VARIATIONS OF $\bar{C}_{21}, \bar{S}_{21}$ GEOPOTENTIAL COEFFICIENTS FROM SLR DATA OF LAGEOS SATELLITES

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ABSTRACT. SLR data of Lageos 1 and Lageos 2 satellites on 8-year time span have been processed to analyse long-term variations of $\bar{C}_{21}, \bar{S}_{21}$ geopotential coefficients. The first-degree harmonic coefficients $\bar{C}_{10}, \bar{C}_{11}, \bar{S}_{11}$ which are equivalent to the geocenter offsets and the corrections to $\bar{C}_{20}$ coefficient were also included in 10-day solutions together with orbital parameters and along-track accelerations of satellites. The aim of the work was to verify the adequacy of the dynamic pole tide formulation in the latest issue of IERS Conventions. Monthly averaged values of corrections to $\bar{C}_{21}, \bar{S}_{21}$ coefficients does not show explicit long-term variations. The analysis also allowed to determine corrections to the linear model based on the mean rotational pole path of the Earth.

1. INTRODUCTION

Determination of the geopotential coefficients $\bar{C}_{21}$ and $\bar{S}_{21}$ is essential for relating the Earth gravity field to the reference coordinate system. Monitoring of these coefficients can also provide with important information about the Earth core-mantle dynamics (Wahr,1987), (Wahr, 1991), (Greiner-Mai and Barthelmes, 2001), but for a long time the accessibility of their variations from satellite observations was not obvious (Gegout and Cazenave, 1993). A number of investigations of last years devoted to analysis of temporal variations of the geopotential contain results on coefficients $\bar{C}_{21}, \bar{S}_{21}$ as well (Cheng et al, 1997), (Eanes et al, 1997), (Pavlis, 2002).

In this connection it should be particularly mentioned the analysis by the research group from CSR of Texas university (Eanes et al, 1997), where the temporal variations of the second-degree geopotential coefficients were determined from SLR data of Lageos satellites. It was found that the $\bar{C}_{21}$ and $\bar{S}_{21}$ time series on 5-year time interval have a variability correlated to polar motion. This result could be explained by errors in the nominal model of the Earth rotational deformations. We were interested in analysing this effect since the following innovations have been made by IERS:

1) The formulation of mean $\bar{C}_{21}, \bar{S}_{21}$ coefficients and the rotational deformations of the Earth has been revised in IERS Conventions 2000 (McCarthy, 2000);

2) New geopotential model EGM96 (Lemoine et al., 1998) has been released and recommended by IERS Conventions instead of JGM3.

2. DATA ANALYSIS

It is expected that the implementation of the new satellite projects based on new observation techniques will result in essential increasing the accuracy of the geopotential coefficients up to level of 10^{-12}. However, this optimistic prognosis concerns mainly the coefficients in high-frequency area. As for the lowest degree harmonic coefficients first results from GRACE project (GRACE, 2003), (Reigber et.al., 2003) show (Table 1), that the accuracy of their determination is not better than in models derived by traditional satellite geodesy methods.

Table 1. Errors of low-degree harmonic coefficients
in the recent gravity models (units: 10^{-10}).

l m	EGM-96 (GSFC, NIMA)	GGM01-GRACE (CSR)	EIGEN-GRACE (GFZ)
2 0	0.36	2.65	3.11
2 1	-	0.77	1.57
2 2	0.54	0.85	1.26
3 0	0.18	0.45	0.44
3 1	1.40	0.64	0.52
3 2	1.11	0.90	0.71
3 3	0.95	1.24	0.69

Thus, the analysis of laser observations of geodynamic satellites on long time intervals can be still considered as a reasonable tool for determination of low-degree harmonic coefficients of the geopotential and their variations.

GRAPE program package (Gayazov et al, 2000) developed at the IAA for processing GPS and SLR data has been used for the data analysis. All dynamic and kinematic models in this package follow the IERS Conventions (McCarthy, 2000). Coefficients of the gravity model EGM96 (Lemoine et al, 1998) up to degree and order (20 x 20) were taken into account in orbital calculations. For accounting the ocean tide effects on satellite orbits the combination of EGM96S and GOT99.2b (Ray R., 1999) models has been used. Considering that the short-term tides are given more accurately in the GOT99.2b model, we used it for diurnal and semidiurnal tides, whereas the long-term ocean tide coefficients were taken from EGM96S model. Fixed IERS C04 series for Earth rotation parameters and ITRF2000 station coordinates were used in all calculations.

Lageos 1 and Lageos 2 laser observations on 8-year time span from 49800 MJD to 52800 MJD have been processed. The data analysis was performed in the following two steps:

1) adjustment of parameters for 10-day orbital arcs;

2) forming the series of 30-day averages of harmonic coefficients and their analysis.

The set of free parameters for each orbital arc included:

– Initial state vectors of satellites;

– Along-track accelerations;

– 6 harmonic coefficients $(\bar{C}_{10}, \bar{C}_{11}, \bar{S}_{11}, \bar{C}_{20}, \bar{C}_{21}, \bar{S}_{21})$.

Including of the first-degree coefficients to the set of adjusted parameters is natural, because they are responsible for translation of the gravity field to the Earth fixed system, while $\bar{C}_{21}$ and $\bar{S}_{21}$ coefficients determine its orientation.

The model values of coefficients $\bar{C}_{21}, \bar{S}_{21}$, were calculated according to IERS Conventions

$$\begin{aligned}\bar{C}_{21}(t) &= \bar{C}_{21}(t_0) + \dot{\bar{C}}_{21}(t - t_0) + \Delta\bar{C}_{21}(RD) + \Delta\bar{C}_{21}(TD),\\ \bar{S}_{21}(t) &= \bar{S}_{21}(t_0) + \dot{\bar{S}}_{21}(t - t_0) + \Delta\bar{S}_{21}(RD) + \Delta\bar{S}_{21}(TD),\end{aligned}$$

where mean values of coefficients are determined by

$$\bar{C}_{21}(t) = -2.23 \cdot 10^{-10} - 0.337 \cdot 10^{-11}(t - t_0),$$
$$\bar{S}_{21}(t) = 14.48 \cdot 10^{-10} + 1.606 \cdot 10^{-11}(t - t_0),$$

for the reference epoch $t_0 = 2000.0$ and t is time in years. Terms with RD and TD refer to rotational and tidal deformations correspondingly. Different components of these coefficients give rise to various perturbations in satellite orbits. The characteristic values of orbital perturbations for Lageos satellites are given in Table 2.

Table 2. Perturbations in Lageos orbits due to $\bar{C}_{21}, \bar{S}_{21}$ coefficients.

Components in $\bar{C}_{21}, \bar{S}_{21}$	Periods	Amplitudes
Constant, linear	1 d	10–12 cm
Long-term (rotational deformations)	1 d	5–6 cm
Short-term (lunar and solar tides)	10–6000 d	up to 500 m

When analysing the observations on 10-day orbital arcs we intended to use the sensitivity of short-term perturbations to linear and long-term components of the coefficients and determined corrections to their model values.

3. RESULTS

Common characteristics of 10-day arc solutions are summarised in Table 3.

Table 3. Summary of SLR data analysis.

Number of observations per arc	1500 - 5000
RMS of SLR data residuals	2 - 6 cm
Formal errors of determined coefficients	$(1-5) \cdot 10^{-11}$

Figures 1 and 2 present monthly averages of determined corrections to $\bar{C}_{21}, \bar{S}_{21}$ coefficients and results of their spectral analysis. As it could be seen from Fig. 2, there are no dominating peeks in the vicinity of chandler period. It can be considered as a result of adequate accounting the effect of rotational deformations according to the latest IERS Conventions.

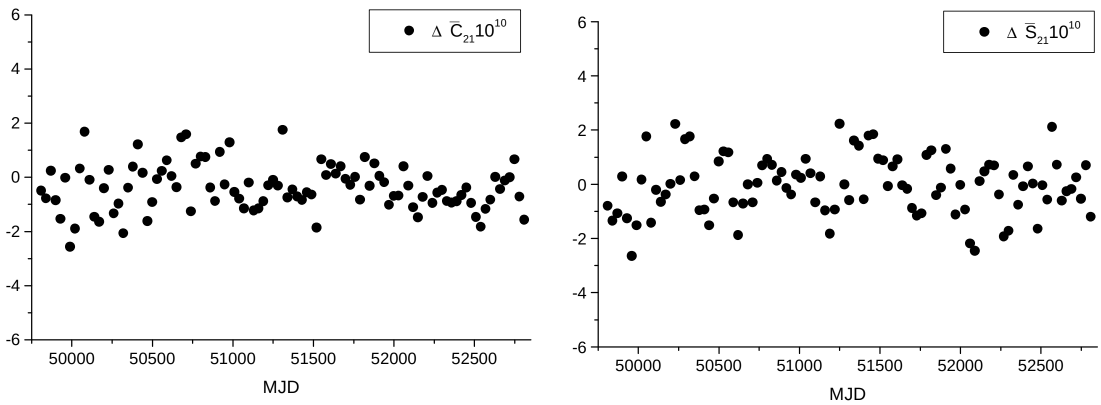

Figure 1: Monthly averages of corrections to $\bar{C}_{21}, \bar{S}_{21}$. coefficients

Averaging results on the 8-year time interval we have found the following corrections to the mean coefficients at epoch 2000.0

$$\Delta\bar{C}_{21}(2000.0) = (-4.1 \pm 0.8) \cdot 10^{-11},$$
$$\Delta\bar{S}_{21}(2000.0) = (-0.5 \pm 1.0) \cdot 10^{-11},$$

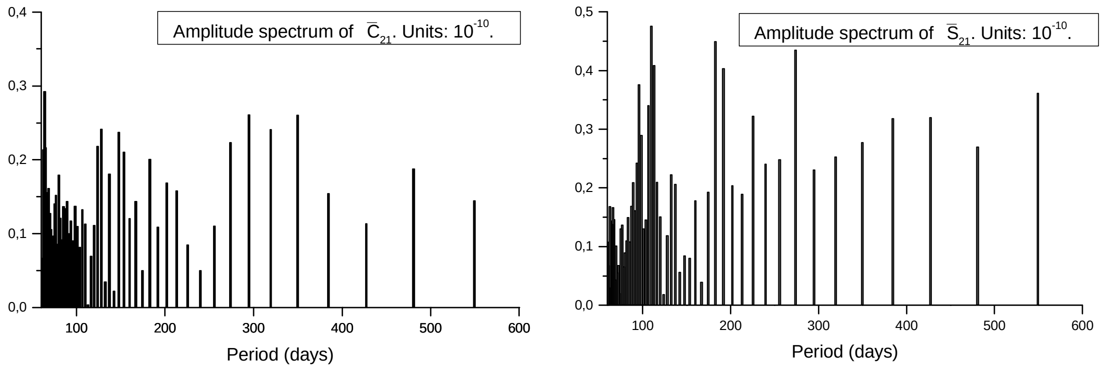

Figure 2: Amplitude spectra of $\Delta\bar{C}_{21}$ and $\Delta\bar{S}_{21}$.

and to their rates

$$\Delta\dot{\bar{C}}_{21} = (-0.26 \pm 0.23) \cdot 10^{-11}\ y^{-1},$$
$$\Delta\dot{\bar{S}}_{21} = (+0.08 \pm 0.29) \cdot 10^{-11}\ y^{-1}.$$

Obtained coefficients as compared with other recent results are presented in Fig. 3.

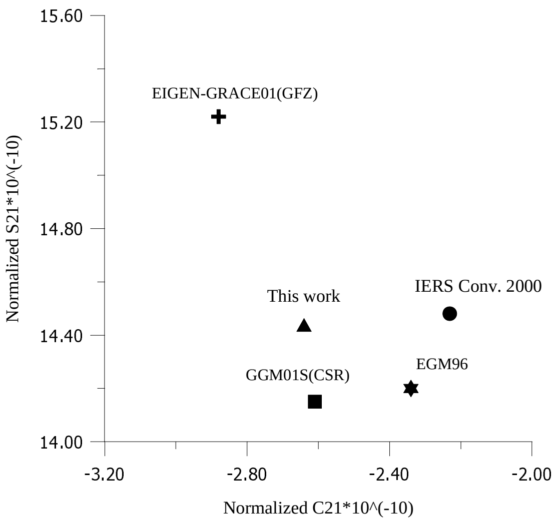

Figure 3: Comparing results for $\bar{C}_{21}, \bar{S}_{21}$ coefficients.

It should be mentioned that the coefficients $\bar{C}_{21}, \bar{S}_{21}$ derived from the satellite motion analysis correspond to the mean figure axis but not to the mean rotation pole. Using the relation between mean pole coordinates and $\bar{C}_{21}, \bar{S}_{21}$ (McCarthy, 2000) we transformed our coefficients to linear trends in the mean figure axis. They are shown in Fig. 4 against the background of mean rotation pole path from IERS Conventions. The significant difference in X-coordinate of the figure axis can be analysed in further investigations.

We also present here results for the geocenter offsets T_x, T_y, T_z. They were calculated from our series of harmonic coefficients $\bar{C}_{11}, \bar{S}_{11}, \bar{C}_{10}$ using the following well known relations

$$T_x = \sqrt{3}R_e\bar{C}_{11},$$
$$T_y = \sqrt{3}R_e\bar{S}_{11},$$
$$T_z = \sqrt{3}R_e\bar{C}_{10},$$

where $R_e = 6.378 \cdot 10^9$ mm.

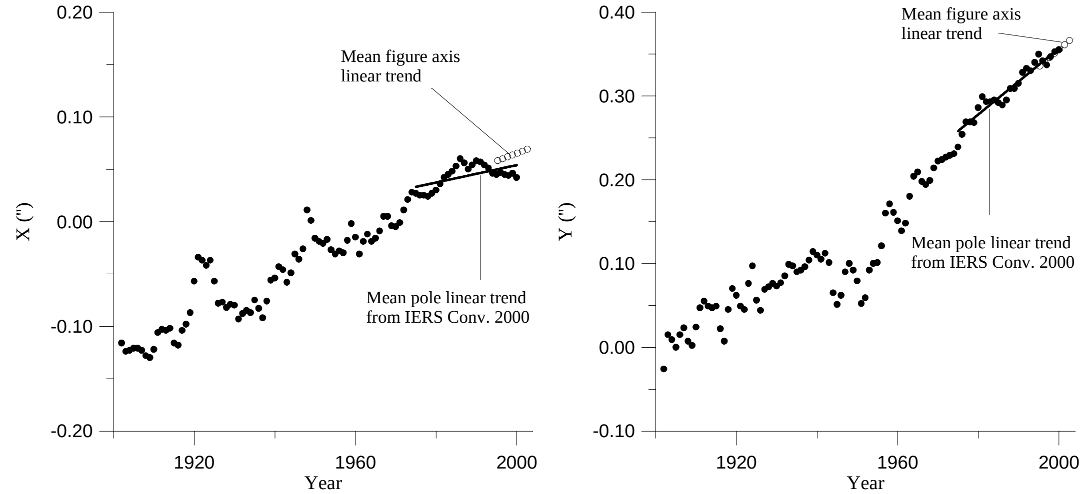

Figure 4: Coordinates of the determined mean figure axis and the IERS mean pole.

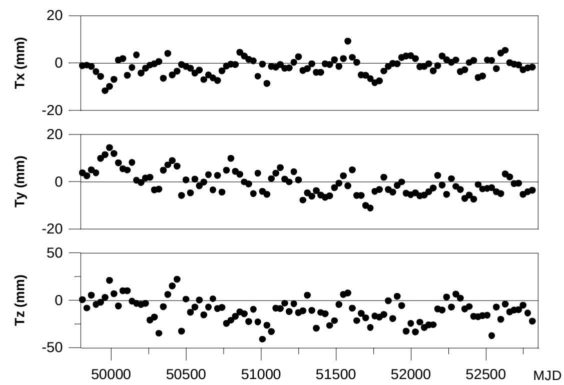

Figure 5: Geocenter offsets.

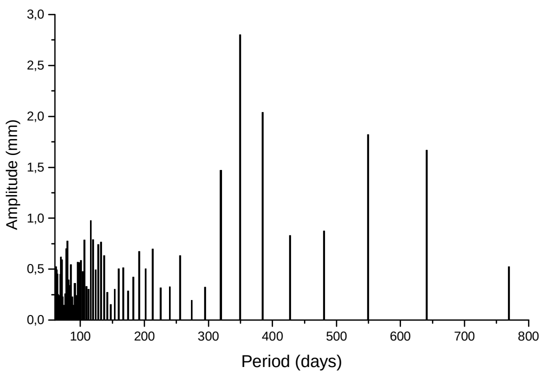

Figure 6: Amplitude spectrum of T_x.

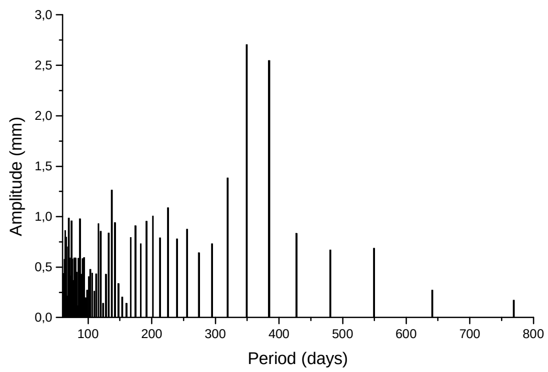

Figure 7: Amplitude spectrum of T_y.

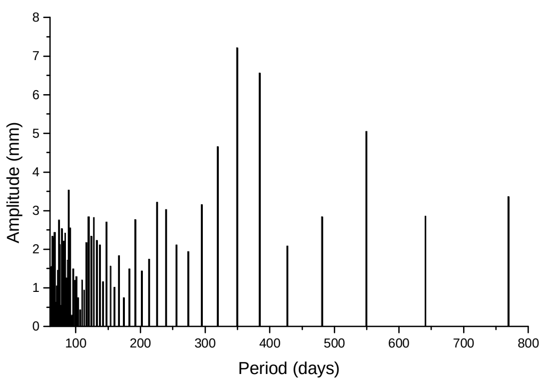

Figure 8: Amplitude spectrum of T_z.

Monthly averaged geocenter offsets are presented in Fig. 5 and their amplitude spectra are shown in Fig. 6 - 8. Amplitudes of annual period found from this analysis are 2.5 ± 0.4 mm, 2.8 ± 0.5 mm, 7.1 ± 1.6 mm for T_x, T_y, T_z correspondingly. They are in good agreement with other results obtained during the geocenter motion analysis campaign (Ray J., 1999).

4. REFERENCES

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