

## 1. INTRODUCTION

The Earth's precession-nutation models describe the long-term and long-periodic changes of the Celestial Intermediate Pole (CIP) direction in the Geocentric celestial reference system (GCRS). In this work, we focus on the Celestial Pole Offsets (CPO), representing the differences between observations and theoretical predictions of the CIP locations (i.e.  $dX$ ,  $dY$ ).

By far, VLBI has been playing the most crucial role in this domain given its micro-arcsecond level accuracy. However, the relatively short time span of VLBI observations can be a disadvantage in revealing effects with long periods such as the deficiencies in the Earth's precession model with a period of about 26 000 years.

Lunar Laser Ranging (LLR), being operated since 1969, is the only space-geodetic technique now capable to realize a stable dynamical reference system, with sufficient accuracy of determinations of the lunar orbit (Zerhouni & Capitaine 2009, denoted as ZC09 hereafter). ZC09 provided a basic method of determining CPO from LLR observations using data in the interval of 1969-2008. Hofmann et al. (2018) (denoted as H18 hereafter) have estimated of reflector coordinates, station coordinates and velocities and EOP at the same time, with LLR data during 1969-2016.

The history of optical observations is much longer than that of both VLBI and LLR observations. Vondrák & Štefka (2010) have constructed a series of Earth Orientation Catalogs (EOC) based on a combination of Hipparcos/Tycho Catalogs and long lasting ground-based astrometric observations. These catalogs have been used to derive the CPO during time interval 1899.7-1992.0, referred to the Hipparcos Celestial Reference Frame (HCRF), which can be regarded as the optical realization of the ICRS.

In this work, we aim to improve the determination of precession-nutation corrections based on analyses of VLBI observations solely, and also a joint analysis of VLBI and LLR data. We follow the basic method of ZC09 for the analyses of LLR and the joint analysis of LLR and VLBI, and make a comparison with the results of H18. Further discussions and the future potential of LLR are presented. The time distribution of CPO series derived from LLR, VLBI (opa2018a) and optical observations are plotted in Figure ?? to show the differences in both the number of observations and in time span.

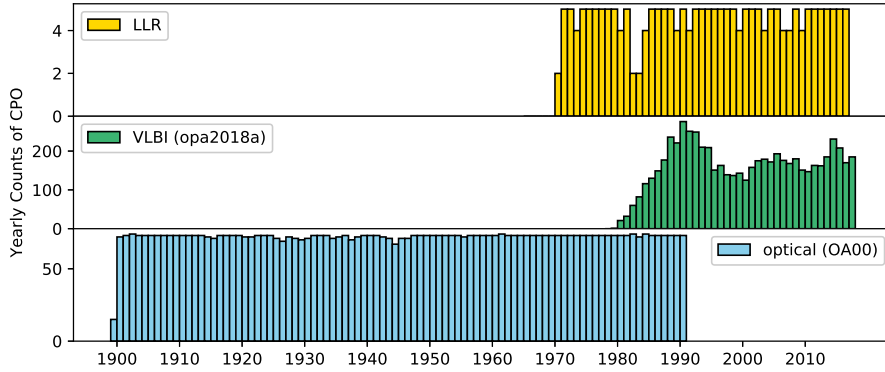


Figure 1: Time distributions of LLR, VLBI (opa2018a) and optical CPO series. Time span of every CPO series: LLR 1970.6-2017.7; VLBI (opa2018a) 1979.9-2018.5; optical (OA00) 1899.7-1991.9.

## 2. MODELS USED IN CPO ANALYSES

Three empirical models are fitted to the CPO series in the following analysis:

1. A parabola, namely a quadratic function of  $t$ .
2. A linear term and 18.6-year nutation term.
3. A linear term, 18.6-year and 9.3-year nutation terms.

All models are functions of  $t$  (centuries from the basic epoch). The basic epoch was set to be J2000.0 for VLBI and LLR series, but J1956.0 for optical series, which is almost the central epoch of the time coverage of optical observations.

## 3. CPO RESIDUALS WITH VLBI OBSERVATIONS

We use CPO series from 1979 to 2018 derived by the Calc/Solve software at the Paris Observatory analysis center (OPA) and also the series derived by the Goddard Space Flight Center (GSF) in the framework of the International VLBI Service for geodesy and astronomy (IVS, Nothnagel et al. 2017). The quasi-periodic free core nutation (FCN), with a period of 430.23 days, is removed before analyzing the residuals. We remove this effect according to the model recommended by the IERS conventions 2010 (Petit & Luzum 2010).

We fit all of the three models mentioned to the residuals, and show the results in Table ??, with both sources of data presented for comparison. The fitted coefficients of the secular terms show an underestimation in the IAU model of the precession rate in  $X$  about  $0.3 \text{ mas cy}^{-1}$ . Capitaine et al. (2009) have pointed out that the 18.6-year nutation term is the most sensitive to the error in the precession-nutation model, with VLBI data up to 2008. With the accumulated high-precision data, we here obtain more consistent results of the corrections of the 18.6-year nutation term. All fit

Table 1: Weighted fits of three models to VLBI residuals (1979.6-2018.6), corresponding to IAU 2006/2000 model.

Term		$t^0$	$t^1$	$t^2$	$\sin(18.6 \text{ yr})$	$\cos(18.6 \text{ yr})$	$\sin(9.3 \text{ yr})$	$\cos(9.3 \text{ yr})$	WRMS <sub>pre</sub>	WRMS <sub>post</sub>
Unit		$\text{mas}$	$\text{mas/cy}$	$\text{mas/cy}^2$	$\text{mas}$	$\text{mas}$	$\text{mas}$	$\text{mas}$	$\text{mas}$	$\text{mas}$
opa2018a	dX	$18 \pm 1$	$266 \pm 21$	$-4338 \pm 143$					126	119
	dY	$-86 \pm 1$	$-519 \pm 22$	$5325 \pm 145$					147	130
gsf2016a	dX	$49 \pm 1$	$215 \pm 19$	$540 \pm 120$					134	104
	dY	$-101 \pm 1$	$-477 \pm 20$	$5492 \pm 122$					131	109
opa2018a	dX	$13 \pm 1$	$381 \pm 12$		$36 \pm 1$	$-18 \pm 1$			126	116
	dY	$-71 \pm 1$	$-34 \pm 12$		$-35 \pm 1$	$40 \pm 1$			147	128
gsf2016a	dX	$49 \pm 1$	$441 \pm 10$		$43 \pm 1$	$-11 \pm 1$			134	101
	dY	$-87 \pm 1$	$61 \pm 10$		$-34 \pm 1$	$47 \pm 1$			131	107
opa2018a	dX	$13 \pm 1$	$334 \pm 12$		$34 \pm 1$	$-24 \pm 1$	$-19 \pm 1$	$-3 \pm 1$	126	116
	dY	$-70 \pm 1$	$4 \pm 12$		$-38 \pm 1$	$46 \pm 1$	$22 \pm 1$	$23 \pm 1$	147	126
gsf2016a	dX	$49 \pm 1$	$413 \pm 11$		$41 \pm 1$	$-14 \pm 1$	$-13 \pm 1$	$3 \pm 1$	134	101
	dY	$-84 \pm 1$	$40 \pm 11$		$-43 \pm 1$	$48 \pm 1$	$20 \pm 1$	$23 \pm 1$	131	105

coefficients of the 18.6-year nutation term reveal that the amplitude is underestimated by about 35 mas. However, the formal errors of the CPO derived from VLBI data are probably underestimated by about a factor 2 according to Herring et al. (2002).

#### 4. ANALYSES OF LLR RESIDUALS

LLR observations are presented as so-called normal points. They refer to lines of data containing the emission time of the laser, observed round trip time in UTC, telescope and reflector ID, and some atmospheric parameters of each observation. These data can be used to calculate the round-trip times, then the residuals of the round-trip time [observation minus calculation (O−C)], which can be converted to residuals in one-way distance in centimeters. Finally we obtain CPO series based on these residuals.

We used the residuals spanning 1970-2017 (O−C of the one-way distance in centimeters) provided by Pavlov et al. (2016)<sup>1</sup>. The rejection procedure is taken in advance to exclude data with relatively poor quality. The rejection criterion is such that O−C values that are higher than three times the respective formal error and the total WRMS of all normal points of the same station are excluded.

ZC09 offered a method of calculating the CPO from LLR observations. We generally follow their method in converting LLR residuals into CPO. The method of getting LLR residuals from the observational data (also known as “normal points”) in ZC09 (Chapront et al. 1999) and Pavlov et al. (2016) are basically the same. Among all the effects which have to be considered in the process of deriving dX and dY from LLR observations, only the transformation matrix for precession and nutation is related to X and Y. In the calculation of partial derivatives  $\frac{\partial \Delta t}{\partial X}$  and  $\frac{\partial \Delta t}{\partial Y}$ , we ignored minor effects such as solid tide deformations of the Earth and the Moon, since we do not need an accuracy of the moment of time as high as that in the calculation residuals of one-way distance. The relativistic transformations between terrestrial and selenocentric reference systems to the BCRS Petit & Luzum (2010) are applied. The relativistic gravitational delay (Kopeikin 1990) and the tropospheric delay (Mendes & Pavlis 2004, Mendes et al. 2002) are also taken into account.

We use JPL (Jet Propulsion Laboratory) planetary ephemerides DE430 (Folkner et al. 2014) for the geocentric and barycentric positions of the Sun, Earth, Moon and major planets, and also for the lunar libration angles and TT−TDB transformations.

Uncertainties are presented as two times and three times the formal error, in ZC09 and H18 respectively, after fitting the CPO series to the models to account for unmodeled effects and

<sup>1</sup><http://iaaras.ru/en/dept/ephemeris/llr-oc/>

Table 2: Weighted fits of model 2 and 3 to LLR residuals.

Term		$t^0$	$t^1$	$\sin(18.6 \text{ yr})$	$\cos(18.6 \text{ yr})$	$\sin(9.3 \text{ yr})$	$\cos(9.3 \text{ yr})$	WRMS <sub>pre</sub>	WRMS <sub>post</sub>
Unit		mas	mas/cy	mas	mas	mas	mas	mas	mas
This work	dX	$-0.38 \pm 0.02$	$1.43 \pm 0.18$	$-0.26 \pm 0.02$	$-0.37 \pm 0.01$			0.526	0.463
	dY	$-0.36 \pm 0.03$	$-0.54 \pm 0.19$	$-0.81 \pm 0.02$	$-0.30 \pm 0.01$			0.672	0.581
ZC09	dX	$0.27 \pm 0.13$	$5.77 \pm 3.25$	$0.00 \pm 0.22$	$0.01 \pm 0.13$				
	dY	$-0.17 \pm 0.13$	$1.07 \pm 3.11$	$-0.02 \pm 0.21$	$-0.22 \pm 0.12$				
This work	dX	$-0.28 \pm 0.03$	$1.77 \pm 0.19$	$-0.14 \pm 0.04$	$-0.26 \pm 0.02$	$0.20 \pm 0.02$	$0.01 \pm 0.02$	0.526	0.458
	dY	$-0.12 \pm 0.04$	$0.02 \pm 0.19$	$-0.25 \pm 0.05$	$-0.24 \pm 0.02$	$-0.05 \pm 0.02$	$-0.40 \pm 0.03$	0.672	0.562
ZC09	dX	$0.16 \pm 0.15$	$3.52 \pm 3.84$	$0.17 \pm 0.27$	$0.12 \pm 0.14$	$0.12 \pm 0.16$	$0.32 \pm 0.14$		
	dY	$-0.22 \pm 0.14$	$-0.16 \pm 3.67$	$0.08 \pm 0.26$	$-0.24 \pm 0.14$	$0.10 \pm 0.15$	$-0.01 \pm 0.14$		
H18	$d\psi \sin \epsilon$			$0.58 \pm 0.18$	$-0.09 \pm 0.13$	$0.04 \pm 0.12$	$-0.01 \pm 0.12$		
	$d\epsilon$			$-0.12 \pm 0.17$	$-0.36 \pm 0.16$	$-0.49 \pm 0.12$	$0.17 \pm 0.13$		

further model deficiencies. The factor three was checked by analyses of sub-sets of used LLR normal points.

Here we estimate several possible sources of uncertainty:

1. Signal propagation through the troposphere and stratosphere (for 523 nm): 0.82 cm (Mendes & Pavlis 2003)
2. Differences in ephemerides (DE430–INPOP17a): 0.11 cm

The sum of the two estimations above is approximately two times the formal error of LLR observations (0.61 cm). Therefore we multiply the formal errors of residuals in one-way distance with a factor three before obtaining CPO with weighted least square fits. In this way, the weighted average uncertainty of CPO determined from LLR observations are 0.061 mas, but it is necessary to stress that the quality of the CPO series also depends on the resolution (70 days for LLR instead of 1 day for VLBI).

We present estimations using empirical models 2 and 3 for the long period components of nutation after removing the FCN. In our results, the fitted coefficients of the 18.6-year nutation term with and without the 9.3-year term are not consistent. This feature is different from the results obtained by VLBI analyses. Meanwhile, the correlation coefficients between two nutation terms are over 0.7, revealing the incapability of LLR data to separate the components effectively. This is probably because that LLR observations are directly related with the motions of the Moon, which is also the most important excitation of the 18.6-year and 9.3-year nutations. Nevertheless, the correlation coefficients between the secular term and the nutation terms are generally smaller than those of VLBI, probably benefiting from the longer time span of ten years. Furthermore, the correlation coefficient between the secular term and the sine term of 18.6yr nutation remains larger than its counterparts (0.4), which may reveal a common problem shared by VLBI and LLR technique.

Further discussion of obtaining more CPO from LLR data and a joint analysis of VLBI and LLR can be seen in the published paper (Cheng et al. 2019).

## 5. FUTURE IMPROVEMENTS OF LLR OBSERVATIONS FOR A BETTER DETERMINATION OF CPO

Accuracy of CPO determined from LLR observations can be affected by many aspects. Of all, the observational error and frequency are most directly related to the observation itself. The observational accuracy, though suffering from instability, is improving. Comparing number of normal points and corresponding observation errors (see Figure ??), especially between 2000 and 2010, we can draw a conclusion that the dispersion and larger uncertainties of CPO in this period are the consequences of the lower frequency of observations. Therefore, making LLR observations

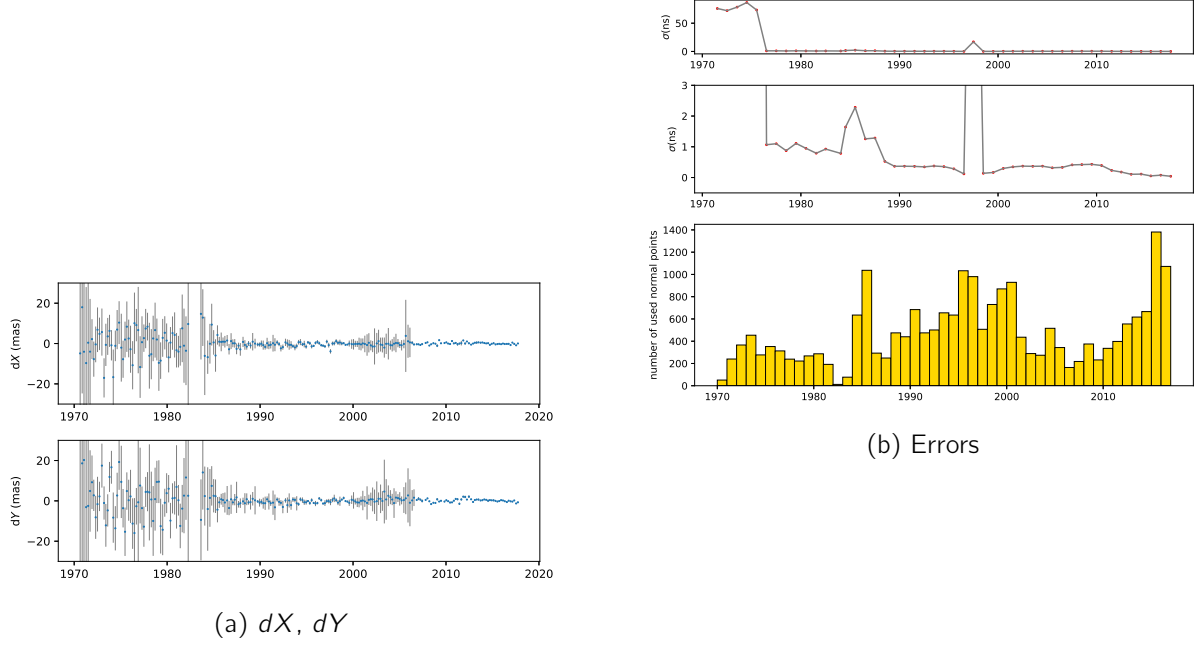


Figure 2: (a)  $dX$  and  $dY$  residuals of LLR observations in mas, using separated windows of 70 days. (b) Time series of observational errors and number of used normal points, the second figure being details of the first one. Each point/bin represents the weighted average/total within a year.

regular and sufficiently frequent to achieve a more uniform time distribution of normal points is quite essential in the future, before other necessary developments of related theories becoming conceivable.

## 6. ANALYSES OF THE OPTICAL RESIDUALS

The OA00 series provides time series of nutation offsets  $d\psi$  and  $d\epsilon$  with respect to the IAU 1976/1980 precession-nutation models (Lieske et al. 1977; Wahr 1981; Seidelmann 1982) between 1899 and 1972. Therefore, we followed the method described by Capitaine & Wallace (2006) to transform them into the same system of VLBI series, i.e.  $dX$  and  $dY$  with respect to IAU 2006/2000 precession-nutation models.

Models 1 and 2 are fitted to the transformed celestial pole offsets (CPO) to estimate the possible long term corrections. In both models  $t$  refers to the number of centuries from a reference epoch J1956.0, which is almost the central epoch of the time coverage of optical observations. Contrary to the VLBI results, there are no significant difference between the constant and secular terms in fitting results of the two models, and the correlation coefficients between secular term and nutation terms are negligible. These indicate that a time span long enough (over ninety years) may be sufficient to separate the quadratic term and the 18.6-year nutation term, whereas optical data cannot help in improving the precession-nutation model nowadays due to the poor accuracy.

**Acknowledgement.** This research is funded by the National Natural Science Foundation of China (NSFC) No. 11473013 and No. 11833004.

## 7. REFERENCES

Capitaine, N., Mathews, P. M., Dehant, V. Wallace, P.T., and Lambert, S.B., 2009, “On the IAU2000/2006 precession nutation and comparison with other models and VLBI observations”,

- Celest. Mech. 103, pp. 179–190.
- Capitaine, N. and Wallace, P. T., 2006, “High precision methods for locating the celestial intermediate pole and origin”, A&A 450, pp. 855–872.
- Chapront, J., Chapront-Touzé, M., and Francou, G., 1999, “Determination of the lunar orbital and rotational parameters and of the ecliptic reference system orientation from LLR measurements and IERS data”, A&A 343, pp. 624–633.
- Cheng, Y. T., Liu, J. C., and Zhu, Z., 2019, “Analyses of celestial pole offsets with VLBI, LLR, and optical observations”, A&A 627, A81.
- Folkner, W. M., Williams, J. G., Boggs, D. H., Park, R. S., and Kuchynka, P., 2014, “The Planetary and Lunar Ephemerides DE430 and DE431”, Interplanetary Network Progress Report 196, pp. 1–81.
- Herring, T. A., Mathews, P. M., and Buffett, B. A., 2002, “Modeling of nutation-precession: Very long baseline interferometry results”, J. Geophys. Res. (Solid Earth) 107, pp. 2069.
- Hofmann, F., Biskupek, L., and Müller, J., 2018, “Contributions to reference systems from Lunar Laser Ranging using the IfE analysis model”, J. Geodesy 92, pp. 975–987.
- Kopeikin, S. M., 1990, “Theory of Relativity in Observational Radio Astronomy”, Soviet Ast. 34, p. 5.
- Lieske, J. H., Lederle, T., Fricke, W., and Morando, B., 1977, “Expressions for the precession quantities based upon the IAU /1976/ system of astronomical constants”, A&A 58, pp. 1–16.
- Mendes, V. B. and Pavlis, E. C., 2003, “Atmospheric refraction at optical wavelengths: problems and solutions” in Proceedings of the 13th International Laser Ranging Workshop, Washington D.C., Noomen, R., Klosko, S., Noll, C., and Pearlman, M. (eds.), NASA/CP-2003- 212248.
- Mendes, V. B. and Pavlis, E. C., 2004, “High-accuracy zenith delay prediction at optical wavelengths”, Geophys. Res. Lett. 31, L14602.
- Mendes, V. B., Prates, G., Pavlis, E. C., Pavlis, D. E., and Langley, R. B., 2002, “Improved mapping functions for atmospheric refraction correction in SLR”, Geophys. Res. Lett. 29, pp. 1414.
- Nothnagel, A., Artz, T., Behrend, D., and Malkin, Z., 2017, “International VLBI Service for Geodesy and Astrometry. Delivering high-quality products and embarking on observations of the next generation”, J. Geodesy 91, pp. 711–721.
- Pavlov, D. A., Williams, J. G., and Suvorkin, V. V., 2016, “Determining parameters of Moon’s orbital and rotational motion from LLR observations using GRAIL and IERS-recommended models” Celest. Mech. 126, pp. 61–88.
- Petit, G. and Luzum, B., 2010, IERS Conventions (2010), IERS Technical Note, 36.
- Seidelmann, P. K., 1982, “1980 IAU theory of nutation - The final report of the IAU Working Group on Nutation”, Celest. Mech. 27, pp. 79–106.
- Vondrák, J. and Štefka, V., 2010, “The Earth Orientation Catalog 4 . An optical reference frame for monitoring Earth’s orientation in the 20th century”, A&A 509, pp. A3.
- Wahr, J. M., 1981, “The forced nutations of an elliptical, rotating, elastic and oceanless earth”, Geophys. J. 64, pp. 705–727.
- Zerhouni, W. and Capitaine, N., 2009, “Celestial pole offsets from lunar laser ranging and comparison with VLBI”, A&A 507, pp. 1687–1695.