COMPARISON BETWEEN THE VARIABLES AND PARAMETERS USED FOR HIGH ACCURACY PRECESSION AND NUTATION

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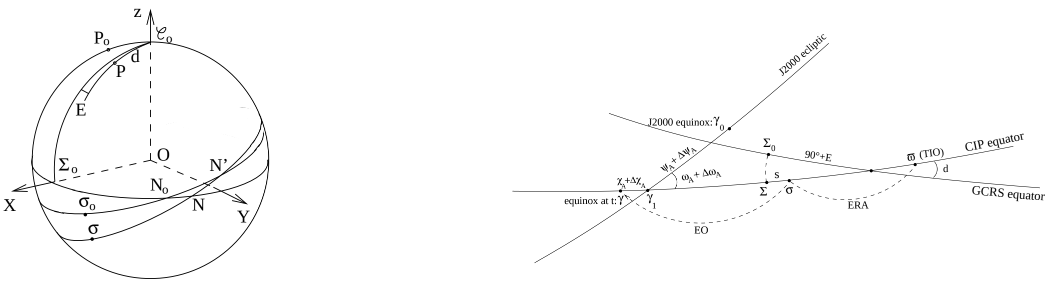
ABSTRACT. Different forms of variables and parameters, which refer to various celestial reference systems, have been used in the development of high accuracy equations and models for the Earth's precession and nutation, as well as for the estimation of the time-dependent celestial pole offsets from the most accurate astrometric observations. The purpose of this presentation is to compare and relate rigorously those various forms of variables and parameters and recommend the most appropriate ones to be used for providing the best accuracy for the celestial motion of the Celestial intermediate pole.

1. THE PRECESSION-NUTATION PARAMETERS

The CIO based precession-nutation parameters reflect the motion of the Celestial intermediate pole, CIP, in the Geocentric celestial reference system (GCRS); they consist in the coordinates of the CIP unit vector, either in their polar form (E and d), or their rectangular form ($X = \sin d \cos E,\ Y = \sin d \sin E$). These quantities contain precession and nutation of the CIP, frame bias, plus the cross terms; they are directly related to the GCRS and independent of the ecliptic and the equinox (Capitaine et al. 2003).

Figure 1a shows the position of the CIP (P) in the GCRS (of pole $\mathcal{C}_0$ and origin Σ_0 on the GCRS equator) and the celestial intermediate origin, CIO (σ), on the CIP equator (P_0 and σ_0 being the CIP and CIO at epoch t_0). Figure 1b shows the link of the quantities E, d and the Earth rotation angle, ERA, with the corresponding quantities referred to the J2000.0 ecliptic and the equinox. The point Σ is distant from the CIO by the quantity s, called the "CIO locator" (Capitaine & Wallace 2006) and the "equation of the origins", EO, is the difference ERA−GST between ERA and Greenwich sidereal time, GST.



1a 1b

Figure 1: The CIO based precession-nutation parameters and link to the J2000.0 ecliptic and the equinox.

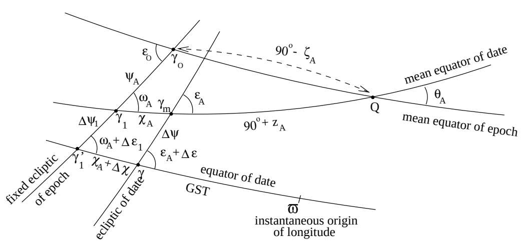


Figure 2: Equinox based precession-nutation parameters referred to the ecliptic (of epoch, or date).

The equinox based precession-nutation parameters refer either to the ecliptic of date, or the ecliptic of epoch (e.g. J2000.0). Figure 2 shows a number of those parameters, e.g. ψ_A, ω_A, which refer to the ecliptic and equinox of epoch, ϵ_A, z_A and the usual nutation angles, $\Delta\psi$, $\Delta\epsilon$, which refer to the ecliptic and (mean or true) equinox of date, and $\chi_A + \Delta\chi_A$ that measures the precession of the ecliptic on the CIP equator (Lieske et al. 1977). It also shows the Euler angles between the J2000 ecliptic system and the terrestrial intermediate reference system, namely $\psi = \psi_A + \Delta\psi_1$, $\omega = \omega_A + \Delta\epsilon_1$, and $\varphi = \text{GST} + \chi_A + \Delta\chi_A$, from γ_1' (intersection of the fixed ecliptic with the CIP equator) to the terrestrial intermediate origin, TIO; γ_1' is distant from the equinox, γ, by the quantity $-\chi_A - \Delta\chi_A$ and from the point Σ, by EO $-\chi_A - \Delta\chi_A - s$.

The rigorous relationship between the CIO and equinox based quantities, are, if ξ_0, η_0 and $d\alpha_0$ are the celestial pole offsets and the frame bias in right ascension, respectively, at J2000.0 (Capitaine 1990):

$$\begin{aligned} X &= \bar{X} + \xi_0 - d\alpha_0\ \bar{Y}, \qquad & Y &= \bar{Y} + \eta_0 + d\alpha_0\ \bar{X} \\ \bar{X} &= \sin\omega\sin\psi, & \bar{Y} &= -\sin\epsilon_0\cos\omega + \cos\epsilon_0\sin\omega\cos\psi, \end{aligned} \tag{1}$$

and for quantities referred to the ecliptic and equinox of date (developments at the 4th order in ψ_A):

$$\begin{aligned} \bar{X} &= \psi_A\sin\epsilon_0 - (\psi_A^3/6)\sin\epsilon_0 + \psi_A(\omega_A - \epsilon_0)\cos\epsilon_0 + \Delta\psi\sin\epsilon_0 + \Delta\psi\Delta\epsilon\ \cos\epsilon_0 \\ &\quad +(\psi_A\cos\epsilon_0 - \chi_A)\ \Delta\epsilon + (\epsilon_A - \epsilon_0)\ \Delta\psi\cos\epsilon_0 - (\psi_A^2/2)\ \Delta\psi\sin\epsilon_0, \\ \bar{Y} &= (\omega_A - \epsilon_0) - (\psi_A^2/2)\sin\epsilon_0\cos\epsilon_0 + (\psi_A^4/24)\sin\epsilon_0\cos\epsilon_0 + \Delta\epsilon \\ &\quad -(\Delta\psi^2/2)\sin\epsilon_0\cos\epsilon_0 - (\psi_A\cos\epsilon_0 - \chi_A)\ \Delta\psi\sin\epsilon_0 - (\psi_A^2/2)\cos\epsilon_0^2\ \Delta\epsilon. \end{aligned} \tag{2}$$

It should be noted that the equinox of date results from two different phenomena: the precession of the ecliptic, due to planetary perturbations, and the precession-nutation of the equator, due to the effect of the luni-solar and planetary torques on the oblate Earth. Such a reference should therefore be avoided for high accuracy developments. Furthermore, as there is not a unique and clear way to define an ecliptic (of date, or epoch), especially in a geocentric reference system of General relativity, relating equinox based precession-nutation parameters to the GCRS is a complex issue.

Therefore the CIO based precession-nutation parameters, which are related in a direct and rigorous way to the GCRS (the geocentric reference system recommended by the IAU 2000 resolutions for being used for the Earth), are the most appropriate parameters to be used for high accuracy precession-nutation.

2. THE PRECESSION-NUTATION EQUATIONS

The basis for the equations of the rotation of a rigid Earth are the Euler dynamical equations that express the angular momentum balance in the terrestrial reference system as functions of the components of the instantaneous rotation vector and external torque, and the Earth's principal moments of inertia, A, B, C. For an axially symmetric Earth, $A = B$ and the third component of the torque is equal to 0. The precession-nutation equations are obtained from these equations by appropriate transformations. The new variables can be either quantities defined in Section 1 (cf. Capitaine et al. 2006), or the precession and nutation angles referred to the ecliptic and equinox of date (cf. Kinoshita 1977, Williams 1994, Souchay et al. 1999), or the Euler angles ψ, ω, φ, defined in Section 1 (cf. Woolard 1953, Bretagnon et al. 1997). The components of the torque are in an intermediate celestial reference system defined by the CIP, and either the point Σ (cf Figure 1b), or the equinox of date, or the point γ_1' (cf Figure 2).

The precession-nutation equations for a rigid axially symmetric Earth are thus as follows:

(i) CIO based approach (Capitaine et al. 2006):

$$\begin{aligned} -\ddot{Y} + (C/A)\Omega\,\dot{X} &= L_\Sigma/A + F'' \\ \ddot{X} + (C/A)\Omega\,\dot{Y} &= M_\Sigma/A + G'', \end{aligned} \tag{3}$$

Ω being the mean Earth's angular velocity, L_Σ, M_Σ the equatorial components of the torque referred to Σ, and F'', G'' functions of X, Y and of their first and second time derivatives;

(ii) equinox based approach using the Euler angles (Bretagnon et al. 1997):

$$\begin{aligned} -\ddot{\omega} + (C/A)\Omega\,\dot{\psi}\sin\epsilon_0 &= L_{\gamma_1'}/A + F' \\ \sin\epsilon_0\ddot{\psi} + (C/A)\Omega\,\dot{\omega} &= M_{\gamma_1'}/A + G' \\ \ddot{\varphi} &= H', \end{aligned} \tag{4}$$

$L_{\gamma_1'}, M_{\gamma_1'}$ being the equatorial components of the torque referred to γ_1', and F', G', H' functions of ψ, ω, φ and of their first and second time derivatives.

Note that, unlike the Equations (4) based on the ecliptic, Equations (3) using the CIO based paradigm are independent of the variations in the Earth rotation angle. It should also be noted that the CIO based precession-nutation equations provide solutions that are directly expressed in the GCRS.

3. THE IAU PRECESSION-NUTATION MODEL

The IAU 2006/2000A precession-nutation is composed of the IAU 2000A Nutation (Mathews et al. 2002, denoted MHB2000), adopted by IAU 2000 Resolution B1.6 and the IAU 2006 precession (Capitaine et al. 2003, Hilton et al. 2006, denoted P03), adopted by IAU 2006 Resolution B1. The IAU 2000A semi-analytical series for nutation is composed of lunisolar and planetary terms with "in-phase" and "out-of-phase" components of the $\Delta\psi$, $\Delta\epsilon$ angles; they are transformed, from the REN2000 solution (Souchay et al. 1999) of these angles for a rigid Earth model, to nutation of a non-rigid Earth with the MHB2000 "transfer function". The IAU 2006 precession provides P03 polynomial expressions up to the 5th degree in time, both for the precession of the ecliptic and the precession of the equator. The IAU 2006 precession of the equator is based on the expressions of the fundamental quantities ψ_A and ω_A, which have been derived from the dynamical precession equations, using integration constants, such as values for the precession rates of the equator at J2000, the J2000 obliquity and the J_2 rate value.

While the IAU 2006 values for the precession rates of the equator are compatible with the IAU 2000 ones, updates have been applied to the J2000 obliquity, and to the J_2 rate, which was neglected in the IAU 2000 model. Consequently, very small changes, described in the following, are needed in a few of the IAU 2000A nutation amplitudes in order to ensure compatibility with the IAU 2006 precession.

(1) Introducing the IAU 2006 J_2 rate value ($dJ_2/dt = -3.0\times10^{-11}$/yr) gives rise to additional Poisson terms in nutation, the coefficients of which are proportional to $\dot{J}_2/J_2$ (*i.e.* -2.7774×10^{-6}/century). The largest corresponding changes (Capitaine and Wallace, 2006) in μas are, in the expressions for X, Y:

$$\begin{aligned} dX_{J2d} &= 18.8\ t\ \sin\Omega + 1.4\ t\ \sin 2(F-D+\Omega) \\ dY_{J2d} &= -25.6\ t\ \cos\Omega - 1.6\ t\ \cos 2(F-D+\Omega), \end{aligned} \quad (5)$$

and similar changes in the expressions for $\Delta\psi$, $\Delta\epsilon$ (F, D, Ω being fundamental arguments of nutations).

(2) The IAU 2006 obliquity ($84381.406''$) is different from the IAU 2000 obliquity ($84381.448''$) that was used when estimating the IAU 2000A nutation amplitudes. To compensate for this change, it is necessary to multiply the amplitudes of the nutation in longitude by (cf. Section 4) $\sin\epsilon_{\rm IAU2000}/\sin\epsilon_{\rm IAU2006}$.

The largest corresponding changes (Capitaine and Wallace, 2006) in μas are:

$$d_\epsilon\psi = \ -8.1\ \sin\Omega - 0.6\ \sin 2(F-D+\Omega). \quad (6)$$

Note that no such adjustment is needed in the case of X, Y. Note also that the periodic terms given by (5) are included in the IAU 2006/2000A version of the X, Y series. This shows that the use of these series ensures the best compatibility between the IAU 2006 precession and the IAU 2000 nutation.

The adjustments above are taken into account in the SOFA implementation of the IAU 2006/2000A precession-nutation as well as in the IERS Conventions 2010, but not in some other implementations. Whenever these corrections are included, a specific label, such as"IAU $2000\rm A_{R06}$", or "IAU $2000\rm A_R$" must be added to specify that the nutation has been revised for use with the IAU 2006 precession.

4. THE OBSERVED PRECESSION-NUTATION

VLBI is the only technique that is currently able to estimate high accuracy corrections to the precession-nutation model on a regular basis, as "celestial pole offsets", with the form of either (δX, δY), or ($\delta\psi$, $\delta\epsilon$). These observations are directly sensitive to the orientation of the Earth's equator (or the CIP) in the GCRS, but they are not sensitive to an ecliptic; therefore, the form used for the "celestial pole offsets" must reflect this property; this is not the case for ($\delta\psi$, $\delta\epsilon$).

The dependence of the precession in longitude, ψ_A (hence $\delta\psi$), on the ecliptic to which it is referred is shown in Figure 3. If such a dependence is ignored and the value for ψ_A is considered as being the "estimated" value, a change from ecliptic 1 (with obliquity at epoch, ϵ_{01}) to ecliptic 2 (with obliquity at epoch, ϵ_{02}) would give a change in the value for $\psi_A\sin\epsilon_0$ to which VLBI is actually sensitive. To compensate for this change, it is necessary to multiply ψ_A by $\sin\epsilon_{\rm IAU2000}/\sin\epsilon_{\rm IAU2006}$. For example, the change in the estimated rate in ecliptic longitude corresponding to the change from the IAU 2000 to IAU 2006 ecliptics is 2.37 mas/cy. This clearly shows that the equinox based precession-nutation

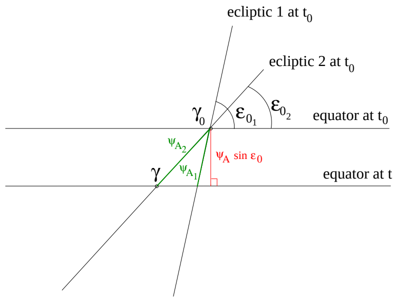


Figure 3: Difference in the precession in longitude referred to two different ecliptics.

parameters are dependent on the conventional ecliptic at epoch. Consequently, there is a big risk, when using such estimated quantities, of introducing inconsistencies between different determinations of nutation offsets. In contrast, the X, Y parameters, and hence (δX, δY), are independent of an ecliptic and are directly related to the motion of the CIP in the GCRS to which VLBI observations are sensitive.

5. CONCLUDING REMARKS

The precision goals for the motion of the CIP in the GCRS are a few μas over a time span of a few centuries. In order to achieve the corresponding accuracy in precession-nutation, it is necessary that the variables used in the equations, the parameters used in the models and the celestial pole offsets estimated from the observations be best compliant with the GCRS and the CIO based paradigm. The coordinates of the CIP unit vector in the GCRS that ensure all these conditions are the appropriate quantities to be used; the general use of the (δX, δY) celestial pole offsets is recommended to avoid various complexities.

In order to ensure the best consistency between the IAU 2006 precession and IAU 2000A nutation, it is necessary to take into account slight adjustments to the IAU 2000A nutation; a label should be given (e.g. "IAU 2006/2000$\mathrm{A_{R06}}$", or "IAU 2006/2000$\mathrm{A_R}$") to the precession-nutation implementations that contain these adjustments in order to clearly distinguish them from those without any adjustment.

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