# GEODETIC (RELATIVISTIC) ROTATION OF THE MARS SATELLITES SYSTEM

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**ABSTRACT.** The relativistic effect of the geodetic rotation (which consist of two effects: the geodetic precession and the geodetic nutation) in the rotation of Mars satellites system for the first time was investigated. The most essential terms of the geodetic rotation were computed by the algorithm of Pashkevich (2016), which is applicable to the study of any bodies of the Solar system that have long-time ephemeris. As a result, a new high-precision values of the geodetic rotation for Mars dynamically adjusted to JPL DE431/LE431 ephemeris (Folkner et al., 2014) in Euler angles and for its satellites dynamically adjusted to Horizons On-Line Ephemeris System (Giorgini et al., 2001) in Euler angles and in the perturbing terms of its physical librations were calculated.

## 1. INTRODUCTION

The geodetic rotation of a body, first considered by Willem de Sitter in 1916 (De Sitter, 1916), is the most essential relativistic effect of its rotation. This effect consist of two effects: the geodetic precession, which is the systematic effect, and the geodetic nutation (Fukushima, 1991), which is the periodic effect. These effects have a formal similarity with the phenomena of precession and nutation, which are better-known events on the classical mechanics. In contrast to the above-mentioned classical events their emergence are not depend on from influences of any forces to body and represents only the effect of the curvature of space-time, predicted by general relativity, on a vector of the body rotation axis carried along with an orbiting body.

The main objectives of the present research are for the first time to study the effect of the geodetic rotation in the rotation of Mars satellites system and to obtain a new high-precision values of the geodetic rotation for Mars dynamically adjusted to JPL DE431/LE431 ephemeris (Folkner et al., 2014) in Euler angles and for its satellites dynamically adjusted to Horizons On-Line Ephemeris System (Giorgini et al., 2001) in Euler angles and in the perturbing terms of its physical librations.

For these purposes the algorithm of Pashkevich (2016), which is applicable to the study of any bodies of the Solar system that have long-time ephemeris will be used.

### 1.1 Mathematical model of the problem

The problem of the geodetic (relativistic) rotation for Mars and for his satellites (Phobos and Deimos) is studied over the time span from AD1600 to AD2500 with one hour spacing with respect to the kinematically non-rotating (Kopeikin et al., 2011) proper coordinate system of the studied bodies. Body orientation parameters for Mars taken from Seidelmann et al., (2005) and for Mars satellites from Archinal et al., (2018).

The positions, velocities, physical parameters and orbital elements for Phobos and Deimos are taken from the Horizons On-Line Ephemeris System (Giorgini et al., 2001) and ones for the disturbing bodies: the Sun, the Moon, Pluto and the major planets are calculated using the fundamental ephemeris JPL DE431/LE431 (Folkner et al., 2014).

## 2. RESULTS

As a result of this investigation, in the perturbing terms of the physical librations and in Euler angles for the Martian satellites (Phobos and Deimos), and in Euler angles for Mars the most significant systematic $\Delta x_s$ (Table 1) and periodic $\Delta x_p$ (Table 3) terms of the geodetic rotation are calculated:

$$\Delta x_s = \sum_{n=1}^{N} \Delta x_n t^n, \qquad (1)$$

$$\Delta x_p = \sum_{j} \sum_{k=0}^{M} [\Delta x_{Sjk} \sin(\nu_{j0} + \nu_{j1} t) + \Delta x_{Cjk} \cos(\nu_{j0} + \nu_{j1} t)] t^k, \qquad (2)$$

where $\Delta x = x_{relativistic} - x_{Newtonian}$, $x = \psi, \theta, \varphi, \tau, \rho, \mathrm{I}\sigma$; $\Delta x_n$ are the coefficients of the systematic terms; $\Delta x_{Sjk}, \Delta x_{Cjk}$ are the coefficients of the periodic terms for sine and for cosine, respectively; $\nu_{j0}, \nu_{j1}$ are phases and frequencies of the body under study, which are combinations of the corresponding Delaunay arguments (Smart, 1953) and the mean longitudes of the perturbing bodies; the summation index $j$ is the number of added periodic terms, and its value changes for each body under study; $t$ is the time in the Julian days.

*Notes to tables.* In Tables 1–3: $T$ is the Dynamical Barycentric Time (TDB) measured in thousand Julian years (tjy) (of 365250 days) from J2000; $a$ is orbital semi-major axis of Mars satellites taking from the Horizons On-Line Ephemeris System (Giorgini et al., 2001); $\Omega_{L41}$ , $\Omega_{L42}$ are longitudes of the ascending node (Mars satellites orbits) on the Laplace plane for Phobos and Deimos, respectively; $D_{41} = \lambda_{41} - \lambda_4 + 180^o$, $D_{42} = \lambda_{42} - \lambda_4 + 180^o$ are mean elongations of Phobos and Deimos from the Sun, respectively; $\lambda_4$ is mean longitude of Mars; $\lambda_{41}$, $\lambda_{42}$ are marsocentric longitudes of Phobos and Deimos, respectively. The mean longitude of Mars was taken from (Brumberg and Bretagnon, 2000). The mean longitudes of the Martian satellites, their longitudes of the ascending node on the Laplace plane and mean elongations from the Sun are calculated using data from the Horizons On-Line Ephemeris System (Giorgini et al., 2001).

| | Mars | Phobos ($a = 9376km$) | | Deimos ($a = 23458km$) | |
|---|---|---|---|---|---|
| tjy | $\Delta\psi_s$ ($\mu$as) | $\Delta\psi_s$ ($\mu$as) | $\Delta\tau_s$ ($\mu$as) | $\Delta\psi_s$ ($\mu$as) | $\Delta\tau_s$ ($\mu$as) |
| $T$ | −7113935.6683 | −209314864.7430 | −95713236.3800 | −27680096.2268 | −15836815.1715 |
| $T^2$ | 9758.6588 | 43043.9996 | 22074.5862 | 14436.8795 | 1970.5567 |
| $T^3$ | 1328.3085 | | | | |
| tjy | $\Delta\theta_s$ ($\mu$as) | $\Delta\theta_s$ ($\mu$as) | $\Delta\rho_s$ ($\mu$as) | $\Delta\theta_s$ ($\mu$as) | $\Delta\rho_s$ ($\mu$as) |
| $T$ | 119866.5547 | 109821.3069 | 109821.3069 | 118932.5546 | 118932.5546 |
| $T^2$ | −1065.6036 | −79913.4426 | −79913.4426 | −5802.8941 | −5802.8941 |
| $T^3$ | −57.9607 | | | | |
| tjy | $\Delta\varphi_s$ ($\mu$as) | $\Delta\varphi_s$ ($\mu$as) | $\Delta(\mathrm{I}\sigma)_s$ ($\mu$as) | $\Delta\varphi_s$ ($\mu$as) | $\Delta(\mathrm{I}\sigma)_s$ ($\mu$as) |
| $T$ | 405134.4944 | 113601628.3630 | 94088505.4932 | 11843281.0553 | 12076398.3007 |
| $T^2$ | −11482.6140 | −20969.4134 | 32232.9894 | −12466.3227 | 640.6748 |
| $T^3$ | −280.3423 | | | | |

Table 1: The secular terms of geodetic rotation of Mars and its satellites

The secular terms of geodetic rotation of some bodies of Solar system (Pashkevich and Vershkov, 2019) represented in Euler angles (Table 2).

As can be seen from Tables 1 and 2, the values of the geodetic rotation of Mars satellites decrease with increasing their distance from Mars and the values of the geodetic rotation of planets

| | The Sun | Mercury | Venus | The Earth | The Moon |
|---|---|---|---|---|---|
| tjy | $\Delta\psi_s$ ($\mu$as) | $\Delta\psi_s$ ($\mu$as) | $\Delta\psi_s$ ($\mu$as) | $\Delta\psi_s$ ($\mu$as) | $\Delta\tau_s$ ($\mu$as) |
| $T$ | −870.0219 | −426451032.8798 | −156030839.3400 | −19198865.6280 | −19494124.5472 |
| $T^2$ | 1.3770 | −39215.8785 | −687024.3196 | 50432.5497 | −12.3454 |
| $T^3$ | −0.2568 | 14420.2934 | 78660.6535 | −657.0605 | 565.0905 |
| tjy | $\Delta\theta_s$ ($\mu$as) | $\Delta\theta_s$ ($\mu$as) | $\Delta\theta_s$ ($\mu$as) | $\Delta\theta_s$ ($\mu$as) | $\Delta\rho_s$ ($\mu$as) |
| $T$ | −1.8891 | 36028.3827 | −740880.9685 | −10.7322 | −297.2493 |
| $T^2$ | 0.0809 | −2910.6802 | 60179.7955 | −1951.6003 | −1779.1546 |
| $T^3$ | −0.0080 | −193.9063 | 627.4990 | −4125.4000 | −3128.5299 |
| tjy | $\Delta\varphi_s$ ($\mu$as) | $\Delta\varphi_s$ ($\mu$as) | $\Delta\varphi_s$ ($\mu$as) | $\Delta\varphi_s$ ($\mu$as) | $\Delta(\mathrm{I}\sigma)_s$ ($\mu$as) |
| $T$ | 179.5703 | 214756196.8118 | 113009422.3955 | −17.8008 | 6536.9172 |
| $T^2$ | −1.3915 | 2268.1428 | 687231.8895 | −54775.6865 | −36208.8512 |
| $T^3$ | 0.0433 | −12967.5814 | −78746.0736 | 1245.0101 | 27296.6113 |

Table 2: The secular terms of geodetic rotation of some bodies of Solar system (Pashkevich and Vershkov, 2019)

decrease with increasing their distance from the Sun. The values of the geodetic rotation of the Earth and the Moon are very close. It is due to the fact that the Earth and the Moon have very close heliocentric orbit and the Sun has a greater influence on the geodetic rotation of the Moon than the Earth. At the same time, despite the fact that the heliocentric orbits of Mars and its satellites are also very close, the values of the geodetic rotation of Phobos and Deimos far exceed the value of the geodetic rotation of Mars. This is because Mars has a greater influence on the geodetic rotation of its satellites than the Sun on its and Mars by reason of the close distances between Mars and its satellites, than between the Earth and the Moon. The value of the geodetic rotation of Phobos is greater than ones values of the Earth and Venus; the value of the geodetic rotation of Deimos is greater than one value of the Earth. It is due to the fact that Mars has a greater influence on the geodetic rotation of its satellites than the Sun on some near planets by reason of very close distances between Mars and its satellites.

Mars and its satellites Phobos and Deimos (like the Earth and the Moon) are on average at the same distance from the Sun. As a result, their coefficients in $\Delta\psi_p$ and $\Delta\tau_p$ for periodic with argument $\lambda_4$ (Table 3) components are quite close to each other.

The geodetic rotations of Phobos and Deimos are determined not only by the Sun, but also by Mars. This fact is confirmed by the appearance of a harmonic with the argument $D_{41}$ for Phobos and $D_{41}$ for Deimos (Table 3).

In contrast to Phobos, located closer to the planet, the Sun has a greater influence on the geodetic rotation of Deimos. It is easy to see that the closer a satellite is located to the planet, the more the harmonic contribution depends on the mean longitude of the planet (see Phobos in Table 3). Therefore, the harmonic with a period of 1.881 years and the argument of $\lambda_4$ (Table 3) becomes predominant for Phobos. If the satellite is farther away from the planet, then more harmonic contribution depends on the precession orbit node in the Laplace plane[1] (see Deimos in Table 3). Therefore, the harmonic with a period of 54.537 years and the argument of $\Omega_{L42}$ (Table 3) becomes predominant for Deimos.

[1] *The Laplace plane is the plane normal to the satellite's orbital precession pole. It is a kind of "average orbital plane" of the satellite (between their planet's equatorial plane and the plane of its solar orbit), around which the instantaneous orbital plane of the satellite precesses, and to which it has a constant additional inclination (P. Kenneth Seidelmann (ed.), 1992).*

| Body | Angle | Period | Arg | Coefficient of sin(Arg) ($\mu$as) | Coefficient of cos(Arg) ($\mu$as) |
|---|---|---|---|---|---|
| Mars | $\Delta\psi_p$ | 1.881 yr | $\lambda_4$ | $-543.438-22.455T+\ldots$ | $-241.415+40.433T+\ldots$ |
| | $\Delta\theta_p$ | 1.881 yr | $\lambda_4$ | $9.157+0.241T+\ldots$ | $4.068-0.742T+\ldots$ |
| | $\Delta\varphi_p$ | 1.881 yr | $\lambda_4$ | $30.949-0.392T+\ldots$ | $13.748-3.045T+\ldots$ |
| Phobos | $\Delta\psi_p$ | 1.881 yr | $\lambda_4$ | $-537.291+\ldots$ | $-238.028+\ldots$ |
| | | 2.262 yr | $\Omega_{L41}$ | $-125.461+\ldots$ | $680.758+\ldots$ |
| | | 7.657 h | $D_{41}$ | $-58.679+0.204T+$<br>$+0.880T^2+\ldots$ | $59.450-0.022T+$<br>$+0.783T^2+\ldots$ |
| | $\Delta\theta_p$ | 1.881 yr | $\lambda_4$ | $9.448+\ldots$ | $3.305+\ldots$ |
| | | 2.262 yr | $\Omega_{L41}$ | $4.099+\ldots$ | $1.339+\ldots$ |
| | | 7.657 h | $D_{41}$ | $-8.480+0.017T-$<br>$-0.098T^2+\ldots$ | $-9.228-0.036T-$<br>$-0.392T^2+\ldots$ |
| | $\Delta\varphi_p$ | 1.881 yr | $\lambda_4$ | $-29.698+\ldots$ | $14.013+\ldots$ |
| | | 2.262 yr | $\Omega_{L41}$ | $-1.139+\ldots$ | $-3.219+\ldots$ |
| | | 7.657 h | $D_{41}$ | $21.359-0.129T-$<br>$-1.005T^2+\ldots$ | $-22.722-0.036T-$<br>$-0.319T^2+\ldots$ |
| | $\Delta\tau_p$ | 1.881 yr | $\lambda_4$ | $-507.594+\ldots$ | $-224.015+\ldots$ |
| | | 2.262 yr | $\Omega_{L41}$ | $-126.600+\ldots$ | $677.538+\ldots$ |
| | | 7.657 h | $D_{41}$ | $-37.319+0.074T-$<br>$-0.125T^2+\ldots$ | $36.728-0.058T+$<br>$+0.464T^2+\ldots$ |
| | $\Delta\rho_p$ | 1.881 yr | $\lambda_4$ | $9.448+\ldots$ | $3.305+\ldots$ |
| | | 2.262 yr | $\Omega_{L41}$ | $4.099+\ldots$ | $1.339+\ldots$ |
| | | 7.657 h | $D_{41}$ | $-8.480+0.017T-$<br>$-0.098T^2+\ldots$ | $-9.228-0.036T-$<br>$-0.392T^2+\ldots$ |
| | $\Delta(\mathrm{I}\sigma)_p$ | 1.881 yr | $\lambda_4$ | $250.815+\ldots$ | $114.078+\ldots$ |
| | | 2.262 yr | $\Omega_{L41}$ | $-176.168+\ldots$ | $935.392+\ldots$ |
| | | 7.657 h | $D_{41}$ | $26.377-0.044T-$<br>$-0.408T^2+\ldots$ | $-26.723-0.035T-$<br>$-0.358T^2+\ldots$ |
| Deimos | $\Delta\psi_p$ | 1.881 yr | $\lambda_4$ | $-544.598+\ldots$ | $-241.408+\ldots$ |
| | | 54.537 yr | $\Omega_{L42}$ | $-2879.646+\ldots$ | $757.953+\ldots$ |
| | | 1.265 d | $D_{42}$ | $-51.777+0.142T-$<br>$-1.000T^2+\ldots$ | $11.009+0.007T+$<br>$+0.532T^2+\ldots$ |
| | $\Delta\theta_p$ | 1.881 yr | $\lambda_4$ | $9.220+\ldots$ | $3.792+\ldots$ |
| | | 54.537 yr | $\Omega_{L42}$ | $28.678+\ldots$ | $106.025+\ldots$ |
| | | 1.265 d | $D_{42}$ | $-1.214+0.009T+$<br>$+0.231T^2+\ldots$ | $-7.571-0.090T+$<br>$+0.675T^2+\ldots$ |
| | $\Delta\varphi_p$ | 1.881 yr | $\lambda_4$ | $32.026+\ldots$ | $13.676+\ldots$ |
| | | 54.537 yr | $\Omega_{L42}$ | $195.066+\ldots$ | $-216.931+\ldots$ |
| | | 1.265 d | $D_{42}$ | $19.149-0.045T-$<br>$-0.069T^2+\ldots$ | $-4.576-0.042T-$<br>$-0.310T^2+\ldots$ |
| | $\Delta\tau_p$ | 1.881 yr | $\lambda_4$ | $-512.573+\ldots$ | $-227.733+\ldots$ |
| | | 54.537 yr | $\Omega_{L42}$ | $-2684.581+\ldots$ | $541.022+\ldots$ |
| | | 1.265 d | $D_{42}$ | $-32.628+0.097T-$<br>$-1.069T^2+\ldots$ | $6.433-0.035T+$<br>$+0.223T^2+\ldots$ |
| | $\Delta\rho_p$ | 1.881 yr | $\lambda_4$ | $9.220+\ldots$ | $3.792+\ldots$ |
| | | 54.537 yr | $\Omega_{L42}$ | $28.678+\ldots$ | $106.025+\ldots$ |
| | | 1.265 d | $D_{42}$ | $-1.214+0.009T+$<br>$+0.231T^2+\ldots$ | $-7.571-0.090T+$<br>$+0.675T^2+\ldots$ |
| | $\Delta(\mathrm{I}\sigma)_p$ | 1.881 yr | $\lambda_4$ | $237.604+\ldots$ | $105.749+\ldots$ |
| | | 54.537 yr | $\Omega_{L42}$ | $-5381.398+\ldots$ | $1058.129+\ldots$ |
| | | 1.265 d | $D_{42}$ | $22.601+0.174T-$<br>$-1.924T^2+\ldots$ | $-4.805-0.053T+$<br>$+0.261T^2+\ldots$ |

Table 3: The periodic terms of geodetic rotation of Mars and its satellites

In this investigation it was also carried out a study on the mutual relativistic influence of Mars satellites on each other and on Mars (i.e., the inclusion of another satellite in the number of perturbing bodies).

So, the change in Deimos geodetic rotation from Phobos relativistic influence: in the longitude of the node $\psi$ is $-0.22$ $\mu$as/tjy, in the longitude $\tau$ is $-9.5 \cdot 10^{-2}$ $\mu$as/tjy, in the inclination $\theta$ is $-9.3 \cdot 10^{-6}$ $\mu$as/tjy, in the inclination $\rho$ is $-9.3 \cdot 10^{-6}$ $\mu$as/tjy, in the proper rotation angle $\varphi$ is 0.12 $\mu$as/tjy; in the node longitude $\mathrm{I}\sigma$ is $9.4 \cdot 10^{-2}$ $\mu$as/tjy.

The change in Phobos geodetic rotation from Deimos relativistic influence: in the longitude of the node $\psi$ is $-5.3 \cdot 10^{-2}$ $\mu$as/tjy, in the longitude $\tau$ is $-2.4 \cdot 10^{-2}$ $\mu$as/tjy, in the inclination $\theta$ is $6.2 \cdot 10^{-6}$ $\mu$as/tjy, in the inclination $\rho$ is $6.2 \cdot 10^{-6}$ $\mu$as/tjy, in the proper rotation angle $\varphi$ is $2.9 \cdot 10^{-2}$ $\mu$as/tjy; in the node longitude $\mathrm{I}\sigma$ is $2.4 \cdot 10^{-2}$ $\mu$as/tjy.

The change in Mars geodetic rotation from its satellites relativistic influence: in the longitude of the node $\psi$ is $-0.62$ $\mu$as/tjy, in the inclination $\theta$ is $-1.2 \cdot 10^{-4}$ $\mu$as/tjy, in the proper rotation angle $\varphi$ is 0.35 $\mu$as/tjy.

## 3. CONCLUSION

1. New high-precision values with the additions periodic terms of the geodetic rotation for Mars in Euler angles were obtained. These values are the dynamically adjusted to the DE431/LE431 ephemeris.

2. The systematic (Table. 1) and periodic (Table. 3) terms of the geodetic rotation of Martian satellites (Phobos and Deimos) are computed for the first time in the Euler angles and a perturbing terms of the physical libration. The mutual relativistic influence of the Mars satellites on each other in comparison with the Sun and Mars influences is insignificant. The obtained analytical values for the geodetic rotation of Phobos and Deimos can be used for the numerical study of their rotation in the relativistic approximation.

3. The secular terms of geodetic rotation of Mars satellites depend on their distance from the Sun and Mars, which masses are dominant in the Solar and Mars system, respectively. Mars has a greater influence on the geodetic rotation of its satellites than the Sun on the geodetic rotation of Phobos, Deimos and Mars.

4. The main periodic parts of the geodetic rotations for Mars satellites are determined not only by the Sun but also by Mars, which is the nearest planet to their satellites.

5. The values of the geodetic rotation of Mars satellites decrease with increasing their distance from Mars.

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