POLAR MOTION RESONANCE IN THE RETROGRADE DIURNAL BAND

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ABSTRACT. The period of the polar motion resonance as estimated from luni-solar nutation terms is not equal to 433 days, namely the Chandler wobble period, but about 380 days. This puzzling estimate, first obtained by Mathews et al (2002), is caused by the dynamical response of the ocean in the nutation band, corresponding to the retrograde diurnal polar motion with respect a crust-fixed frame. The complex part of the resonance frequency is also strongly modified, and mostly results from the phase shit introduced by the ocean response to the pole tide potential. These conclusions are based upon our knowledge of the diurnal ocean tides, from which we can deduce the ocean pole tide in the same frequency band. Moreover, it seems that we have detected the effect of the free core nutation on the polar resonance parameter in the vicinity of the free core nutation frequency at -1.005 cycle/day.

1. INTRODUCTION

The resonance parameters (T_{PM}, Q_{PM}) of the polar motion (PM) are generally considered as the Chandler wobble period $T_c = 430 - 432$ days and its quality factor $Q_c = 56 - 255$ respectively (Nastula and Gross, 2015). But this coincidence is only insured in the broad frequency band surrounding the Chandler wobble, including annual period. Actually, in the retrograde diurnal band of the polar motion, that is the nutation band in a non-rotating frame, these resonance parameters become $T_{PM} \approx 383$ d, $Q_{PM} \approx -11$ (Nurul Huda et al, 2019). We propose a modelling of this phenomenon by accounting the response of a dynamical ocean and anelastic solid Earth to the pole tide potential.

2. RESONANT PERIOD OF THE COMMON POLAR MOTION

Let C be the Earth axial principal moment of inertia, A and A_m the equatorial principal moments of inertia of the Earth and of the mantle respectively, $e = (C-A)/A \approx 1/304.5$ the Earth dynamical flattening, and $\sigma_e = e\Omega$ the frequency of the Euler free wobble. The resonant angular frequency of the polar motion is given by (see e.g. Dehant and Mathews, 2015)

$$\sigma_{PM} = \sigma_e \frac{A}{A_m} \left(1 - \frac{\tilde{k}}{k_s} + O(e^2)\right) = \frac{A}{A_m} \Omega(e - \kappa + O(e)) , \tag{1}$$

where $k_s = 0.938$ is the secular Love number, $\tilde{k}$ is the coefficient accounting for Earth response to the pole tide potential, and $\kappa = e\tilde{k}/k_s$ is the compliance. Actually $\tilde{k}$ is composed of two parts:

$$\tilde{k} = \tilde{k}_2 + \tilde{k}_o . \tag{2}$$

Here $\tilde{k}_2$ means the body Love number of degree 2 accounting for the solid Earth response to the pole tide tesseral potential, and $\tilde{k}_o$ the oceanic Love number describing the ocean response to the same potential. For the common polar motion (beyond 2 days) the ocean response is considered at equilibrium. This leads to $\tilde{k}_o = 0.0477$. The solid Earth response is assumed as quasi-elastic, described by the body Love number $\tilde{k}_2 = 0.307 - i\,0.0035$ (Petit and Luzum, 2010). These

Q_1	$-0.037+i\,0.039$	O_1	$-0.030+i\,0.038$		
P_1	$-0.023+i\,0.042$	K_1	$-0.023+i\,0.042$	J_1	$-0.022+i\,0.047$

Table 1: Oceanic Love number for some prominent waves of the retrograde diurnal band.

values determine the resonance parameters ($T_{PM}=433.6$, $Q_{PM}=85$) in conformity with the observations.

3. CONTRIBUTION OF THE DYNAMICAL OCEAN RESPONSE

Below 10 days the ocean response is no more hydrostatic, and k_o changes accordingly. In the diurnal band, this issue can be solved in light of the diurnal ocean tides. For, as the pole tide potential has the same form than the luni-solar tesseral potential and concerns the same frequency band, the Earth response should be formally the same. The tidal height variation produces an equatorial component $H(t)$ of the ocean angular momentum. The observed diurnal ocean tide height is smaller than the theoretical equilibrium tide ξ, and strongly out-of-phased with respect to the tidal potential. Meanwhile, dynamical processes produce currents, in turn a relative angular momentum $h(t)$. At a location of colatitude θ and longitude λ the tesseral tidal potential is

$$W=-\frac{\Omega^2 r^2}{3}\ \mathrm{Re}\left[\tilde{\phi}(t)\mathcal{Y}_2^{-1}\right] \quad , \quad \mathcal{Y}_2^{-1}=3\sin\theta\cos\theta e^{-i\lambda}\ , \tag{3}$$

where

$$\tilde{\phi}(t)=\frac{3gN_2^1}{\Omega^2 R_e^2}\sum_{\sigma\geq 0}\xi_\sigma e^{-i(\theta_\sigma(t)-\pi/2)} \quad , \quad N_2^1=\sqrt{\frac{5}{24\pi}}\ . \tag{4}$$

is formally equivalent to $m(t)$ in pole tide potential. So, in Liouville equation, the tidal excitation $\chi_o(t)$ is proportional to $\tilde{\phi}(t)$, as the rotational excitation is proportional to $m(t)$:

$$\chi_o=\tilde{k}_o/k_s\ \tilde{\phi}\ . \tag{5}$$

The components of χ_o are computed from FES 2012 reported in (Madzak, 2016). For tesseral tides J_1, K_1, P_1, O_1, Q1, we extract the retrograde diurnal components H^- and h^-. Then, accounting for loading effect through the loading Love number k_2', for a tidal constituent at frequency σ we have

$$\chi_o(t)=\frac{H_\sigma^-(t)(1+k_2')+h_\sigma^-(t)}{(C-A)\Omega}\ =\frac{H_\sigma^-(1+k_2')+h_\sigma^-}{(C-A)\Omega}\ e^{-i(\theta+\chi)}\ , \tag{6}$$

where θ is the tidal argument. It results

$$\tilde{k}_o=k_s\frac{H_\sigma^-(t)(1+k_2')+h_\sigma^-(t)}{(C-A)\Omega\tilde{\Phi}_\sigma}\ =-k_s\frac{H_\sigma^-(1+k_2')+h_\sigma^-}{C-A}\ \frac{\Omega R_e^2}{3gN_2^1\xi_\sigma}\ . \tag{7}$$

We estimate $\tilde{k}_o$ for each tidal components H_σ^-, h_σ^- and corresponding tidal height ξ_σ. The resonance of the loading love number k_2' at FCN frequency does not impact significantly $\tilde{k}_o$ in retrograde the diurnal band. For $k_2'=-0.3075$, the obtained values differ strikingly from the oceanic Love number $k_o=0.0477$ estimated for an equilibrium pole tide: at K_1 $k_o=-0.023+i\,0.042$ or compliance $\tilde{\kappa}_o=\tilde{k}_o e/k_s=(-7.9+i\,14.6)\,10^{-5}$ in agreement with $(-6.9+i\,11.5)\,10^{-5}$ proposed by Mathews et al (2002). The values of Table **??** allow to model $k_o(\sigma)$ through a degree 2 polynomial of the frequency in the the diurnal retrograde band:

$$k_o(f)=(-0.716+i\,0.721)f^2+(-1.483+i\,1.337)f+(-0.791+i\,0.658)\ , \tag{8}$$

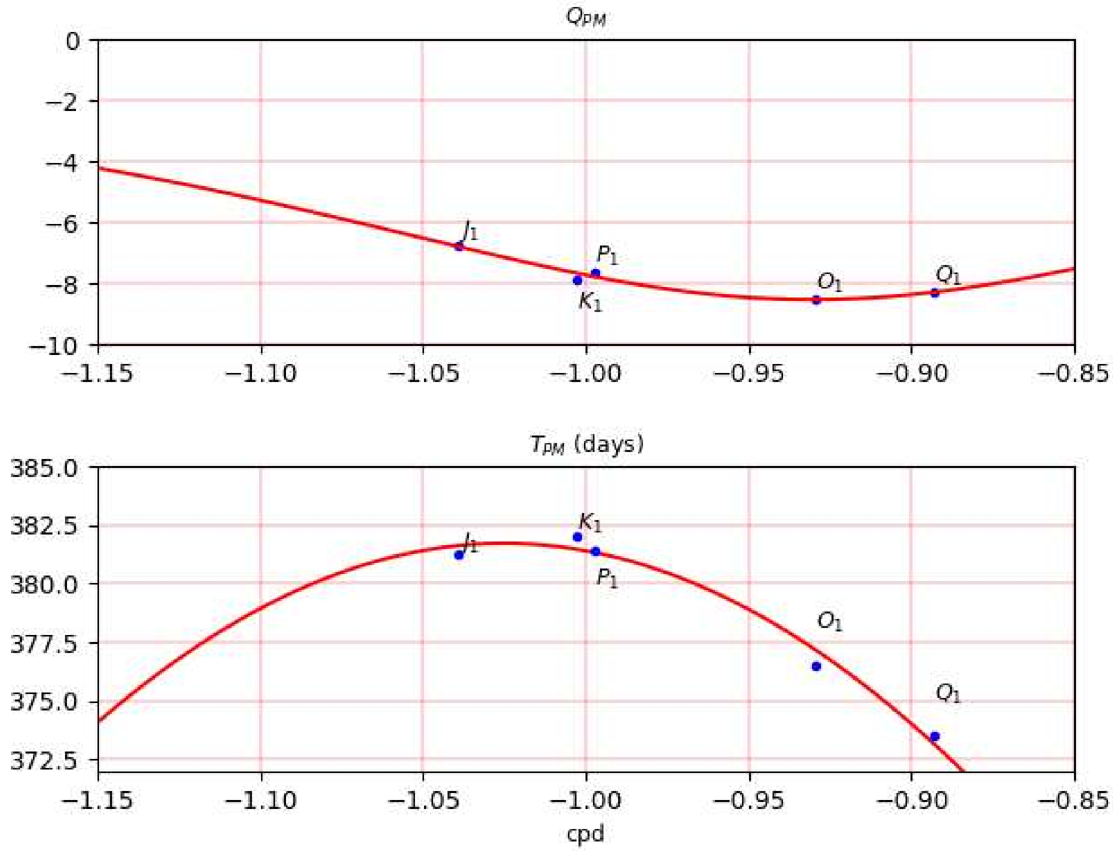

Figure 1: PM Resonance parameters in the diurnal retrograde band for an anelastic solid Earth covered by oceans.

where f is in cpd. Then we see that the resonance frequency (**??**) becomes frequency dependent:

$$\sigma_{PM}(\sigma) = \sigma_e \frac{A}{A_m} \frac{\tilde{k}_2 + \tilde{k}_o(\sigma)}{k_s} \quad , \tag{9}$$

where $\tilde{k}_2$ is taken as the Love number of an anelastic Earth, slightly varying with frequency. In the diurnal domain we have $\tilde{k}_2 = 0.299 - i\,0.00144$ (Petit and Luzum, 2010). The corresponding period and quality factor, namely $T_{PM}(\sigma) = 2\pi/\mathrm{Re}(\sigma_{PM})$ and $Q_{PM}(\sigma) = Re(\sigma_{PM})/(2\,\mathrm{Im}(\sigma_{PM}))$ are displayed in Figure **??**. So, in the frequency band [-1.15 cpd, −0.85 cpd], the dynamical ocean response leads to the resonance parameters lying in the intervals 374 d $< T_{PM} \leq$ 382.5 d and $-4 \leq Q_{PM} \leq -10$, confirmed by $T_{PM} = 382.0 \pm 1.3$ days and $Q_{PM} = -10.4 \pm 0.5$ obtained from nutation analysis in (Nurul Huda et al, 2019).

4. INFLUENCE OF THE FLUID CORE

Close to the retrograde diurnal frequency $\sigma_{FCN} = -1.00506$ cycle/day (cpd) of the free core nutation, the solid Earth tide departs from the one of a quasi-elastic Earth. Other perturbations, of much lesser amplitude (100 times less), occur because of the free inner core nutation (FICN) mode at $\sigma_{FICN} \sim 1.0017$ cpd in the TRF, and because of the Polar motion resonance appearing at the period ~ 380 days, as justified in the former section. From IERS Conventions 2010 (IERS, 2010), Table 6.4, Eq. 6.9 and 6.10, the "diurnal" body Love number has the form

$$k_2(\sigma) = 0.29954 - i\,0.1412\,10^{-2} - \frac{L_{PM}}{\sigma - \tilde{\sigma}_{PM}} - \frac{L_{FCN}}{\sigma - \tilde{\sigma}_{FCN}} - \frac{L_{FICN}}{\sigma - \tilde{\sigma}_{FICN}} \quad , \tag{10}$$

with the dominant term $L_{FCN} = (0.91\,10^{-4} - i\,0.30\,10^{-5})$ cpd. Here $\sigma_{PM} \approx 1/383$ cpd. Replacing in (**??**) the pure anelastic value of k_2 by its resonant version (**??**), we get

$$\sigma_{PM}(\sigma) = \sigma_e \frac{A}{A_m} \, \frac{\tilde{k}_2(\sigma) + \tilde{k}_o(\sigma)}{k_s} \quad . \tag{11}$$

The resonance parameters deduced from (**??**) are plotted in Figure **??** over the band [-1.15 cpd, -0.85 cpd] (denoted band I). This theoretical curve is compared with the estimated values from different sets of dominant luni-solar nutation terms, as reported in (Nurul Huda et al, 2019). In average, far from the resonance at σ_{FCN}, the theoretical curve corresponds grossly to the estimated value obtained for the whole band I ($T_{PM} = 382 \pm 1.3$ d, $Q_{PM} = -10.4 \pm 0.5$).

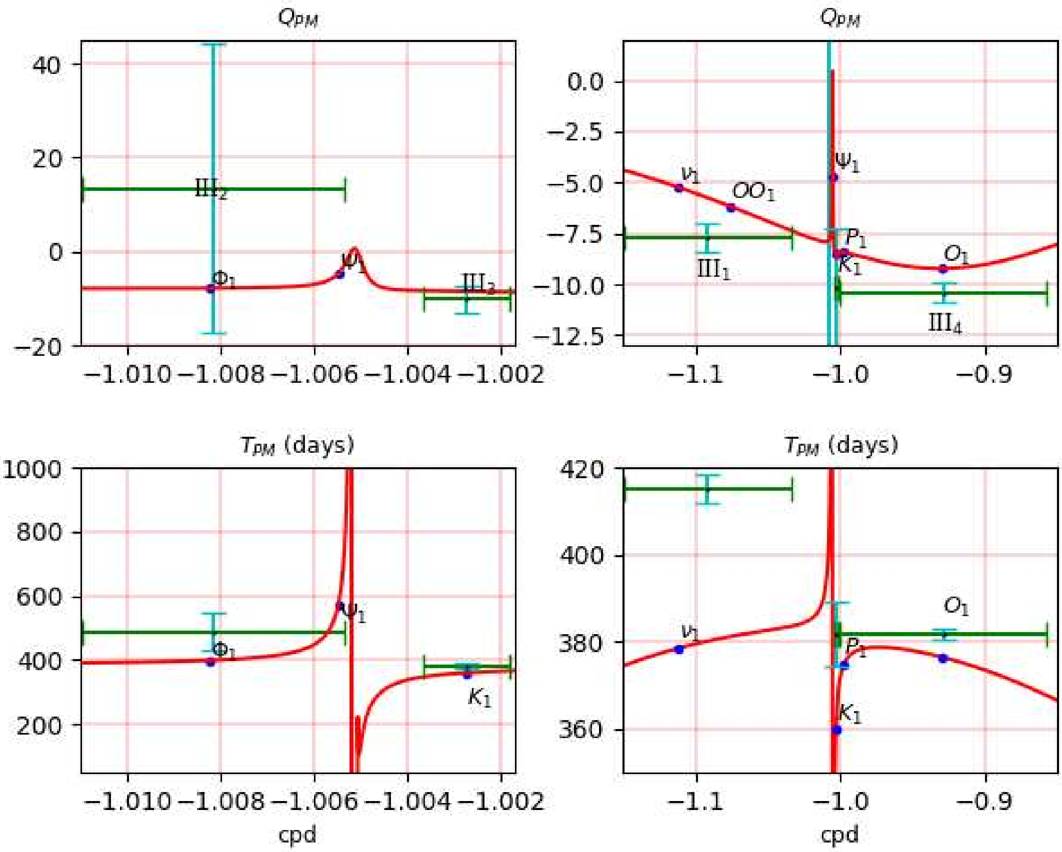

Figure 2: Resonance parameters of the polar motion in the diurnal retrograde band for an anelastic Earth covered by oceans and containing a fluid core. Green crosses specify the values obtained from nutation inversion over the restricted frequency bands III_1 (ν_1,OO_1), III_2 (Φ_1, Ψ_1), III_3 (K_1), III_4 (S_1,P_1,O_1,Q_1): the horizontal bar extension gives the frequency band, and the vertical bar the uncertainty of the estimated value.

The resonance produced by the free core nutation strongly is partly confirmed by nutation inversion for restricted bands:

- K_1 - precession and long period nutation terms (6798 d, 1095 d, 3399 d) in the CRF (band III_3). Modeled values ($T_{PM} = 360$ d, $Q_{PM} = -10$) at K_1 quite well predict the estimated value (382 ± 8 d, -8.5).

- Ψ_1 - retrograde nutation terms in 365.25 d, 386 d in the CRF (band III_2). Enhancement of T_{PM} up to 470 d at Ψ_1, matching $T_{PM}^{observed} = 487 \pm 58$ d. The value $Q_{PM}^{observed} = 13 \pm 31$ too uncertain for confirming the modeled value -5 at Ψ_1.

- far from K_1 and Ψ_1, the resonance parameters rejoin the curves obtained for an anelastic Earth covered by oceans. At the right part of the spectrum corresponding to band III_4, covering tidal lines S_1 and O_1, the estimates ($T_{PM} = 381.8 \pm 1.3$ d, $Q_{PM} = -10.4 \pm 0.5$) are close to the modeled parameters. For the opposite band (III_1), the estimated period is longer (418 days), as expected from the asymmetry of the resonance.

5. CONCLUSION

The dynamical response of the oceans to the pole tide potential is the main factor reducing the polar motion resonance period to about 380 days in the retrograde diurnal band. The associated quality factor ~ -10 reflects the strong phase-shift of the this response with respect to the pole tide. In reason of the free core nutation resonance, the body Love number strongly deviates from its mean value of 0.3 in vicinity of the FCN frequency ($\sigma_{FCN} = -1.0050$ cpd). In turn, in the band [-1.15 cpd, -0.85 cpd] as observed from the Earth, the resonance period of the polar motion increases above 400 days for frequencies smaller than σ_{FCN}, and remains below this threshold for the band above σ_{FCN}.

In contrast to common polar motion, the excitation at stake, namely the diurnal tidal torque through rigid Earth nutation terms, is almost perfectly known. Despite the remoteness of the polar resonance period from the retrograde diurnal nutation terms in the terrestrial frame, the confrontation of observed nutation terms to those of a rigid Earth, as carried out in (Nurul Huda et al, 2019), amazingly confirms the modeled frequency dependence. So, the luni-solar nutation determined by VLBI reflect the dynamical behavior of the ocean and influence of the fluid core on solid Earth deformation in the retrograde diurnal band.

6. REFERENCE

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