WHERE ARE THE SATURN TROJANS ?

H. BAUDISCH, R.DVORAK
Institute for Astronomy, University of Vienna
Türkenschanzstraße 17, A-1180 Vienna, Austria
e-mail: helmut.baudisch@chello.at, e-mail: dvorak@astro.univie.ac.at

ABSTRACT. The gas giants Jupiter and Neptune are known to host Trojan asteroids but up to now no asteroids have been found around the libration points of Uranus and Saturn. With the aid of numerical integrations we checked the stability of fictitious bodies in the 1:1 Mean Motion Resonance with Saturn. Former studies show that around the Lagrange points the bodies escape quite fast whereas a stable ring survives for million of years. Using the results of our investigation in different dynamical models we could show that Jupiter is responsible for the unstable hole around the equilateral equilibrium points.

1. INTRODUCTION

The Lagrange points in the restricted three body problem (built up of two massive bodies – called primary bodies – on circular orbits and a third massless one) mark positions where the centrifugal forces are in equilibrium with the combined gravitational forces exerted by the Sun and a planet. The astronomer Joseph Louis Lagrange (1736 – 1813) was the first who predicted that these points of equilibrium exist. Whereas the points L_1 to L_3 (on the connecting line between the massive bodies) are unstable, the equilibrium points L_4 and L_5 (building an equilateral triangle with the two primaries) are stable[1]. In a rotating coordinate system - the primaries on the x-axes – they are located 60° ahead respectively behind the planet. Consequently the massless body is in a 1:1 Mean Motion Resonance (MMR) and one speaks of coorbital motion. Because of the stability of the two equilateral Lagrange points the area close by allows stable motion and in fact many asteroids are found coorbiting the planet Jupiter (3191 respectively 1743) Neptune (six respectively two) and Mars (one respectively three). Even the Earth has one Trojan close to L_4 (Connors et al, 2011). But why are there no Trojans of Saturn, the second planet in size? Many studies deal with this question for the gas-giants like Baudisch (2010), Dvorak et al (2008), Dvorak et al (2010), Holman and Wisdom (1993), Innanen and Mikkola (1989), Téger (2000), Nesvorný and Dones (2002), Zhou et al (2009) and Zhou et al (2011).

In Figure we show a schematic picture of the restricted problem Sun-Saturn (left graph) and the stable zone around L_4 which is a longitudinal stretched region; we show also the real libration motion of a fictitious Trojan around the Lagrange point L_4 (right picture). One can distinguish two well separated periods namely a large one covering the whole stable region and a small one with many loops.

2. DESCRIPTION OF THE MODELS AND METHODS

To check the stability of Trojans around Saturn we use the results of extensive numerical integrations of a great number of orbits of fictitious massless bodies in the region around L_4. The restricted problem is not a realistic model to answer this question and consequently no analytic approaches (compare Lhotka et al., 2008) can be used for this problem. Five different dynamical models were used: SJUN: all planets of the outer Solar System, SJN: without Uranus, SJU: without Neptune, SUN: without Jupiter, SJ: only with Jupiter as perturber.

In the region around L_4 we equally distributed for a grid with initial conditions the fictitious Trojans and varied the semimajor axes a and the angular distance to the Lagrange point. In additional runs the initial inclination of the fictitious Trojan with respect to the plane of motion of Saturn was set to several values $i < 10°$. As integration method we choose the Lie-integration, a well established tool of computercode to integrate the orbits of Solar System bodies with an automatic stepsize control (Dvorak et al., 2008). We emphasize that for this kind of investigation only point masses were taken for the

---

[1] depending on the mass ratio μ of the primaries which should be $\mu < 1/27$

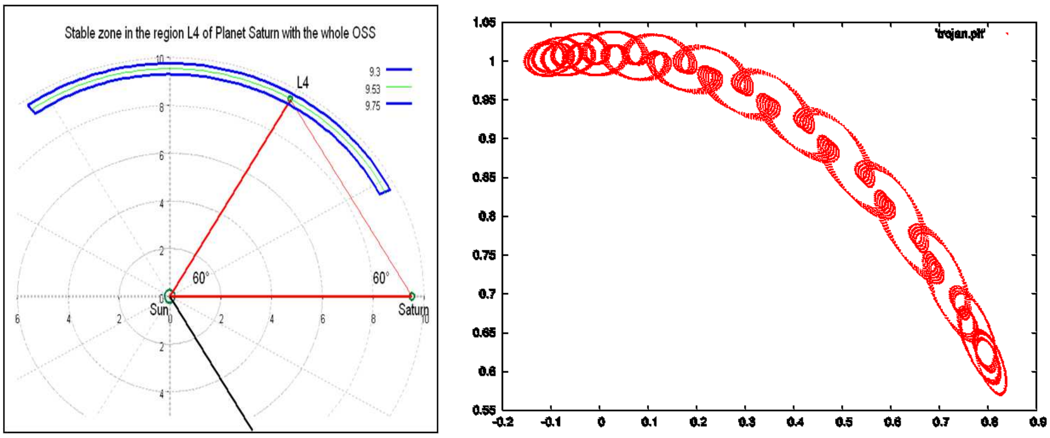

Figure 1: Schematic view of the Sun-Saturn system and the stable region around the Lagrange point L_4 in a rotating coordinate system $x-y$ measured in AU (left picture). Motion of a massless body in the Sun-Jupiter system in the rotating coordinate system; distances measured in units of the semimajor axis of Jupiter (right picture)

planets and no other than the gravitational forces were taken into account.

3. RESULTS

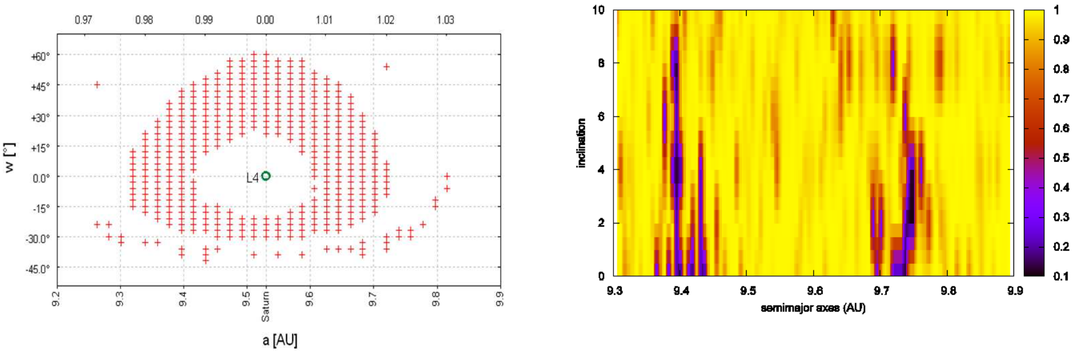

Figure 2: Stable ring (marked by crosses) around the Lagrange point L_4 of Saturn in the model of the outer Solar System after an integration time of 10^6 years semimajor axes versus the elongation around the equilibrium point (left picture). Cut through the equilibrium point L_4 for different initial semimajor axes versus different inclinations for the Trojan integrated for 10^8 years (right picture)

An interesting feature, namely a stable ring around the Lagrange point is visible in Figure (left picture). Close to the equilibrium point the fictitious Trojans escape rather fast, which can be seen in more detail in Figure : on the left graph we plotted the escape times for a cut where we were fixing the semimajor axes ($a = a_{Saturn}$) and varying the initial angular distance to the Lagrange point; on the right graph we fixed the angular distance and just varied the semimajor axis. Comparing the results it is evident that the longer integration time of 10^7 years does not change qualitatively the ring structure with respect to the shorter integration time. In additional runs we changed the integration time to 10^8 years for the cuts where we varied the semimajor axes but we also changed the initial inclination up to 10° (Figure , right graph). Here we can see that even for the plane problem which we treated in the former runs almost no orbits are stable for such a long integration time – the stable ring disappeared! Only some scattered points of stability with orbits of small eccentricity survived (dark blue colors). That means that no Trojans of Saturn can be on stable orbits for the life time of our Solar System. They cannot have

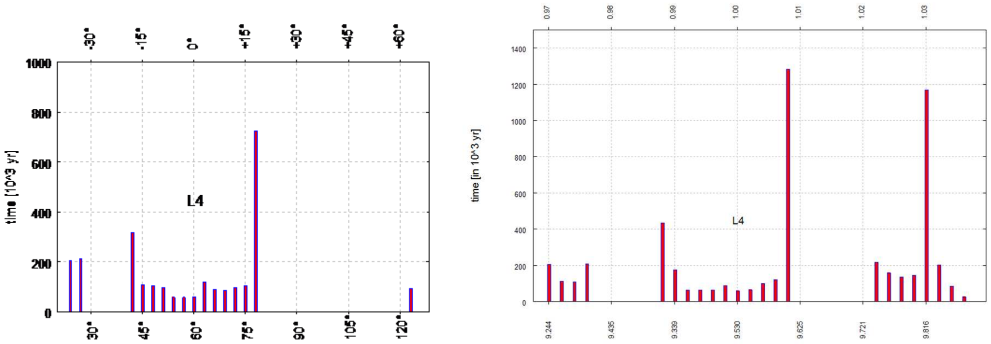

Figure 3: Escape times after an integration time of 10^7 years for cuts through $a = a_{Saturn}$ (left graph) and when the elongation δ is constant (right graph)

formed "in situ" together with their planet like for Jupiter (Robutel et al., 2005) and Neptune (Zhou et al., 2009, Zhou et al., 2011) Trojans.

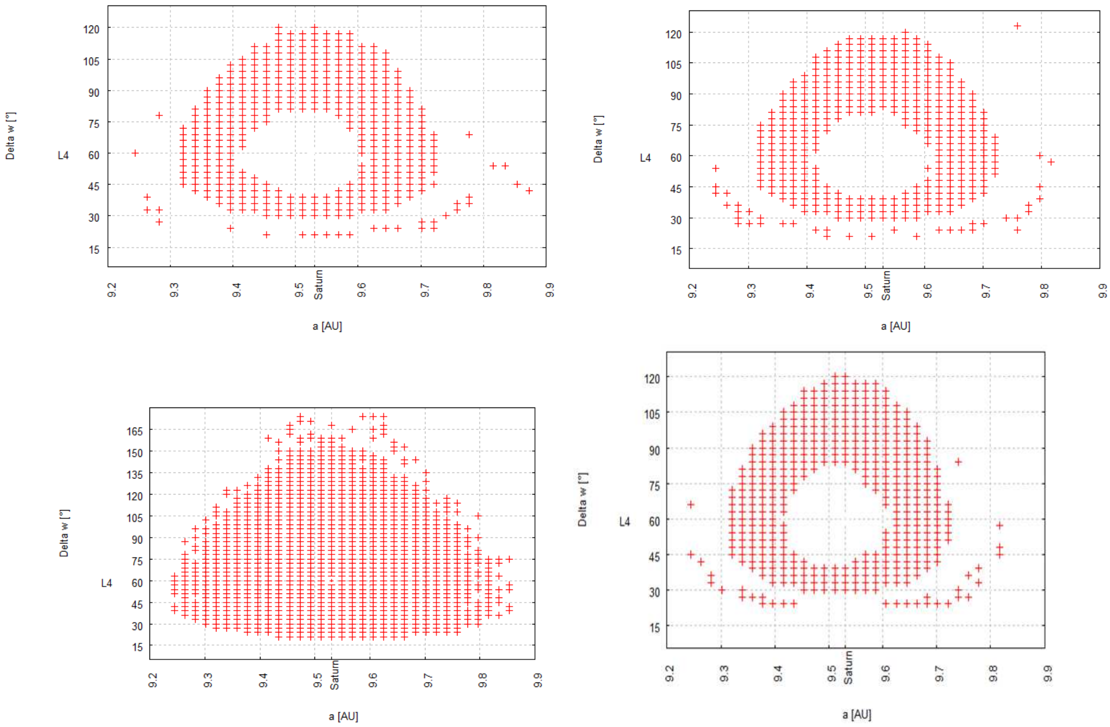

Figure 4: Stable regions for 4 different dynamical models semimajor axis versus elongation from L_4: SJU (upper left), SJN (Upper right), SUN (lower left) and SJ (lower right); for details see text

But what caused the unstable hole? Therefore we investigated different fictive dynamical systems described in Chapter 2. In Figure (upper left) we show the results for a system without Neptune (SJU) and in the same figure (upper right) the results for the dynamical system without Uranus (SJN). Even eliminating Uranus and Neptune, which means a system with only Saturn and Jupiter (SJ) (lower right), we always see nearly the same picture with an unstable hole close to the Lagrange point itself. But the integration without Jupiter (SUN), (lower left graph), shows that the hole is no more here, but a significantly larger stable region around L_4 is visible. With the whole OSS (model SJUN, Figure 2, left picture) the situation is nearly identical with a system with only Saturn and Jupiter.

4. CONCLUSIONS

The influence of Uranus and Neptune as perturbing planets of Trojan asteroids of Saturn is insignificant. The planet Jupiter is solely responsible for the hole of instability for short time integrations ($T < 10^7$) compared to the age of the planetary system. On the long term scale this planet also destabilizes the whole region around the Saturnian libration points. If we find in the future Trojans of Saturn, these Trojans could only be captured asteroids, in orbits in the 1:1 MMR for a short time. However, up to the present day, no Trojans of Saturn have been found. Are they too small for our current instruments or are possible captures too short with respect to the time they stay in the vicinity of L_4 and L_5? The question remains thrilling!

5. REFERENCES

Baudisch, H., 2010, "Investigation on the stability of the Trojans of Saturn around L4 (and L5)", Masterthesis at the Institute of Astronomy, p.95.

Connors, M., Wiegart, P., Veillet, C., 2011, "Earth's Trojan asteroids", Nature 475, 481-483.

Dvorak, R., Bazsó, A., Zhou, L.-Y., 2010, "Where are the Uranus Trojans?", Celest. Mech. Dyn. Astr. 107, 1-15.

Dvorak, R., Lhotka, Ch., Schwarz, R., 2008, "The dynamics of inclined Neptune Trojans", Celest. Mech. Dyn. Astr. 102, 97-110.

Holman, M., Wisdom, J., 1993 "Dynamical stability in the Outer Solar System and the delivery of short period comets", AJ , 105, 5, 1987-1999.

Innanen, K., Mikkola, S., 1989, "Studies on Solar System Dynamics I: The stability of Saturnian Trojans", AJ , 97, 3, 900-908.

Lhotka, C., Efthymiopoulos, C.,Dvorak, R., 2008, "Nekhoroshev stability at L4 or L5 in the elliptic-restricted three-body problem - application to Trojan asteroids" MNRAS 384, 1165-1177.

Nesvorný, D., Dones, L., 2002, "How Long-Lived Are the Hypothetical Trojan Populations of Saturn, Uranus, and Neptune?", Icarus 160, 271-288.

Robutel, P., Gabern, F., Jorba, A., 2005, "The Observed Trojans and the Global Dynamics Around The Lagrangian Points of the Sun Jupiter System", Celest. Mech. Dyn. Astr. 92, 53-69.

Téger, F., 2000, "On the stability of Lagrangian points L4 and L5 of Saturn, Proceedings of the 2nd Austrian Hungarian Workshop", 31-38.

Zhou, L.Y., Dvorak, R., Sun, Y.S., 2009, "The dynamics of Neptune Trojans - I. The inclined orbits", MNRAS 398, 1217-1227.

Zhou, L.Y., Dvorak, R. Sun, Y.S., 2011, "The dynamics of Neptune Trojans - II. Eccentric orbits and observed objects", MNRAS 410, 1849-1860.