GEODETIC RELATIVISTIC ROTATION OF THE SOLAR SYSTEM BODIES

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ABSTRACT. The problem of the geodetic (relativistic) rotation of the major planets, the Moon, and the Sun is studied by using DE404/LE404 ephemeris. For each body the files of the ecliptical components of the vectors of the angular velocity of the geodetic rotation are determined over the time span from AD1000 to AD3000 with one day spacing. The most essential terms of the geodetic rotation are found by means of the least squares method and spectral analysis methods.

1. INTRODUCTION

In every relativistic ephemeris of the Sun, major planets, and the Moon the gravitational interaction is modeled by considering these bodies as non-rotating point masses. However, their geodetic (relativistic) rotation can be determined, as it will be shown here. In accordance with Landau and Lifshitz (1975), a geodetic rotation arises when a body, having non-zero moments of inertia, is orbiting in the Riemannian space of general relativity. It is the most essential relativistic component of the rotational motion of the Earth. A similar statement may be valid for other major bodies of the solar system. An ephemeris of the major bodies of the solar system, based on the relativistic equations of the orbital motion, contains data necessary for the calculation of the secular and periodic components of the vector of the angular velocity of the geodetic rotation of these bodies. It is well known that the vector of the angular velocity of the geodetic rotation of a body i is defined by the following expression

$$\bar{\sigma}_i = \frac{1}{c^2}\sum_{j\neq i}\frac{Gm_j}{\left|\bar{R}_i-\bar{R}_j\right|^3}\left(\bar{R}_i-\bar{R}_j\right)\times\left(\frac{3}{2}\dot{\bar{R}}_i-2\dot{\bar{R}}_j\right) .$$

Here c is the velocity of light; G is the gravitational constant; m_j is the mass of a body$j$; $\bar{R}_i$, $\dot{\bar{R}}_i, \bar{R}_j$, $\dot{\bar{R}}_j$ are the vectors of the barycentric position and velocity of bodies i and j. The symbol $\times$ means a vector product; the subscripts i and j correspond to the Sun, the major planets, and the Moon. Since the mass of the Sun is dominant in the solar system then the main part of $\bar{\sigma}_i$ for the major planets and the Moon is a result of the orbital motion of these bodies. It means that vector $\bar{\sigma}_i$ is almost orthogonal to the plane of the heliocentric orbit. For all planets (except Pluto) and the Moon the vectors $\bar{\sigma}_i$ are practically directed to the north pole of the ecliptic. The geodetic rotation of the Sun depends on the orbital motion of the major planets and the Moon. Only the component σ^Z, orthogonal to the plane of the fixed ecliptic J2000 is studied, because the geodetic rotation in the ecliptic plane is the most interesting phenomenon. For each body the file of the values of the ecliptical component σ^Z is formed over the time span from AD1000 to AD3000 with one day spacing. The secular and periodic components of the geodetic rotation vector are determined by means of the procedure which involves the least-squares method and spectral analysis methods. The result is presented in the form

$\sigma^Z = \sum\limits_{k=0}^{6} \mathrm{T}^k \sum\limits_{m=1} (A_{km}\sin\alpha_m + B_{km}\cos\alpha_m), \quad \alpha_m = \sum\limits_{l=1}^{10} n_l\lambda_l$. The mean heliocentric longitudes of the planets λ_1, …, λ_9 and the mean geocentric longitude of the Moon λ_{10} with respect to the fixed equinox J2000, adjusted to DE404/LE404 ephemeris, are taken from Brumberg and Bretagnon (2002). T means the Dynamical Barycentric Time (TDB) measured in thousand Julian years (tjy). The coefficients A_{km} , B_{km} are to be determined and n_l are some integer numbers. This procedure was earlier used by Pashkevich and Eroshkin (2005) for the construction of the high-precision semi-analytical series describing the rotation of the rigid Earth.

2. ANGULAR VELOCITY OF THE GEODETIC ROTATION

It is quite natural to begin with the determination of the components of the geodetic rotation of the Earth because its high-precision semi-analytical representation is elaborated by Brumberg and Bretagnon (2002). The values of the component σ^Z, orthogonal to the plane of the fixed ecliptic J2000, are calculated by means of DE404/LE404 ephemeris. As it was done in the paper of Bretagnon and co-authors (1998), the equatorial coordinate system of DE404/LE404 ephemeris is put to the fixed ecliptic and dynamical equinox J2000 in the result of two rotations: a rotation in the equator plane at the angle $-0".05294$ to the point of the dynamical equinox J2000 and the rotation at the angle 23°26'21".40928 to the plane of the fixed ecliptic J2000.0. The behavior of the component σ^Z is depicted in Figure 1a.

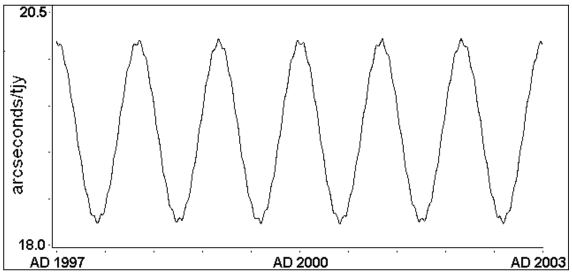

Figure 1a: σ^Z for the Earth (fragment)

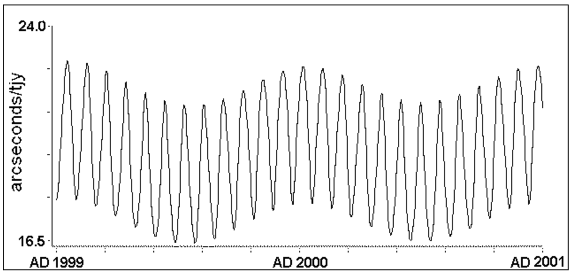

Figure 1b: σ^Z for the Moon (fragment)

In the result of processing 730500 data this component for the Earth is determined:

$$\sigma^Z = \{19".1988821 \ - \ 0".00017685\mathrm{T} + ... + 10^{-6}\dot{\lambda}_3 \left[-34".285\cos\lambda_3 + 149".227\sin\lambda_3 + \right.$$
$$\left. + \mathrm{T}\left(-7".539\cos\lambda_3 - 5".682\sin\lambda_3\right) + \mathrm{T}^2\left(0".261\cos\lambda_3 - 0".291\sin\lambda_3\right) + ...\right] + \ ...\}\,/\mathrm{tjy}.$$

Here λ_3 is the mean longitude of the Earth, $\lambda_3 = 1.75347029148 + 6283.0758511455\mathrm{T}$. All symbols $\dot{\lambda}_i$ mean the time derivatives of the mean longitudes λ_i. From the paper of Brumberg and Bretagnon (2002) one can use the developments representing the geodetic rotation of the Earth in Euler angles for constructing the same component:

$$\sigma^Z_{BB} = \{19."19883018 - 0."00026965T + 10^{-6}\dot{\lambda}_3 \left[(-34."28\cos\lambda_3 + 149."22\sin\lambda_3) + \right.$$
$$\left. + T\left(-7."54\cos\lambda_3 - 5."69\sin\lambda_3\right) + \ T^2\left(0."30\cos\lambda_3 - 0."29\sin\lambda_3\right)\right]\}\,/\mathrm{tjy}.$$

This result is essentially based upon the semi-analytical theory of the rigid Earth rotation SMART97 and the semi-analytical theory of the orbital motion of the major planets VSOP87 by Bretagnon and Francou (1988), while the results of the present paper are based upon DE404/LE404 numerical ephemeris. The remarkable difference in the coefficients of the secular term is explained by the different theories of the orbital motions. This statement is confirmed by the following. In the paper of Eroshkin and co-authors (2004) the high-precision numerical theory of the rigid Earth rotation, consistent to DE404/LE404 numerical ephemeris, was constructed. The results of the comparison of this theory with SMART97 theory are presented in the tables of the mentioned paper. By subtracting the coefficients of the dynamical solution from those of the kinematical solution the comparison of the geodetic rotation components, adequate to DE404/LE404 numerical ephemeris and SMART97 theory, is performed. In particular the difference in the secular term of the component $\sigma^Z$is calculated: $\Delta\sigma^Z = T0."00008611$/tjy. This term is nearly equal to the difference between the secular terms in σ^Z and in σ^Z_{BB}. Taking into account also completely different methods of the determination of the coefficients of σ^Z and σ^Z_{BB}, one can state that they are in a good accordance. Thus the method of the determination of the geodetic rotation velocity vector is very accurate and effective. This method is applied to the other bodies of the solar system. The geodetic rotation of the Moon is determined not only by the Sun but also by the Earth. Figure 1b demonstrates it visually. Since Mercury is the nearest planet to the Sun then it is clear that its geodetic rotation has to be the most significant in the solar system. Some asymmetry in Figure 2a is explained by the relatively large eccentricity of the orbit of Mercury as compared to the other planet orbits (except Pluto's orbit). The sharp peaks of the curve correspond to Mercury's transits via perihelia. The behavior of the Pluto's component σ^Z depicted in Figure 2b is similar to that of Mercury, presented in Figure 2a. It is explained by the fact that the values of the eccentricities of the orbits of Pluto and Mercury are close to each other. There is no semi-analytical theory of the heliocentric motion of Pluto, nevertheless the mean heliocentric longitude of Pluto, with respect to the fixed equinox J2000.0, λ_9 is determined by the least squares method: λ_9 =0.2480488137 + 25.2270056856T.

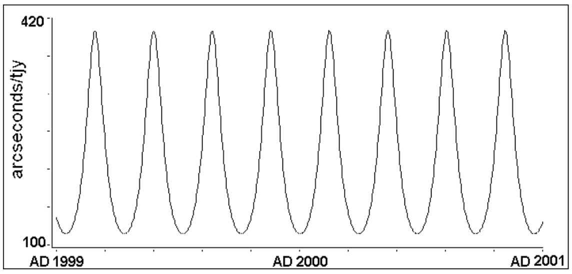

Figure 2a: σ^Z for Mercury (fragment)

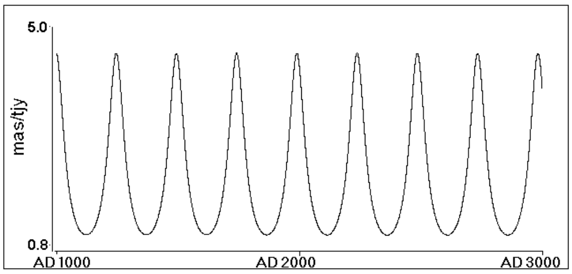

Figure 2b: σ^Z for Pluto

The geodetic rotation of the Sun arises when the Sun is orbiting relatively the barycentre of the solar system in the gravitational field of the major planets and the Moon. The vector of the geodetic rotation of the Sun is determined by the orbital motion of the planets. The axis of the angular velocity of the geodetic rotation is almost perpendicular to the plane of the ecliptic. Since the masses of the planets are essentially less than the mass of the Sun then the geodetic rotation of the Sun is very small. It is interesting to note that the periodic part of the vector $\bar{\sigma}$ for the Sun is determined not only by the motion of Jupiter, the heaviest planet, but also by that of Mercury, the nearest planet to the Sun, which has the largest mean motion.

Object	Secular part	Periodic part	Eccentricity of the orbit
Mercury	214".905T	$1086''.273 \cdot 10^{-6} \sin\lambda_1 - 4882''.196 \cdot 10^{-6} \cos\lambda_1$	0.206
Venus	43".124T	$-56''.907 \cdot 10^{-6} \sin\lambda_2 - 64''.182 \cdot 10^{-6} \cos\lambda_2$	0.007
the Earth	19".199T	$-34''.285 \cdot 10^{-6} \sin\lambda_3 - 149''.227 \cdot 10^{-6} \cos\lambda_3$	0.017
the Moon	19".495T	$-34''.285 \cdot 10^{-6} \sin\lambda_3 - 149''.227 \cdot 10^{-6} \cos\lambda_3$ $30''.212 \cdot 10^{-6} \sin D$	
Mars	6".756T	$516''.062 \cdot 10^{-6} \sin\lambda_4 \quad 229''.326 \cdot 10^{-6} \cos\lambda_4$	0.093
Jupiter	0".312T	$82''.830 \cdot 10^{-6} \sin\lambda_5 - 21''.289 \cdot 10^{-6} \cos\lambda_5$	0.048
Saturn	0".069T	$-2''.710 \cdot 10^{-6} \sin\lambda_6 - 53''.014 \cdot 10^{-6} \cos\lambda_6$	0.056
Uranus	0".012T	$-22''.280 \cdot 10^{-6} \sin\lambda_7 - 3''.492 \cdot 10^{-6} \cos\lambda_7$	0.046
Neptune	0".004T	$1''.847 \cdot 10^{-6} \sin\lambda_8 - 1''.773 \cdot 10^{-6} \cos\lambda_8$	0.009
Pluto	0".002T	$57''.447 \cdot 10^{-6} \sin\lambda_9 \quad 0''.665 \cdot 10^{-6} \cos\lambda_9$	0.249
the Sun	0".001T	$6''.226 \cdot 10^{-6} \sin\lambda_1 - 27''.982 \cdot 10^{-6} \cos\lambda_1$ $55''.823 \cdot 10^{-6} \sin\lambda_5 - 14''.358 \cdot 10^{-6} \cos\lambda_5$	

Table 1: The main secular and periodic terms of the geodetic rotation

The angles of the geodetic rotation of the solar system bodies in the plane of the ecliptic are determined in the result of the analytical integration of the expression for the component σ^Z. The most essential secular and periodic terms are presented in Table 1. It is easy to see that the value of the secular part of the angle of the geodetic rotation of each planet depends on its distance from the Sun. The value of the periodic part depends on the distance from the Sun. It depends essentially on the eccentricity of the orbit. Mars and Pluto are examples of such dependence. The geodetic rotation of a planet varies when it moves in the eccentric orbit.

3. CONCLUSIONS

For the Sun and the superior planets (except Mars) the geodetic rotation is insignificant. Nevertheless in the case of the Sun it is rather interesting from the theoretical point of view. For the terrestrial planets and the Moon the geodetic rotation is considerable and has to be taken into account for the construction of the high-precision theories of the rotational motion. The geodetic rotation has to be taken into account if the influence of the dynamical figure of a body on its orbital-rotational motion is studied in the post-Newtonian approximation. In addition, the lunar laser range processing has to use the relativistic theory of the rotation of the Moon, as well as that of the Earth.

Acknowledgements. The investigation was carried out at the Central (Pulkovo) Astronomical Observatory of Russian Academy of Science and the Space Research Centre of Polish Academy of Science, under a financial support within the program of cooperation between Polish and Russian Academies of Sciences (Theme No.31).

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