

# Analysis of Chandler wobble excitation, reconstructed from observations of the polar motion of the Earth

Leonid Zotov

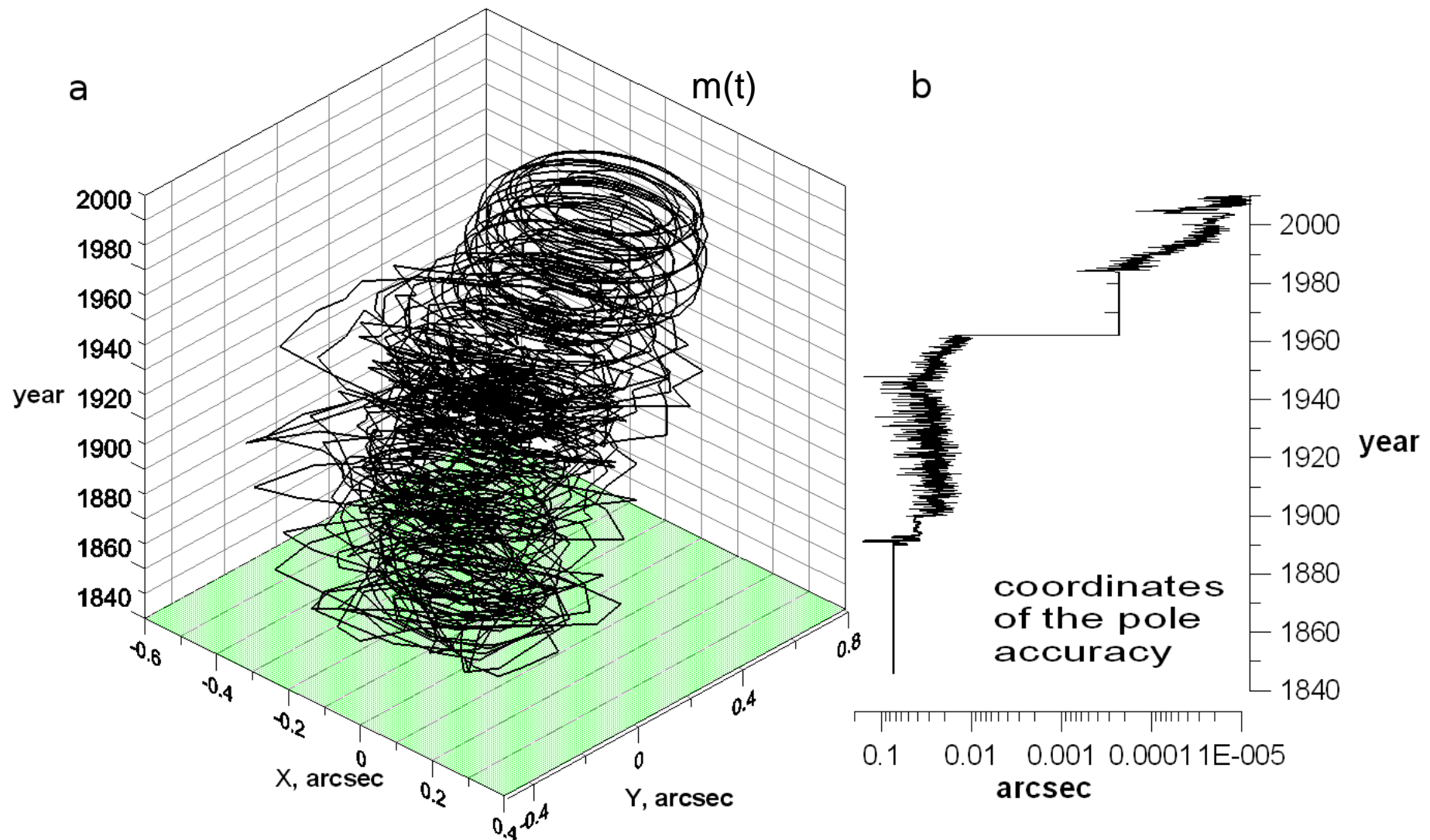
[wolftempus@gmail.com](mailto:wolftempus@gmail.com)

**Sternberg Astronomical Institute  
Lomonosov Moscow State University**

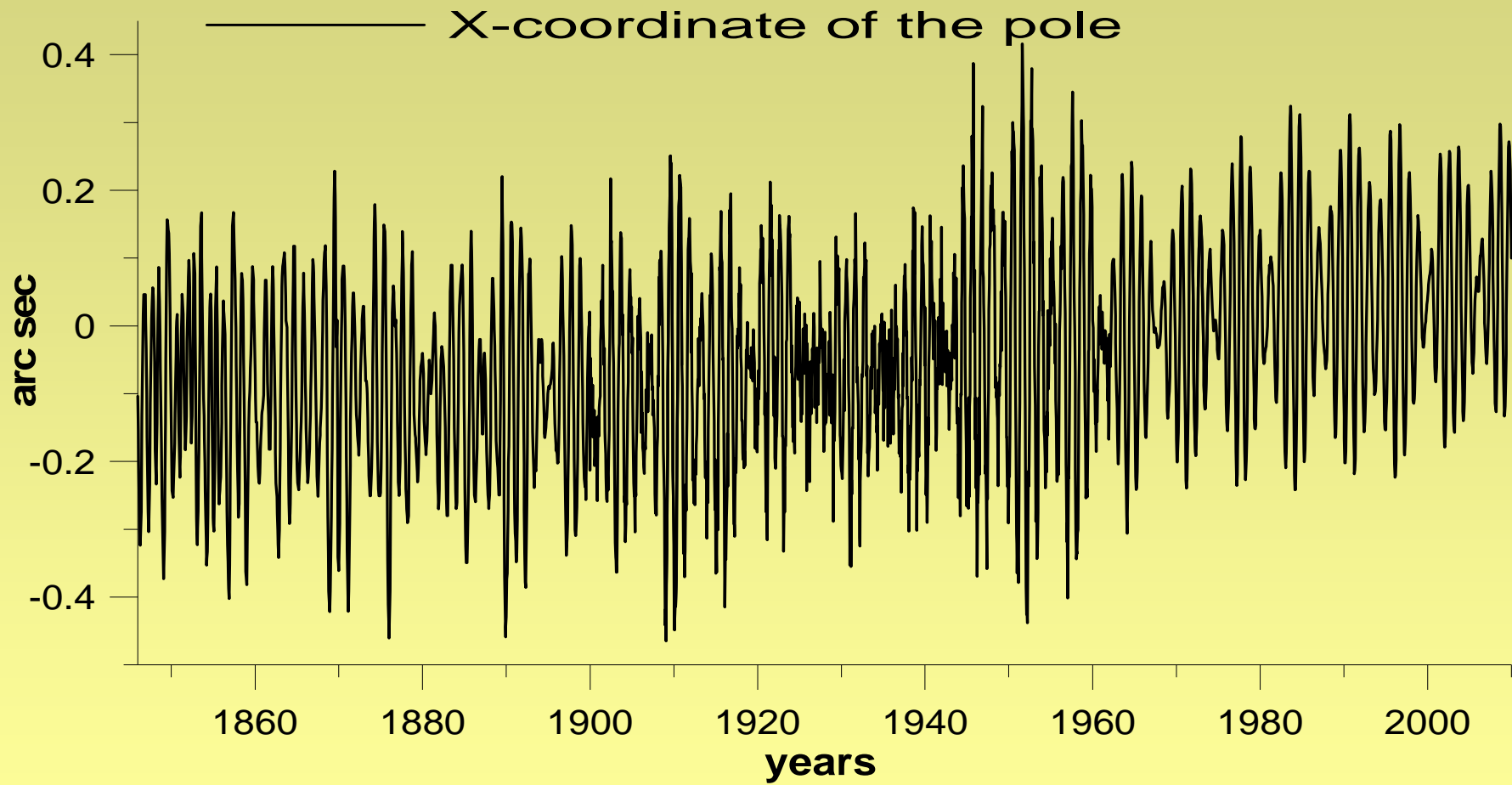
Journées 2010, September 20 – 22, 2010, Paris Observatory

# Earth's pole motion from the bulletin IERS EOP C01

Since 1846 yr, step 0.05 yr

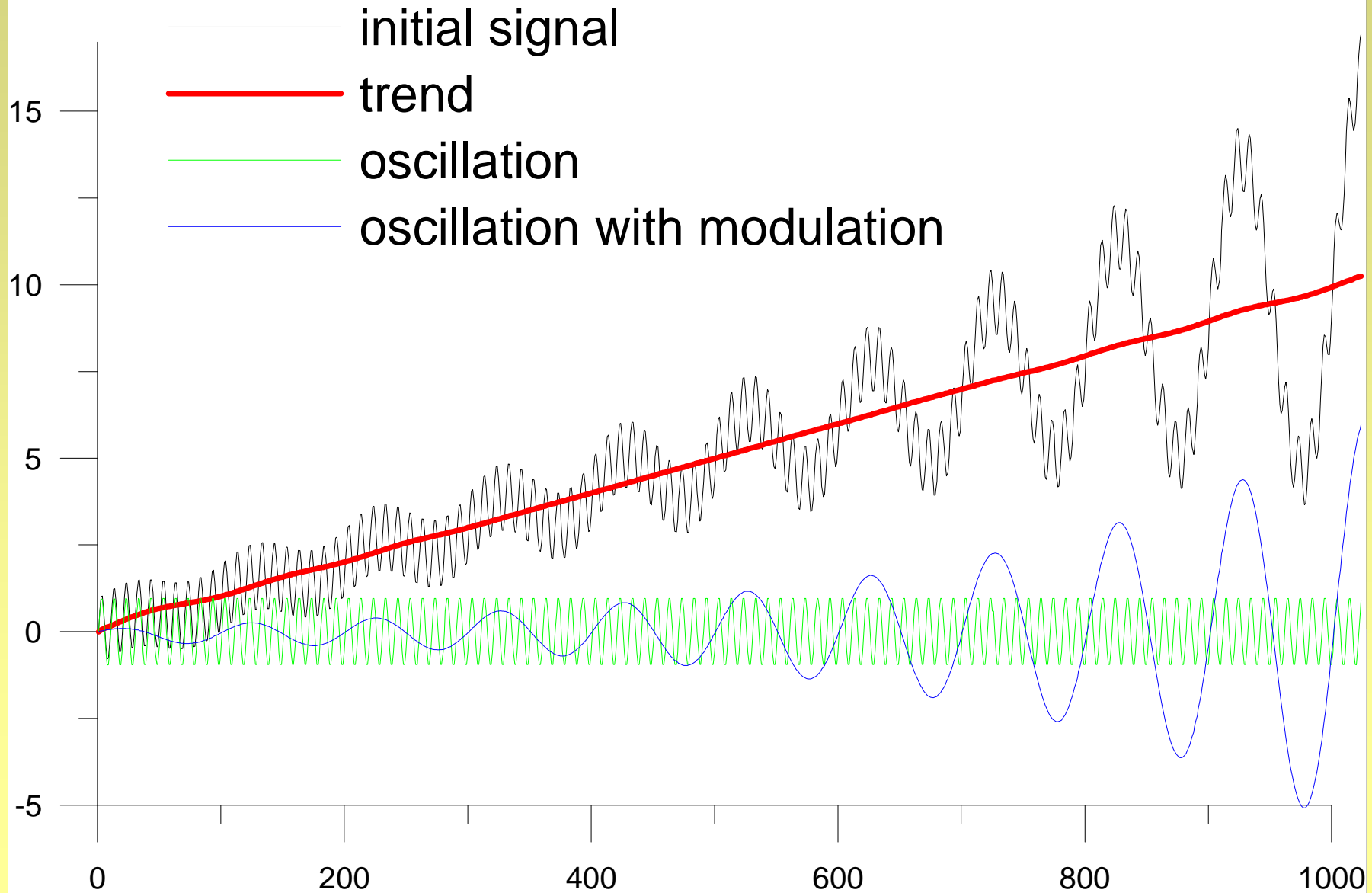


# X-coordinate

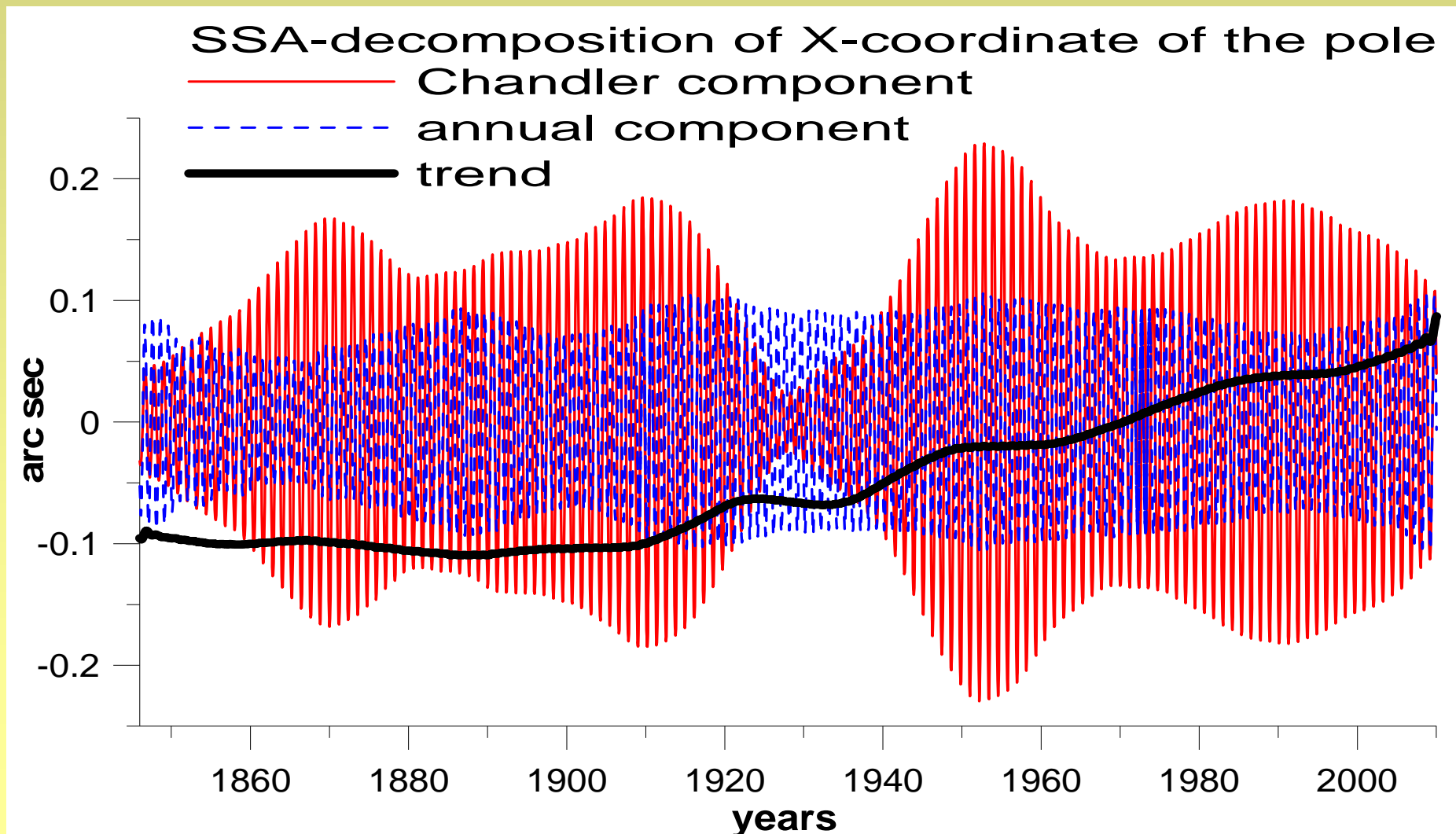


# SSA method – “caterpillar”

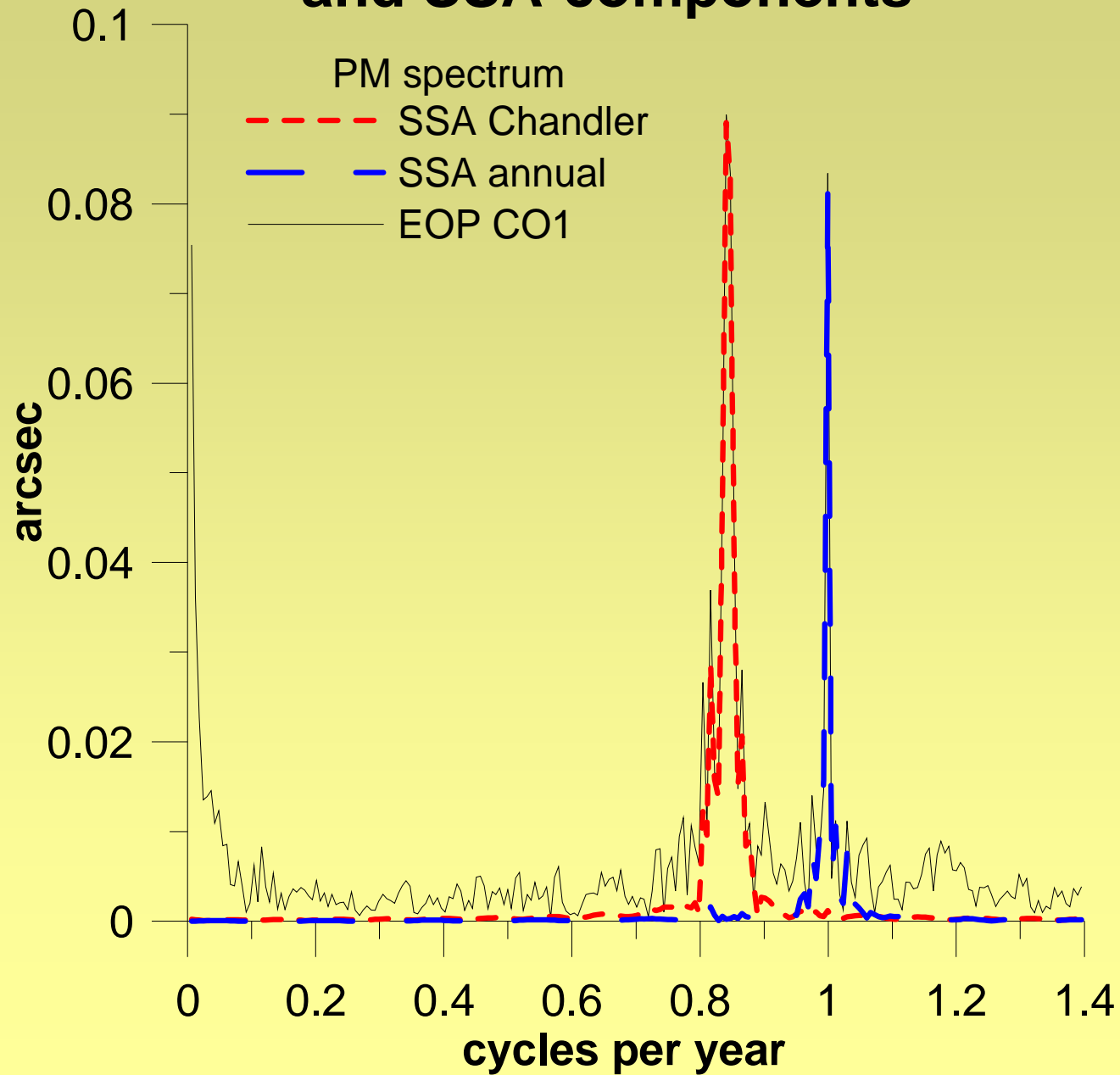
## SSA example



# Result of SSA of the 2-D pole coordinate time series on the example of X-coordinate



# Amplitude spectrum of initial signal and SSA-components



## Dynamical system of the rotating Earth

$$\frac{i}{\sigma_c} \frac{dm(t)}{dt} + m(t) = \chi(t)$$

$$\sigma_c = 2\pi f_c (1 + i/2Q)$$

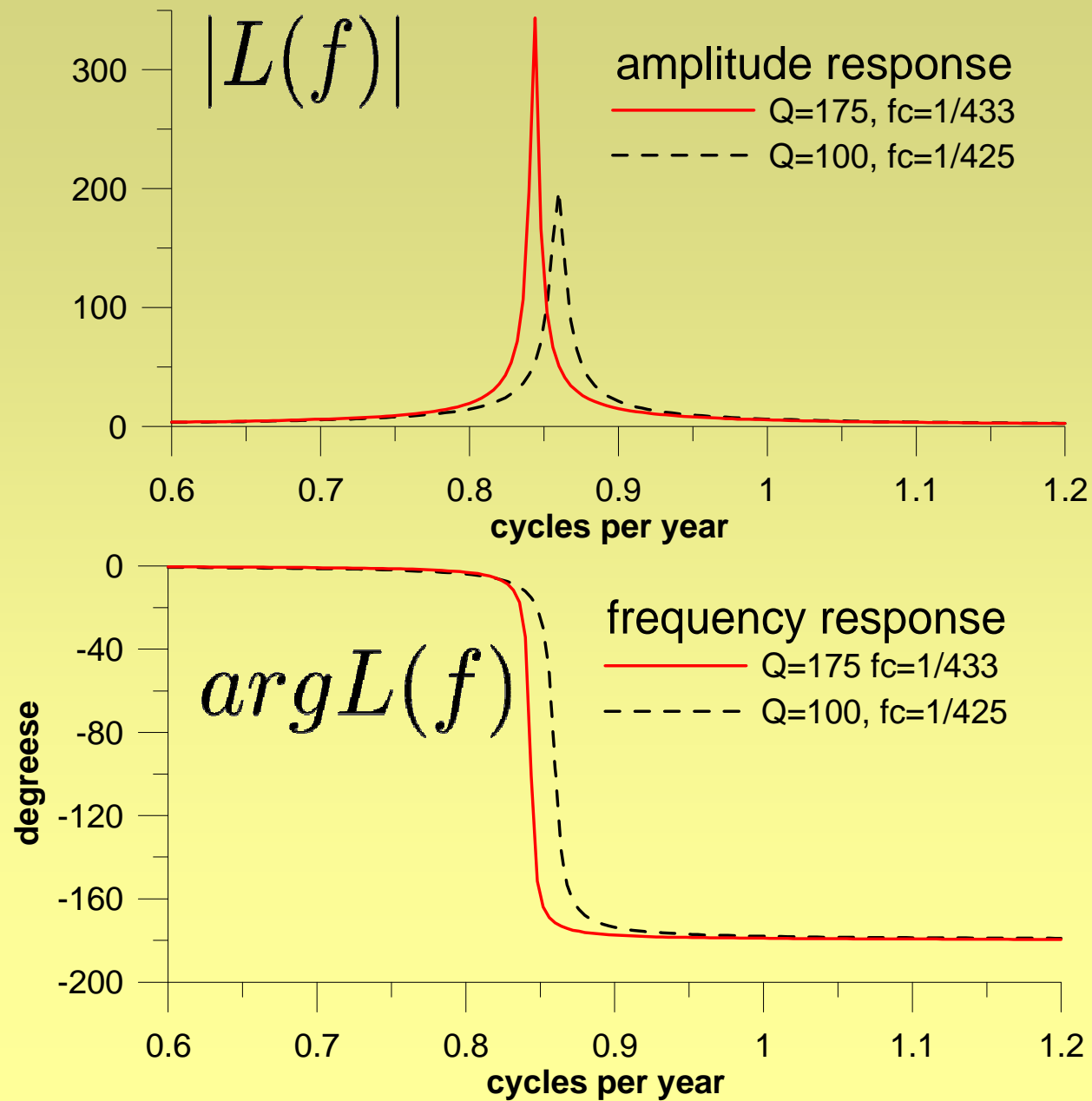
To find  $\chi(t)$  - inverse problem

$$\hat{m}(f) = L(f) \cdot \hat{\chi}(f)$$

$$L(f) = \frac{\sigma_c}{\sigma_c - 2\pi f}$$

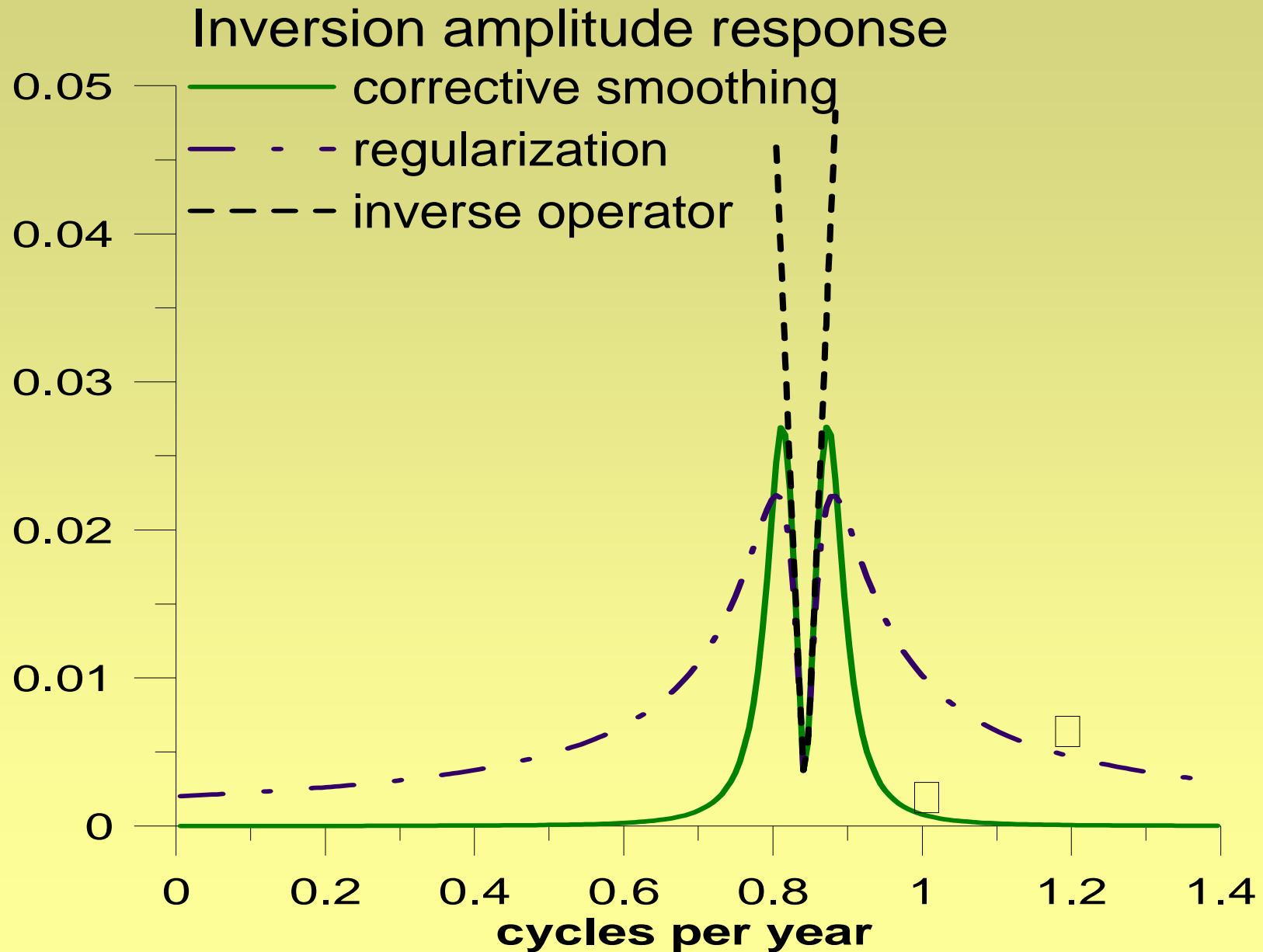
Parameters used  $f_c = \frac{1}{433}$  days<sup>-1</sup>  $Q = 175$

# Frequency response





# Amplitude frequency response of inverse operators



## Wilson-Jeffreys filter

$$\chi(t) = \frac{ie^{-i\pi f_c \Delta t}}{\sigma_c \Delta t} \left[ m_{t+\frac{\Delta t}{2}} - e^{i\sigma_c \Delta t} m_{t-\frac{\Delta t}{2}} \right]$$

$$m = m_X + im_Y$$

$$m_X = \sin X$$

$$m_Y = -\sin Y$$

Jeffreys H. (1940) The variation of latitude, Mon Not Roy Astr. Soc., Vol. 100, 139-155  
Wilson C. (1985), Discrete polar motion equations, Geophys J. Roy. Astr. Soc., Vol. 80, 551-554

# Tikhonov regularization



$$L_{reg}^{-1}(f) = \frac{L^*(f)}{L^*(f)L(f) + \alpha}$$

parameter chosen

$$\alpha = 500$$

LSA-model of the annual oscillation

	amplitude	phase for 1846.0
X-coordinate	$0.088'' \pm 0.005''$	$231^\circ \pm 3^\circ$
Y-coordinate	$0.078'' \pm 0.006''$	$148^\circ \pm 4^\circ$

# Pantelev corrective smoothing



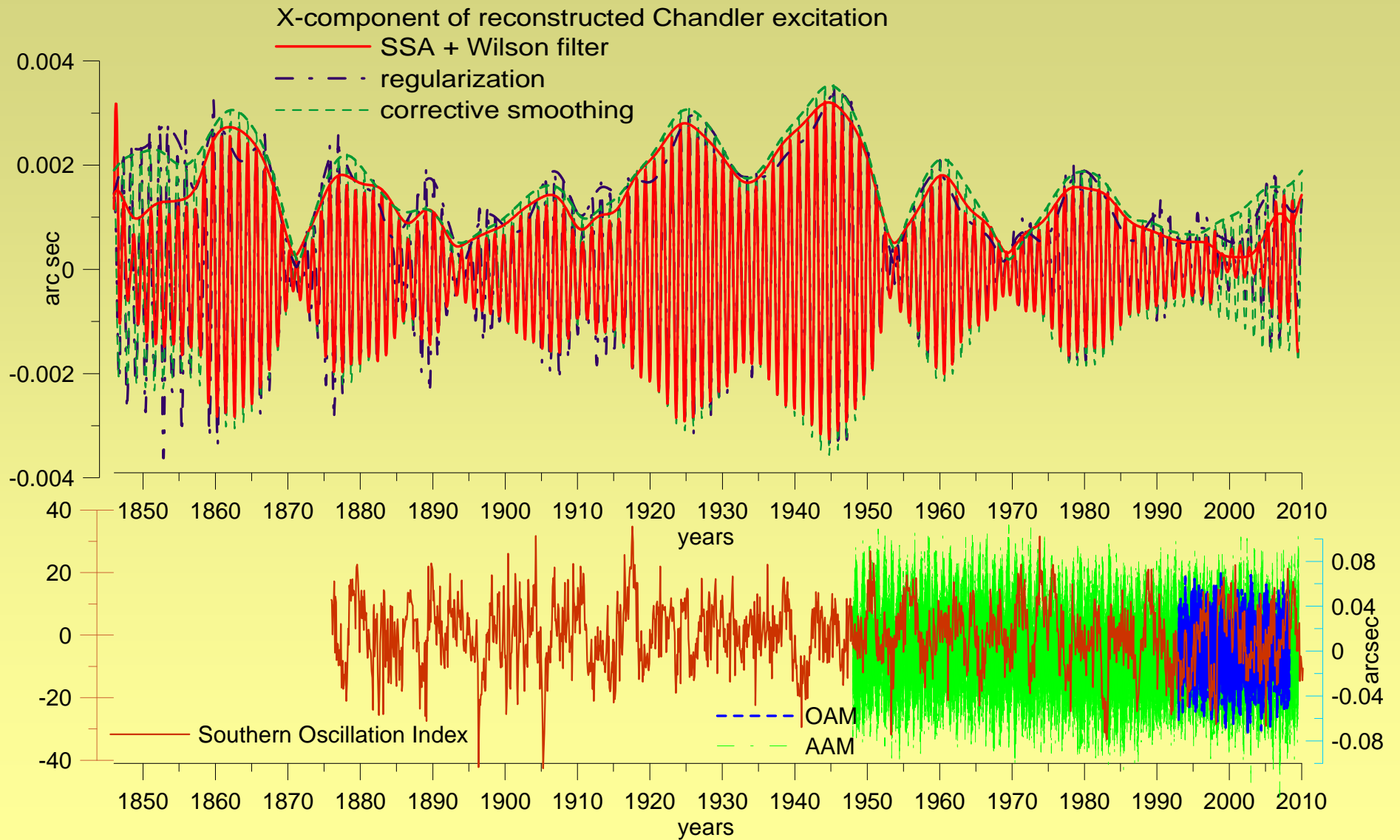
$$L_{corr}^{-1}(f) = \frac{L_{filter}(f)}{L(f)}$$

$$L_{filter}(f) = \frac{f_0^4}{(f - f_c)^4 - f_0^4}$$

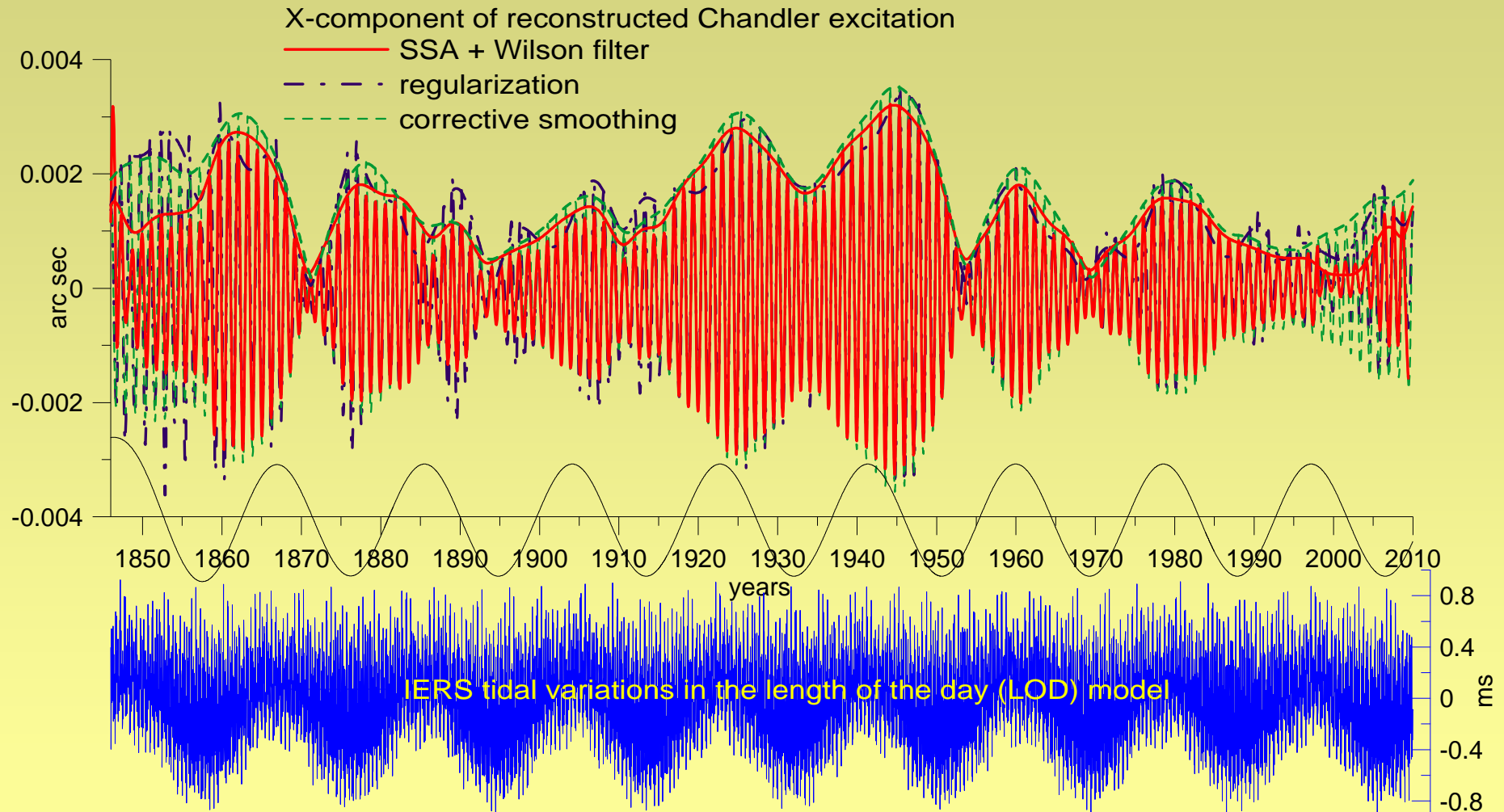
Parameter used

$$f_0 = 0.04$$

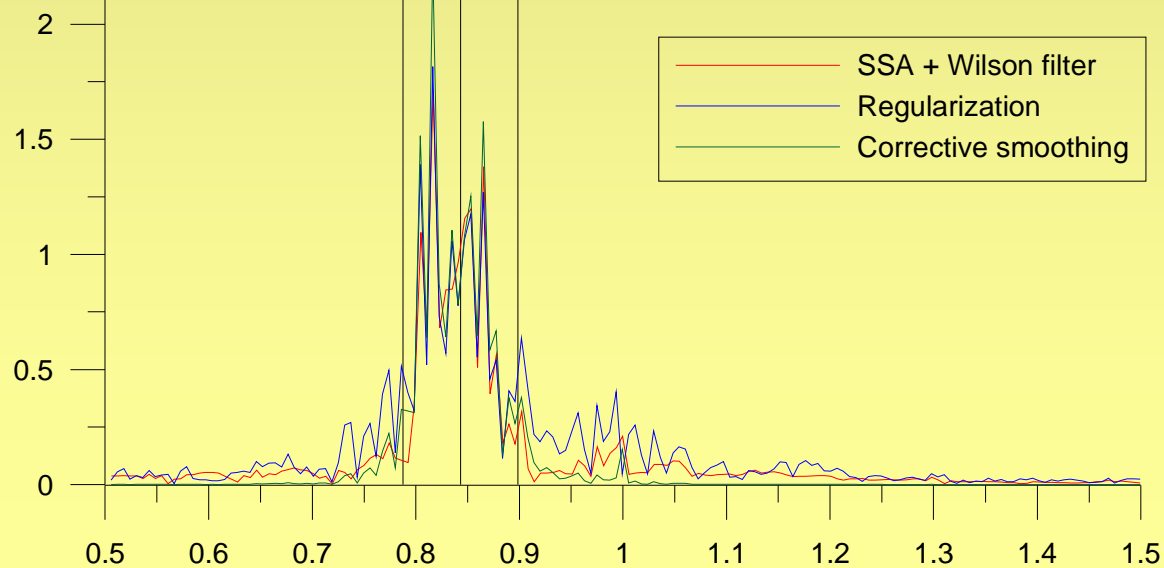
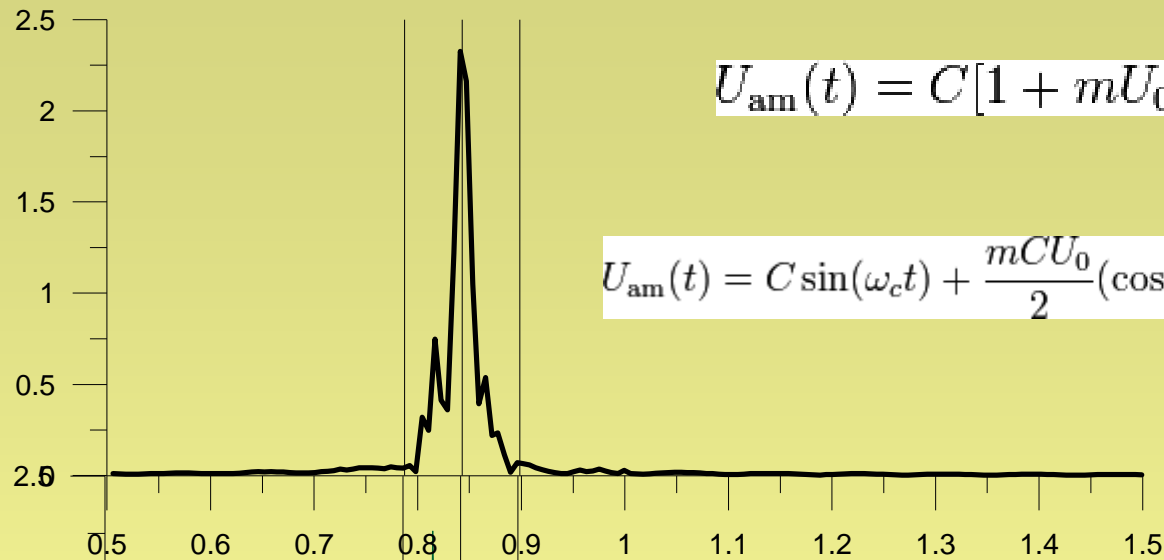
# Comparison with different processes



# Chandler excitation and Tidal model for the Length of day LOD

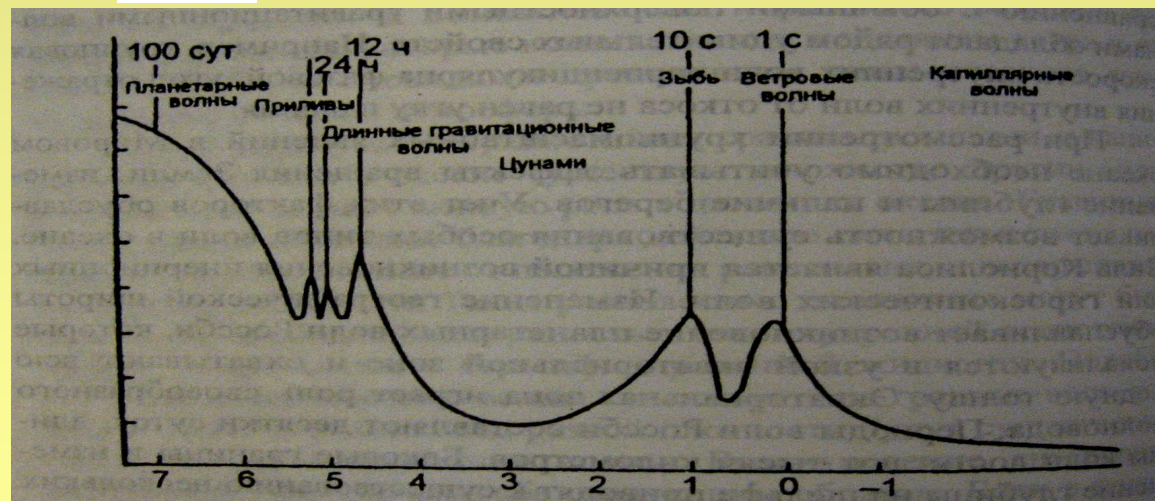
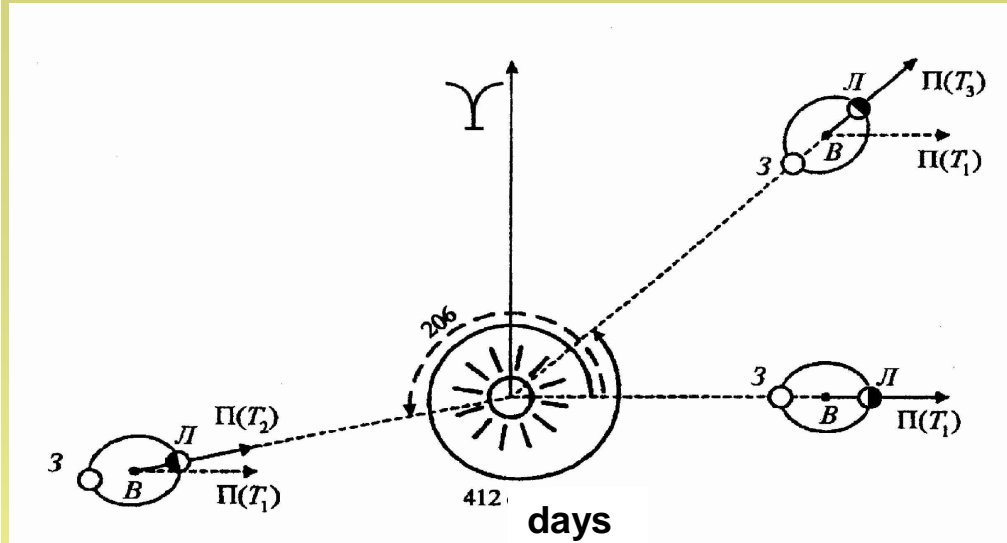


# Spectrum of reconstructed excitation, amplitude modulation





# Full moon in perigee repetition cycle and ocean



Avsyuk Yu. N. Tidal forces and natural processes, 1996, Moscow, Schmidt Institute of Physics of the Earth, Russian Academy of Sciences.

Anisimova E P., Pokazeev K.V. Introduction to physics of hydrosphere. Moscow, MSU, 2002.

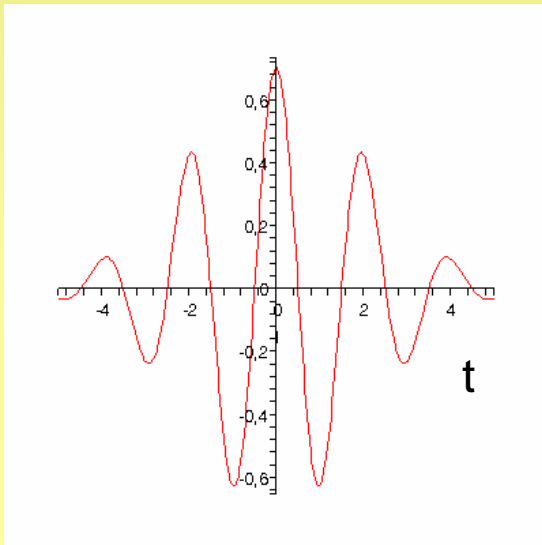


# Gabor window-transform

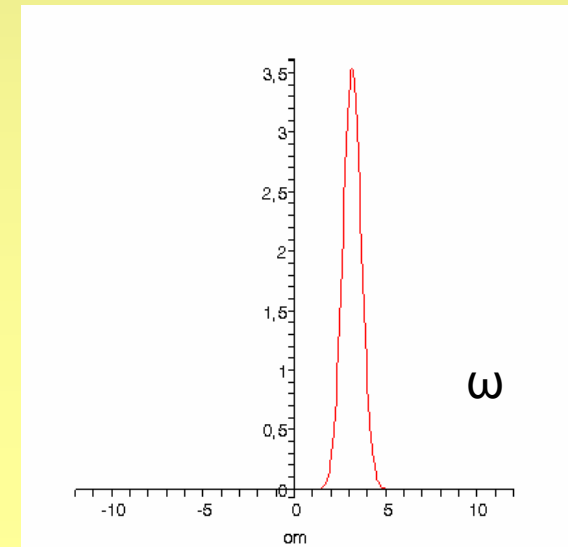
$$S_g f(\omega, t) = \int_{-\infty}^{\infty} f(\tau) g_{\omega, t}^* d\tau = \int_{-\infty}^{\infty} f(\tau) g(\tau - t) e^{-i\omega\tau} d\tau$$

$$g_{\omega, t} = g(\tau - t) e^{i\omega\tau}$$

$$S_g f(\omega_0, t) = \frac{1}{e^{i\omega_0 t}} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) e^{i\omega_0(t - \tau)} d\tau$$



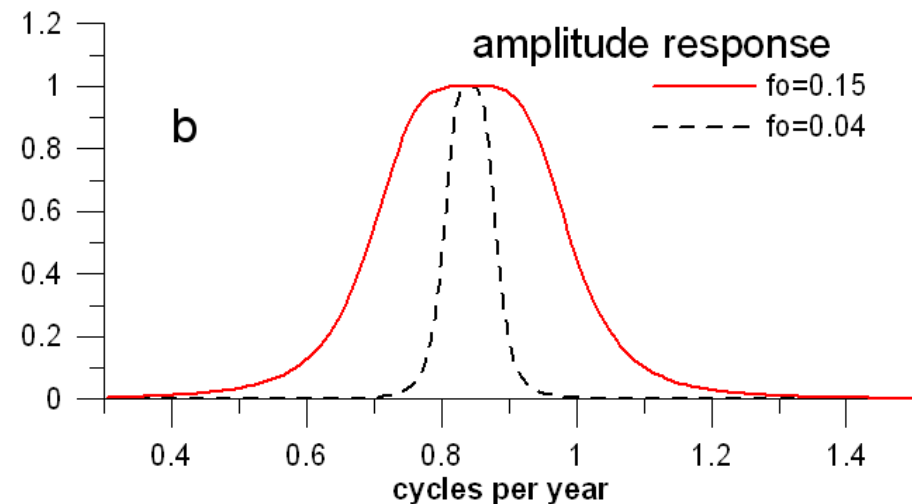
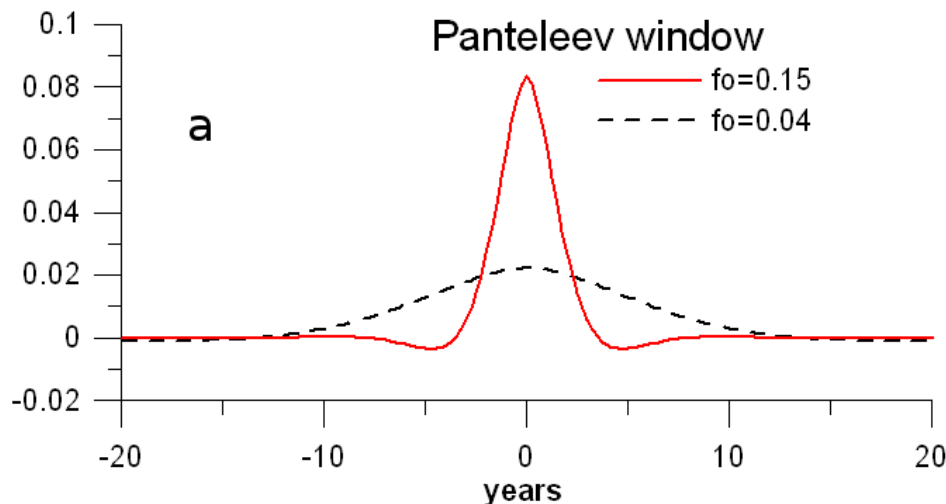
$$h_{\omega_0, t} = g(t) e^{i\omega_0 t}$$



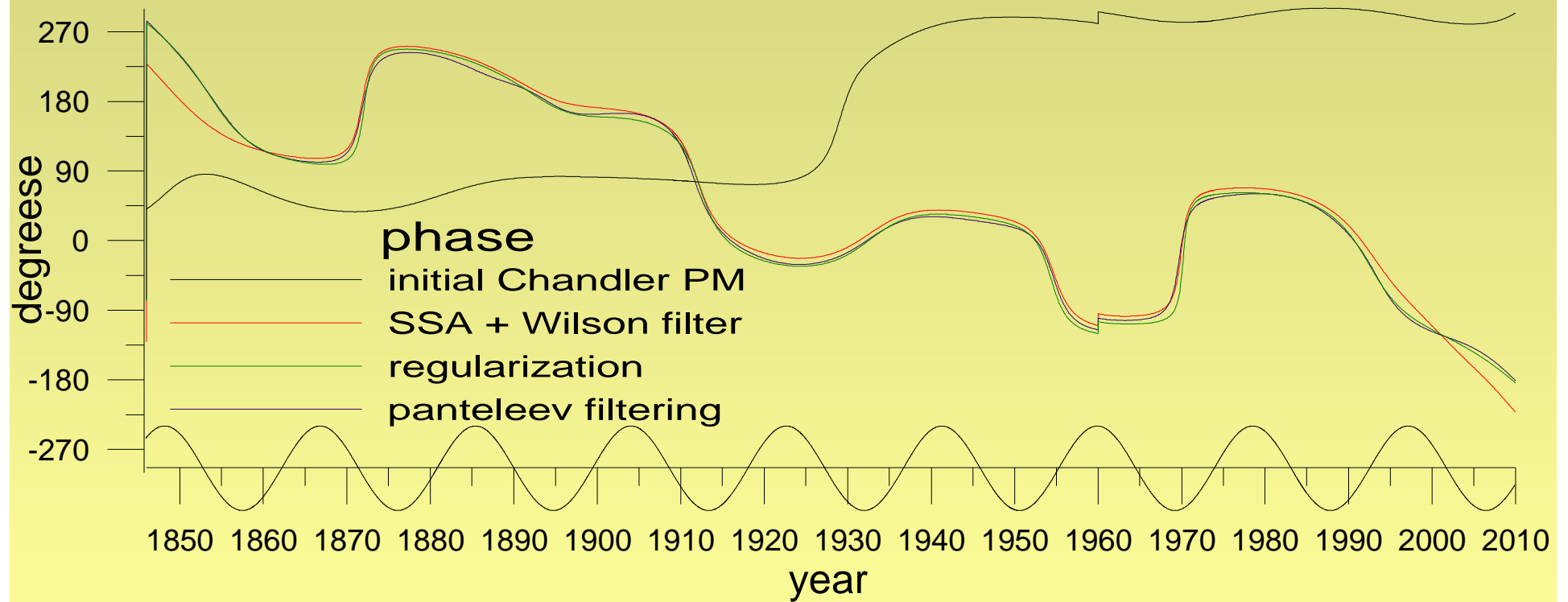
# Pantelev filters impulse response

$$h(t) = \frac{\omega_0}{2\sqrt{2}} e^{-\frac{\omega_0 |t|}{\sqrt{2}}} \left( \cos \frac{\omega_0 t}{\sqrt{2}} + \sin \frac{\omega_0 |t|}{\sqrt{2}} \right)$$

$$\omega_0 = 2\pi f_0$$



# Phase changes



# Conclusions

1) Chandler excitation was reconstructed by three methods for inverse problems solving:

Panteleev corrective smoothing,

Wilson-Jeffreys filter after SSA,

Tikhonov regularization with annual component subtraction

The results are similar. It gives hope they are reliable.

2) 18,6-year modulation of Chandler excitation, synchronous with the saros tidal cycle and related LOD changes has been found. It could prove that tidal energy is transferred to chandler excitation, probably, through the ocean and atmosphere. The mechanism is still to be found.

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