Some new thoughts about long-term precession formula

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Contents:

- ♦ Motivation;
- ♦ Numerically integrated precession in a long time interval;
- ♦ Quasi-periodic representation of different precession parameters;
- ♦ Conclusions.

Motivation:

- ♦ Almost all models of precession in use, including the most recent one IAU2006, are expressed in terms of polynomial development, no matter which precession parameters are used..
- ♦ Model IAU2006 is very accurate, but usable only for a limited time interval (several centuries around the epoch J2000):
  - ♦ its errors rapidly increase with longer time spans!
- ♦ In reality, precession represents a complicated, very long-periodic process, with periods of hundreds of centuries:
  - ♦ this can be seen in numerically integrated equations of motion of the Earth in solar system and is rotation.

Goals:

- We assume that precession covers all periods longer than 100 centuries; shorter ones are included in the nutation;
- The goal of this study is to find relatively simple expressions of different precession parameters, with accuracy comparable to the IAU2006 model near the epoch J2000, and lower accuracy outside the interval ±1000 years (up to several minutes of arc at the extreme epochs ±200 thousand years).

Numerical integration:

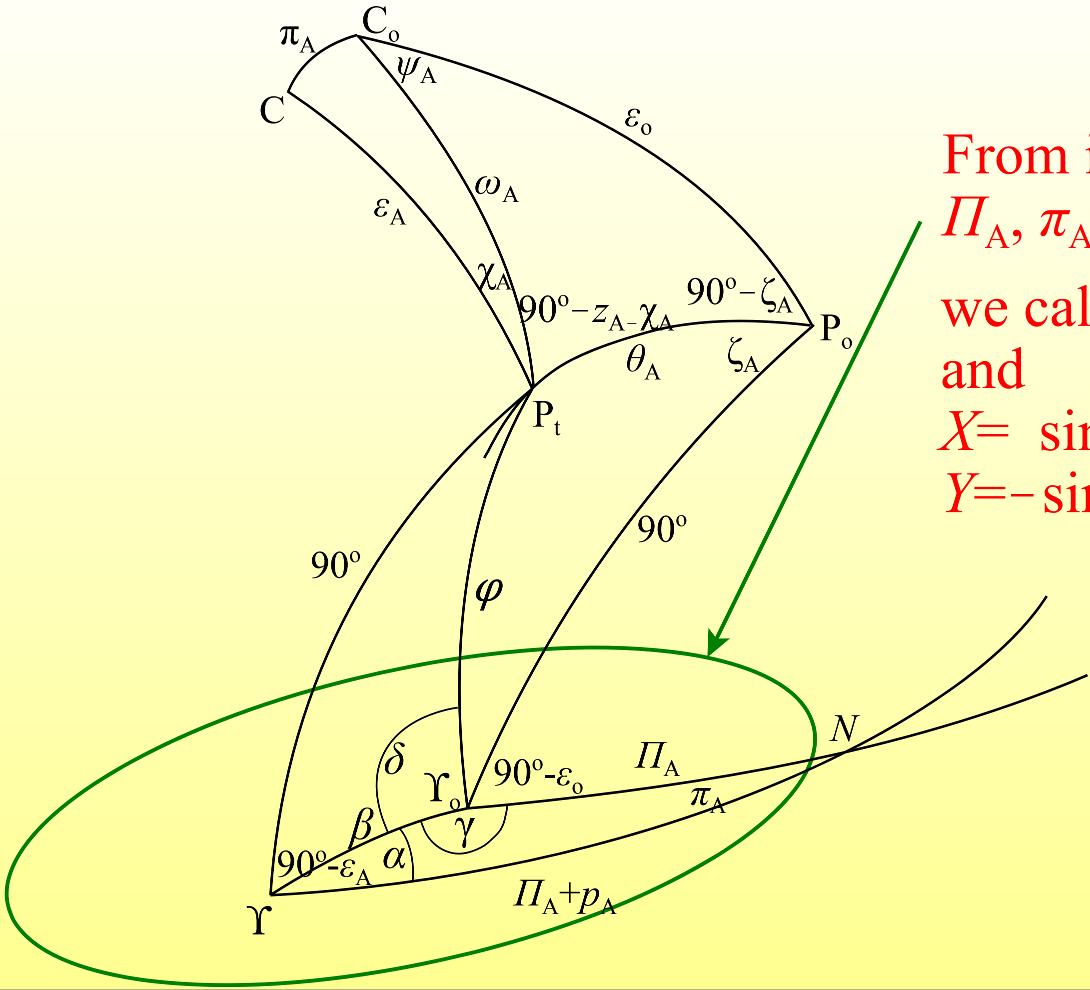
A. Precession of the ecliptic (Π_A, π_A):

Basis: Numerical integration of solar system motion, using the package Mercury 6 (Chambers, MNRAS 1999), in interval ±200 thousand years, with 1-day step. The elements of Earth's orbit are smoothed and interpolated with 100-year step.

B. General precession and obliquity (p_A, ε_A):

Basis: Numerical integration of the general precession and obliquity LA93 (Laskar et al., A&A 1993) in interval ±1 milion years, with1000-year step, interpolated with 100-year step. Additional corrections applied: slightly different value of dynamical ellipticity and its secular change (J2 dot), constant and secular tidal change of the obliquity.

In both cases, inside the interval ±1000 years around J2000 the integrated values are replaced with IAU2006 model, based on JPL DE406 ephemerides.



From integrated vales

Π_A, π_A, ε_A, p_A, and ε_o

we calculate θ_A, ζ_A , ω_A, ψ_A, χ_A, z_A and

$X = \sin\theta_A \cos\zeta_A$

$Y = -\sin\theta_A \sin\zeta_A$

Estimation of periodic terms:

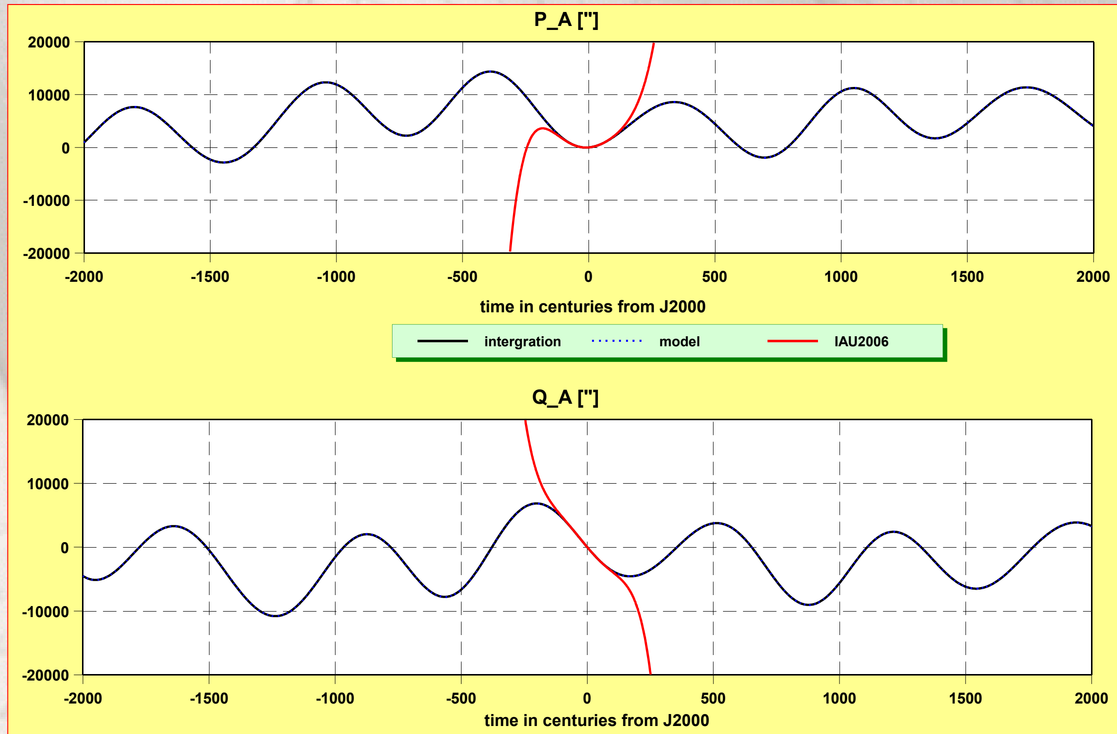
- Following steps are applied:
  - Spectral analysis of integrated values (modified Vaníček method) to find periodicities;
  - Identification of periods found with those found by Laskar (1993, 2004);
  - Estimation of sine/cosine amplitudes + cubic parabola to approximate longer periods;
  - These parameters are fitted to
    - Numerical integration outside the interval ±1000 years, with weights decreasing quadratically;
    - IAU2006 values inside this interval, with much higher weights;
  - Small additional corrections are applied to the constant, linear and quadratic terms, so that the first two derivatives are identical with those of IAU2006.

Long-periodic expressions for P_A, Q_A ["]:

$$P_A = 5851.607687 - 0.1189000T - 0.00028913T^2 + 101\times10^{-9}T^3 + \sum_1^8 \left(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i\right)$$

$$Q_A = -1600.886300 + 1.1689818T - 0.00000020T - 437\times10^{-9}T^3 + \sum_1^8 \left(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i\right)$$

Term		P_A	Q_A	P [cy]
σ_3	C_1	-5486.751211	-684.661560	708.15
	S_1	667.666730	-5523.863691	
s_1	C_2	-17.127623	2446.283880	2309.00
	S_2	-2354.886252	-549.747450	
	C_3	-617.517403	399.671049	1620.00
	S_3	-428.152441	-310.998056	
s_6	C_4	413.855033	-356.652376	492.20
	S_4	376.202861	421.535876	
	C_5	78.614193	-186.387003	1183.00
	S_5	184.778874	-36.776172	
	C_6	-180.732815	-316.800070	622.00
	S_6	335.321713	-145.278396	
	C_7	-87.676083	198.296071	882.00
	S_7	-185.138669	-34.744450	
	C_8	46.140315	101.135679	547.00
	S_8	-120.972830	22.885731	

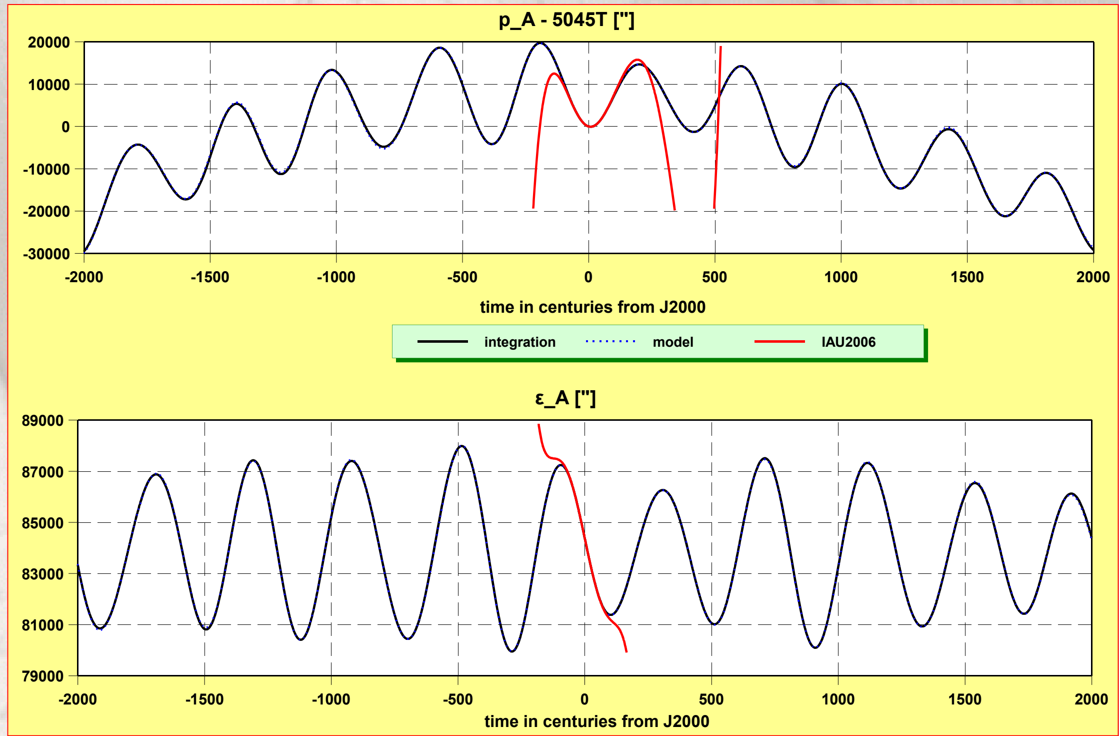


Long-periodic expressions for p_A, ε_A [”]:

$$p_A = 8141.271638 + 5043.0554944T - 0.00711241T^2 + 270 \times 10^{-9} T^3 + \sum_1^{10} \left(C_i \cos 2\pi T / P_i + S_i \sin 2\pi T / P_i \right)$$

$$\varepsilon_A = 84030.395084 + 0.3473890T - 0.00004222T - 106 \times 10^{-9} T^3 + \sum_1^{10} \left(C_i \cos 2\pi T / P_i + S_i \sin 2\pi T / P_i \right)$$

Term		p_A	ε_A	P [cy]
$p+s_3$	C_1	-5999.996067	516.766673	409.90
	S_1	-3890.540763	-1521.747591	
$p+s_4$	C_2	-2845.766039	-336.197888	396.15
	S_2	110.280584	-790.807019	
$p+s_6$	C_3	1455.526061	379.292583	536.91
	S_3	-1257.875168	448.220796	
$p+\nu_6$	C_4	1831.549125	191.094889	402.90
	S_4	1905.697108	-1086.628037	
$p+\nu_{10}$	C_5	882.038629	-10.366677	417.15
	S_5	788.465999	131.735193	
$p+s_1$	C_6	-306.157493	-365.657922	288.92
	S_6	1025.981221	-106.598169	
	C_7	365.085677	-65.344828	4042.97
	S_7	-244.006377	-285.976341	
	C_8	-197.661156	-27.071497	304.90
	S_8	74.590873	-69.931901	
	C_9	322.901212	28.700834	281.46
	S_9	-61.009702	111.399658	
	C_{10}	14.306662	39.794750	204.38
	S_{10}	-174.184581	3.844464	

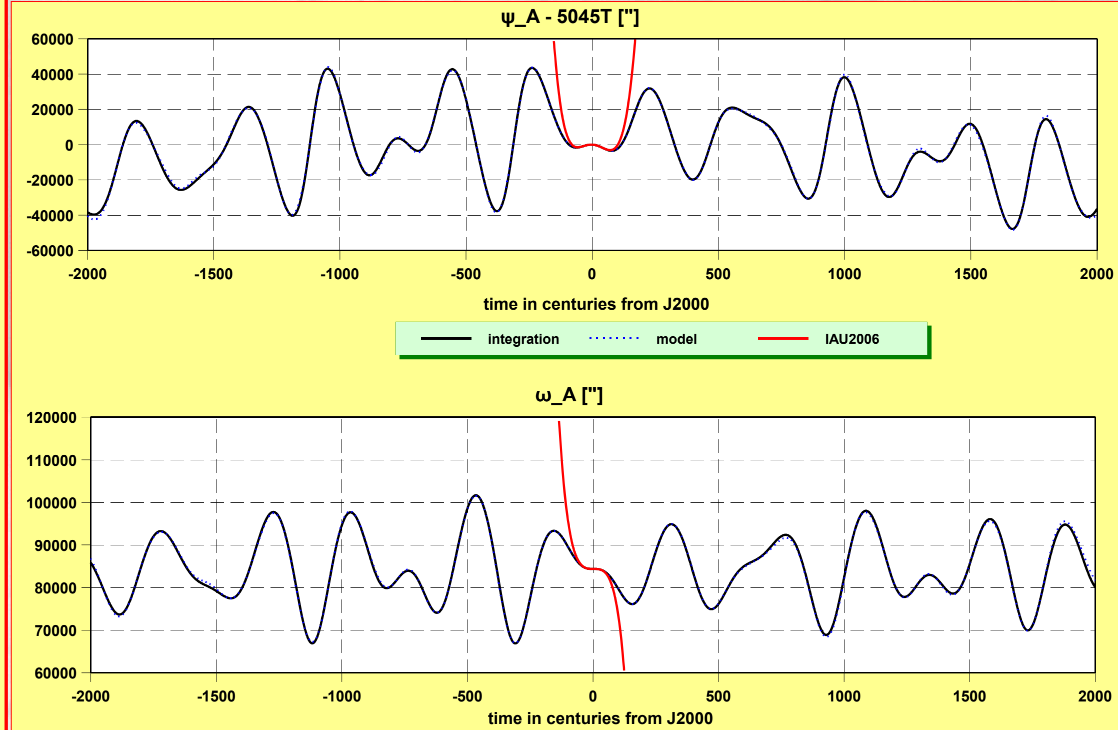


Long-periodic expressions for ω_A, ψ_A [”]:

$$\omega_A = 84283.366108 - 0.4449631T + 0.00000068T^2 + 150\times10^{-9}T^3 + \sum_1^{14}\left(C_i\cos2\pi T/P_i + S_i\sin2\pi T/P_i\right)$$

$$\psi_A = 8472.888973 + 5042.8012257T - 0.00740773T + 285\times10^{-9}T^3 + \sum_1^{14}\left(C_i\cos2\pi T/P_i + S_i\sin2\pi T/P_i\right)$$

Term		ω_A	ψ_A	P [cy]
$p+\nu_6$	C_1	1314.679626	-22420.160932	402.90
	S_1	-8658.248888	-3354.740507	
p	C_2	1698.164478	12364.867916	256.75
	S_2	5359.936261	-3953.468853	
	C_3	-2946.745615	-1855.311803	292.00
	S_3	-717.285550	7053.538527	
$p+s_6$	C_4	691.170703	2501.910635	537.22
	S_4	931.408851	-1895.196678	
$p+g_2-g_5$	C_5	-14.110991	111.451479	241.45
	S_5	-12.736900	143.109393	
	C_6	-534.673649	70.863565	375.22
	S_6	-6.985495	1343.619428	
$2p+s_3$	C_7	-356.790963	389.332023	157.87
	S_7	77.098670	1727.488574	
	C_8	-142.160739	2128.481251	275.90
	S_8	846.285243	316.951469	
	C_9	256.137565	368.139198	203.00
	S_9	83.329986	-1217.037602	
	C_{10}	162.716848	-785.264907	445.90
	S_{10}	-324.406028	-407.953884	
	C_{11}	95.138364	-927.251157	170.72
	S_{11}	-193.842226	-441.696960	
	C_{12}	-332.752312	35.623831	713.37
	S_{12}	-5.493032	-87.277001	
	C_{13}	124.581532	-521.921176	313.90
	S_{13}	-240.668180	-295.259639	
	C_{14}	82.685046	66.351105	128.38
	S_{14}	18.984123	-422.734446	

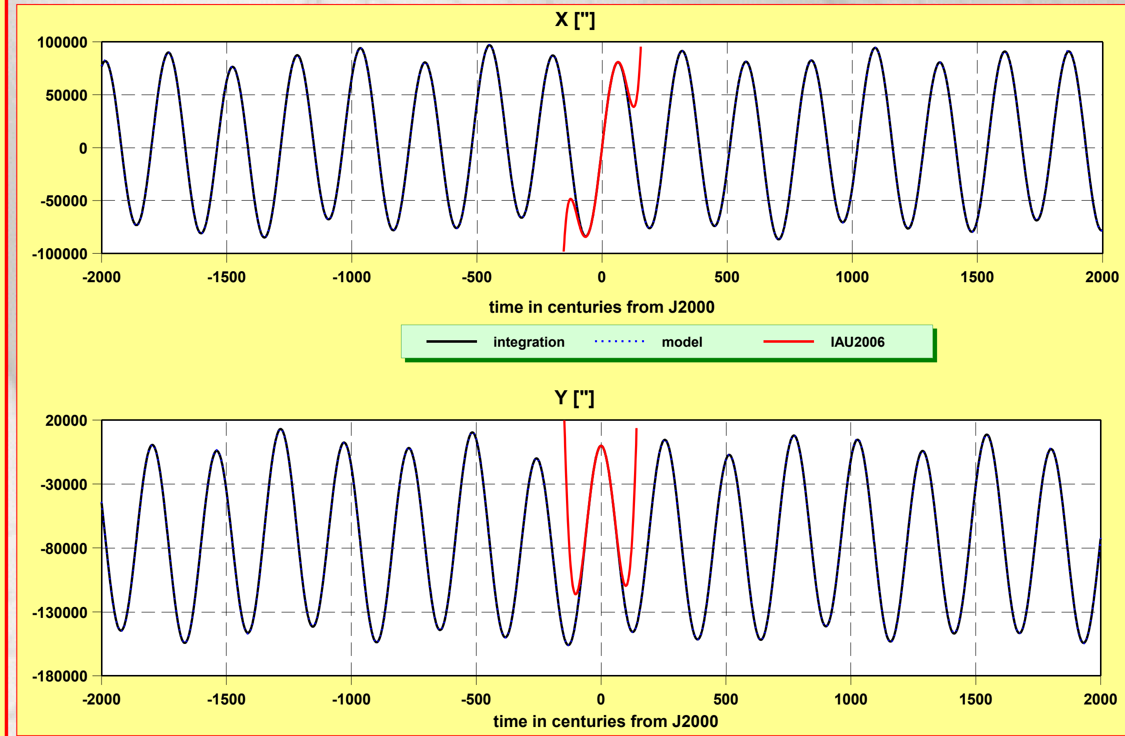


Long-periodic expressions for *X*, *Y* [”]:

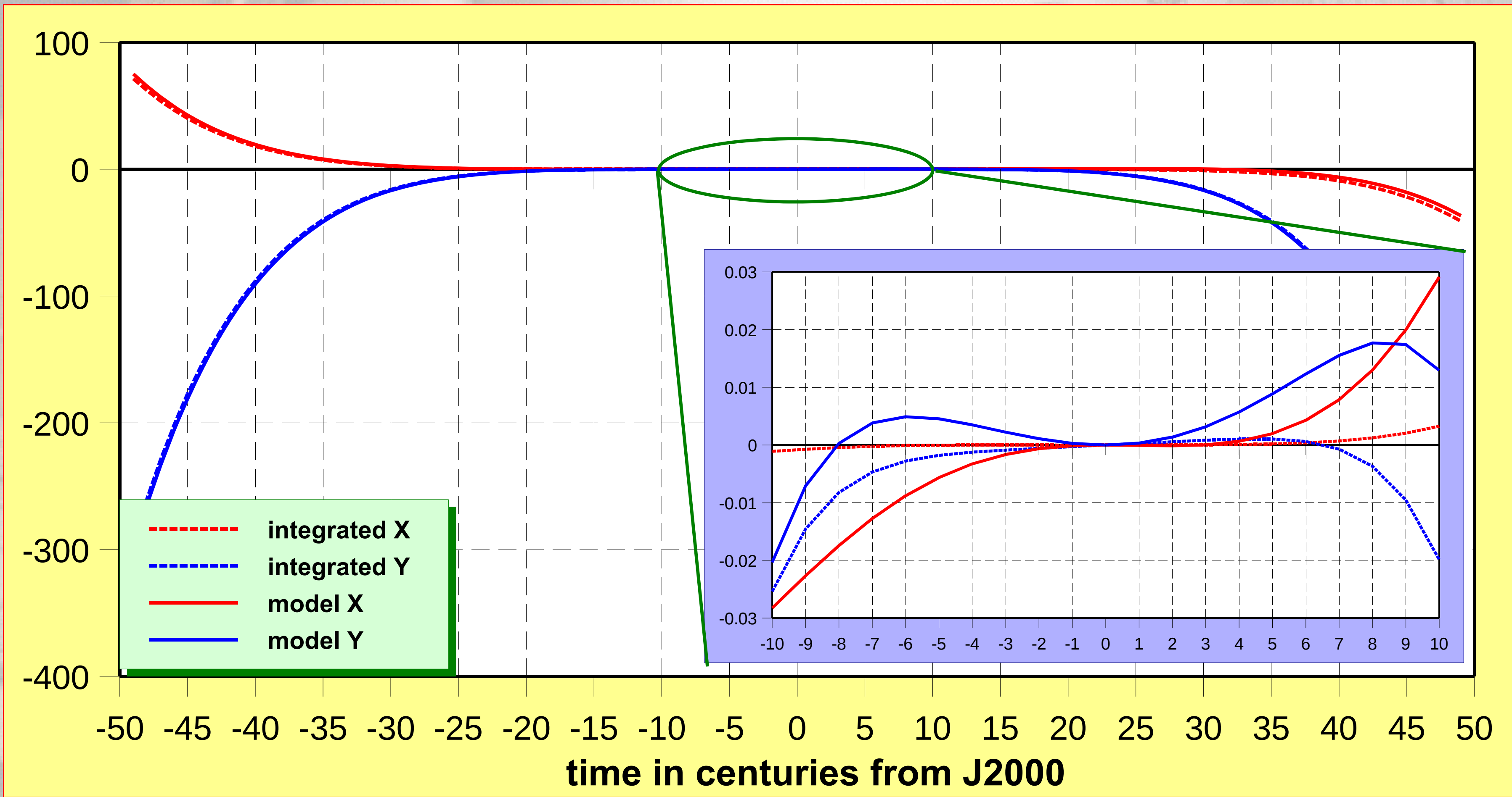
$$X = 5453.270624 + 0.4252850T - 0.00037173T^2 - 152\times10^{-9}T^3 + \sum_1^{14}\left(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i\right)$$

$$Y = -73750.937353 - 0.7675456T - 0.00018725T + 231\times10^{-9}T^3 + \sum_1^{14}\left(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i\right)$$

Term		*X*	*Y*	*P* [cy]
p	C_1	-819.946005	75004.345355	256.75
	S_1	81491.288050	1558.521633	
σ_3	C_2	-8444.676986	624.033815	708.15
	S_2	787.162943	7774.939774	
$p-g_2+g_5$	C_3	2600.009737	1251.136728	274.20
	S_3	1251.296938	-2219.533890	
$p+g_2-g_5$	C_4	2755.175572	-1102.213989	241.45
	S_4	-1257.951746	-2523.969336	
s_1	C_5	-167.659179	-2660.663565	2309.00
	S_5	-2966.800362	247.850562	
s_6	C_6	871.855033	699.292008	492.20
	S_6	639.744569	-846.485543	
$p+s_4$	C_7	44.769702	153.167261	396.10
	S_7	131.600315	-1393.123929	
$p+s_1$	C_8	-512.313270	-950.865460	288.90
	S_8	-445.040719	368.526188	
$p-s_1$	C_9	-819.415456	499.756007	231.10
	S_9	584.524115	749.044958	
	C_{10}	-538.071710	-145.189989	1610.00
	S_{10}	-89.756178	444.704321	
	C_{11}	-189.793616	558.115977	620.00
	S_{11}	524.429711	235.934536	
$2p+s_3$	C_{12}	-402.922967	-23.923094	157.87
	S_{12}	-13.549103	374.049112	
	C_{13}	179.157617	-165.405552	220.30
	S_{13}	-210.157617	-171.329809	
	C_{14}	-9.814377	9.344900	1200.00
	S_{14}	-44.920033	-22.899576	



Central part, differences in *X*, *Y* from IAU2006 [”]



Conclusions:

- Existing models of precession are valid with high accuracy only in interval of several centuries:
  - Their errors grow very quickly outside this interval - to more than 10 degrees ±20 thousand years from the epoch J2000;
- We demonstrate a possiblity of constructing a relatively simple model of precession which:
  - yields the results comparable to IAU2006 model in a short time interval (several centuries) around J2000,
  - follows the periodical character of the integrated precession in long-periodic sense (hundreds thousand years), with the accuracy that detoriates only very slowly (several arcminutes ±200 thousand years from J2000).