SOME NEW THOUGHTS ABOUT LONG-TERM PRECESSION FORMULA

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ABSTRACT. In our preceding study (Vondrák et al. 2009) we formulated developments for the precessional contribution to the CIP X, Y coordinates suitable for use over long time intervals. They were fitted to IAU 2006 close to J2000.0 and to the numerical integration of the ecliptic (using the integrator package Mercury 6) and of the general precession and obliquity (using Laskar's solution LA93) for more distant epochs. Now we define the boundary between precession and nutation (both are periodic) to avoid their overlap. We use the IAU 2006 model (that is based on the Bretagnon's solution VSOP87 and the JPL planetary ephemerides DE406) to represent the precession of the ecliptic close to J2000.0, a new integration using Mercury 6 for more distant epochs, and Laskar's LA93 solution to represent general precession and obliquity. The goal is to obtain new developments for different sets of precession angles that would fit to modern observations near J2000.0, and at the same time to numerical integration of the translatory-rotatory motions of solar system bodies on scales of several thousand centuries.

1. INTRODUCTION

This is a continuation of our preceding study (Vondrák et al. 2009) in which we demonstrated that all models of precession in use, including the most recent one, IAU 2006 (Capitaine et al. 2003, Hilton et al. 2006), lose their accuracy rapidly in time, being expressed in terms of polynomial development, no matter which precession parameters are used. The IAU 2006 model is very accurate, but usable only for a limited time interval (several centuries around the epoch J2000.0); its errors however rapidly increase with longer time spans. In reality, precession is a complicated, very long-periodic process, with periods of hundreds of centuries. This can be seen in numerically integrated equations of motion of the Earth in the solar system and of its rotation.

Here we assume that precession covers all periods longer than 100 centuries; shorter ones are included in the nutation. In this connection, it is necessary to mention that the IAU 2000 model of nutation includes several terms with longer periods: 105 cy, 209 cy for the luni-solar terms and 933 cy, 150 cy, 129 cy, 113 cy for the planetary terms. The amplitudes of these terms are however very small (lower than 4 mas for one term and lower than 0.1 mas for the others).

The goal of the present study is to find relatively simple expressions for different precession parameters, with accuracy comparable to the IAU 2006 model near the epoch J2000.0, and useful accuracy outside the interval ± 10 cy (a few arcminutes at the extreme epochs ± 2000 cy).

2. NUMERICAL INTEGRATIONS

Here we use the following numerical integrations as a basis for all subsequent calculations:

- For the precession of the ecliptic (parameters $P_A = \sin\pi_A \sin\Pi_A$, $Q_A = \sin\pi_A \cos\Pi_A$) the new integration of the solar system motion, using the package Mercury 6 (Chambers 1999), in interval

±2000 cy from J2000.0, with 1-day steps. The elements of the Earth's orbit are then smoothed and interpolated with 1-cy steps.

- For the general precession and obliquity (parameters p_A, ϵ_A) the integration LA93 by Laskar et al. (1993) in the interval ±1 million years, with 10-cy steps, interpolated in 1-cy steps. Additional corrections are applied to account for: slightly different values of the dynamical ellipticity (compatible with the IAU 2006 model) and its secular change $\dot{J}_2$, constant and secular tidal change of the obliquity.

In both cases, inside the interval ±10 cy around J2000.0 the integrated values are replaced with the values computed from the IAU 2006 model which, in turn, is based on Bretagnon's semi-analytical theory VSOP87 (Bretagnon 1987) and JPL DE406 (Standish 1998) ephemerides.

The relations of the four above mentioned angles to other parameters describing precession are shown in Fig. 1. To calculate different precession parameters, we obtain first the auxiliary angles α, β, γ from the triangle $\Upsilon\Upsilon_\circ$N, and then the angles φ, δ by solving the triangle $\Upsilon\Upsilon_\circ$P$_t$ (see Vondrák et al. 2009).

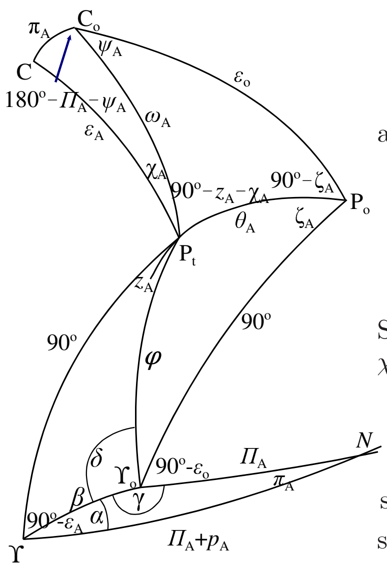

Figure 1: Precession parameters

From the triangle $\Upsilon_\circ$P$_t$P$_\circ$ follow precession parameters θ_A, ζ_A

$$
\begin{aligned}
\cos\theta_A &= -\sin\varphi\sin(\gamma+\delta-\varepsilon_\circ) \\
\sin\theta_A\sin\zeta_A &= -\sin\varphi\cos(\gamma+\delta-\varepsilon_\circ) \\
\sin\theta_A\cos\zeta_A &= \cos\varphi,
\end{aligned}
\tag{1}
$$

and the triangle P$_\circ$P$_t$C$_\circ$ then yields precession parameters ω_A, ψ_A:

$$
\begin{aligned}
\cos\omega_A &= \cos\varepsilon_\circ\cos\theta_A + \sin\varepsilon_\circ\sin\theta_A\sin\zeta_A \\
\sin\omega_A\sin\psi_A &= \sin\theta_A\cos\zeta_A \\
\sin\omega_A\cos\psi_A &= \sin\varepsilon_\circ\cos\theta_A - \cos\varepsilon_\circ\sin\theta_A\sin\zeta_A.
\end{aligned}
\tag{2}
$$

Solving the triangles P$_t$CC$_\circ$, P$_\circ$P$_t$C$_\circ$ we finally obtain the parameters χ_A, z_A:

$$
\begin{aligned}
\sin\varepsilon_A\sin\chi_A &= P_A\cos\psi_A + Q_A\sin\psi_A \\
\sin\varepsilon_A\cos\chi_A &= \cos\pi_A\sin\omega_A - (P_A\sin\psi_A - Q_A\cos\psi_A)\cos\omega_A \\
\sin\theta_A\sin(z_A+\chi_A) &= \sin\omega_A\cos\varepsilon_\circ - \cos\omega_A\sin\varepsilon_\circ\cos\psi_A \\
\sin\theta_A\cos(z_A+\chi_A) &= \sin\varepsilon_\circ\sin\psi_A.
\end{aligned}
\tag{3}
$$

We used these formulas to calculate all above defined precession parameters in the interval ±2000 cy with 1-cy steps. Since the pole coordinates X, Y are referred to the GCRS rather than the mean equator and equinox of J2000.0, they require small additional corrections to account for displacements of the celestial pole and equinox (see Eq.(3) in Vondrák et al. 2009).

3. ANALYTICAL APPROXIMATION

To find the long-term analytical approximation of precession parameters, we apply the following steps:

- Spectral analysis of integrated values is done, using a modified Vaníček method (Vondrák 1977);
- Periods found are identified with those found by Laskar et al. (1993, 2004). In positive cases, Laskar's values are adopted;
- Sine/cosine amplitudes of the terms found in preceding step, plus cubic parabola, are fitted to the numerical integration. The weights used in the fit are very high close to J2000.0, and they decrease quadratically with time;
- Small additional corrections are applied to the constant, linear and quadratic terms, so that the function value and first two derivatives are identical with those of the IAU 2006 model.

Here we show long-term expressions for only some of the precession parameters and their comparison with both integrated values and the IAU 2006 model. In these examples, T is the time in Julian centuries, running from J2000.0, and periodic terms have the general form $\sum(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i)$.

Table 1: Periodic terms in ψ_A, ω_A

term	C/S	$\psi_A['']$	$\omega_A['']$	P[cy]
$p+\nu_6$	C_1	-22420.160932	1314.679626	402.90
	S_1	-3354.740507	-8658.248888	
p	C_2	12364.867916	1698.164478	256.75
	S_2	-3953.468853	5359.936261	
	C_3	-1855.311803	-2946.745615	292.00
	S_3	7053.538527	-717.285550	
$p+s_6$	C_4	2501.910635	691.170703	537.22
	S_4	-1895.196678	931.408851	
$p+g_2-g_5$	C_5	111.451479	-14.110991	241.45
	S_5	143.109393	-12.736900	
	C_6	70.863565	-534.673649	375.22
	S_6	1343.619428	-6.985495	
$2p+s_3$	C_7	389.332023	-356.790963	157.87
	S_7	1727.488574	77.098670	
	C_8	2128.481251	-142.160739	275.90
	S_8	316.951469	846.285243	
	C_9	368.139198	256.137565	203.00
	S_9	-1217.037602	83.329986	
	C_{10}	-785.264907	162.716848	445.90
	S_{10}	-407.953884	-324.406028	
	C_{11}	-927.251157	95.138364	170.72
	S_{11}	-441.696960	-193.842226	
	C_{12}	35.623831	-332.752312	713.37
	S_{12}	-87.277001	-5.493032	
	C_{13}	-521.921176	124.581532	313.90
	S_{13}	-295.259639	-240.668180	
	C_{14}	66.351105	82.685046	128.38
	S_{14}	-422.734446	18.984123	

Long-term expressions for the precession angles ψ_A, ω_A, are given as

$$
\begin{aligned}
\psi_A &= 8472.888973 + 5042.8012257T - \\
&- 0.00740773T^2 + 285 \times 10^{-9}T^3 + \sum\nolimits_\psi \\
\omega_A &= 84283.366108 - 0.4449631T + \\
&+ 0.00000068T^2 + 150 \times 10^{-9}T^3 + \sum\nolimits_\omega,
\end{aligned} \tag{4}
$$

where the cosine/sine amplitudes of the periodic parts $\sum_\psi$, $\sum_\omega$ are given in Table 1. The comparison of the long-term model of precession angles ψ (reduced by a conventional rate 5045″/cy, in order to see more details) and obliquity, $\psi_A - 5045''T$ (top), ω_A (bottom) is shown in Fig. 2a, in which the vertical scale is in arcseconds.

The curves representing the new model and integrated values in Fig. 2 (full and dotted lines, respectively) are very close so that they are graphically indistinguishable. The IAU 2006 precession model (dashed line) fits well to both integrated values and new model near the epoch J2000.0, but it diverges rapidly from both of them for more distant epochs.

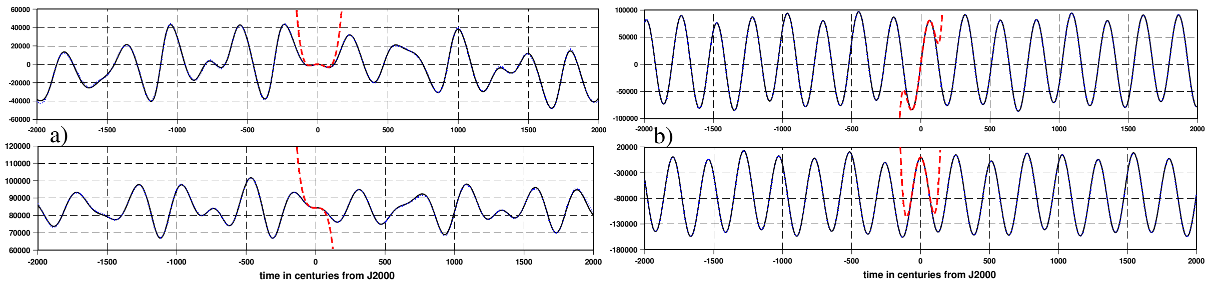

Figure 2: Long-term models of parameters a) ψ_A, ω_A, b) X, Y

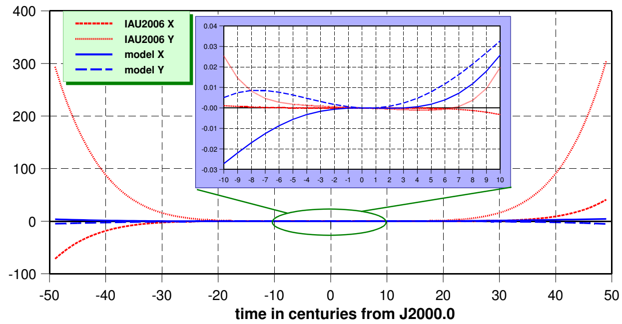

Figure 3: Comparison of X, Y values - closeups

Closeups of the differences between the IAU 2006 and long-term models from the integrated values are depicted in Fig. 3; differences in X, Y of the IAU 2006 model are shown as short dashed and dotted lines, the differences of the long-term model as full and long dashed lines. The vertical scales are again in arcseconds.

Table 2: Periodic terms in X, Y

term	C/S	$X['']$	$Y['']$	P[cy]
p	C_1	-819.946005	75004.345355	256.75
	S_1	81491.288050	1558.521633	
σ_3	C_2	-8444.676986	624.033815	708.15
	S_2	787.162943	7774.939774	
$p - g_2 + g_5$	C_3	2600.009737	1251.136728	274.20
	S_3	1251.296938	-2219.533890	
$p + g_2 - g_5$	C_4	2755.175572	-1102.213989	241.45
	S_4	-1257.951746	-2523.969336	
s_1	C_5	-167.659179	-2660.663565	2309.00
	S_5	-2966.800362	247.850562	
s_6	C_6	871.855033	699.292008	492.20
	S_6	639.744569	-846.485543	
$p + s_4$	C_7	44.769702	153.167261	396.10
	S_7	131.600315	-1393.123929	
$p + s_1$	C_8	-512.313270	-950.865460	288.90
	S_8	-445.040719	368.526188	
$p - s_1$	C_9	-819.415456	499.756007	231.10
	S_9	584.524115	749.044958	
	C_{10}	-538.071710	-145.189989	1610.00
	S_{10}	-89.756178	444.704321	
	C_{11}	-189.793616	558.115977	620.00
	S_{11}	524.429711	235.934536	
$2p + s_3$	C_{12}	-402.922967	-23.923094	157.87
	S_{12}	-13.549103	374.049112	
	C_{13}	179.516279	-165.405552	220.30
	S_{13}	-210.157617	-171.329809	
	C_{14}	-9.814377	9.344900	1200.00
	S_{14}	-44.920033	-22.899576	

Long-term expressions for the precession angles X, Y, are given as

$$\begin{aligned} X &= 5453.270624 + 0.4252850T - \\ &- 0.00037173T^2 - 152 \times 10^{-9}T^3 + \textstyle\sum_X \\ Y &= -73750.937353 - 0.7675456T - \\ &- 0.00018725T^2 + 231 \times 10^{-9}T^3 + \textstyle\sum_Y, \end{aligned} \tag{5}$$

where the cosine/sine amplitudes of the periodic parts X, Y are given in Table 2. The comparison of the long-term model of precession angles X (top) and Y (bottom) is shown in Fig. 2b, in arcseconds.

4. CONCLUSIONS

The present study demonstrates the possibility of constructing a new model of precession that is equivalent to the most recent IAU model of precession in a short-term sense (up to several centuries around J2000.0) and, at the same time, fitting well to modern long-term numerical integrations of the motions of the solar system bodies. The accuracy of this solution is improved, with respect to Vondrák et al. (2009), mainly in the long-term precession of the ecliptic. The long-term expressions are valid only in the interval ±2000 cy from J2000.0; outside this interval their validity rapidly deteriorates. This limitation not only reduces the necessary number of periodic terms, but also avoids the problem of resonances in the solar system mentioned by Laskar et al. (2004). We also derived the expressions for all other precession parameters that are not presented here due to the page limit, but they are available on request from the first author.

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