Post-Newtonian Mechanics of the Earth-Moon System

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Outline

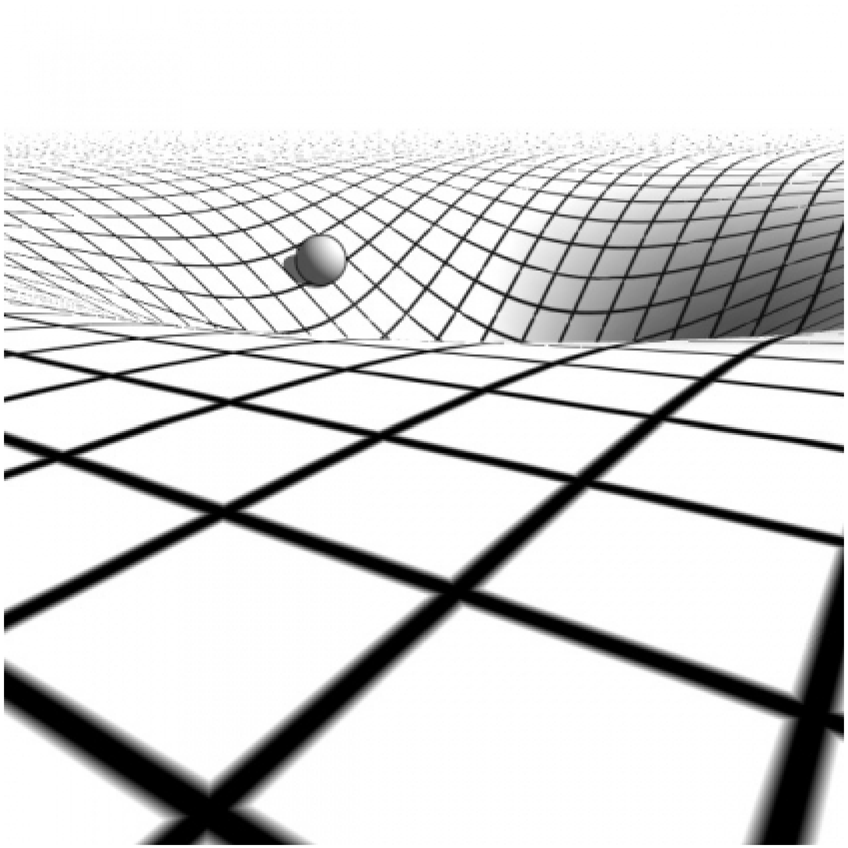
- Motivations
- Post-Newtonian reference frames
- Post-Newtonian mechanics
- Outlook

Motivations

- Distinguish physics from coordinates
- Comprehensive and self-consistent theory of PN Equations of Motion
- Current and future high-precision experiments

Motivations

- Distinguish physics from coordinates



Curved spacetime:

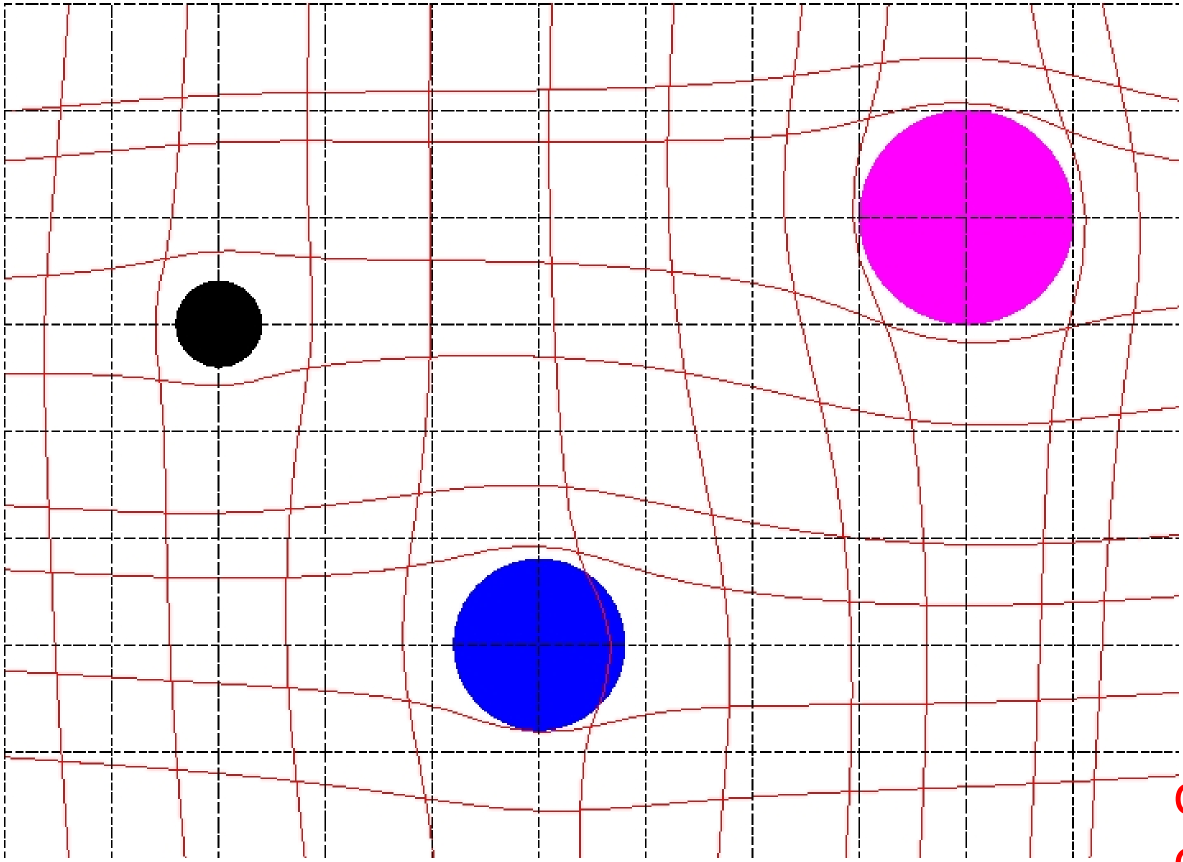
Coordinates

Strongly coupled
Difficult to disentangle

Physics

Extrinsic gravitomagnetic effects:

PPN-EIH: $$\vec{a}_i = \vec{a}_i^{\text{Newton}} + \frac{(2\gamma+2)}{c^2} \sum_{j\neq i} \vec{v}_i \times \left(\vec{v}_j \times \frac{GM_j}{r_{ij}^3} \vec{r}_{ji} \right) + \cdots$$



$$t' = t - \frac{1}{c^4} \sum_B \nu_B \frac{GM_B}{r_B} (\vec{r}_B \cdot \vec{v}_B)$$

$$\vec{x}' = \vec{x} - \frac{1}{c^2} \sum_B \lambda_B \frac{GM_B}{r_B} \vec{r}_B$$

gravitomagnetic force:

$$\sum_{j\neq i} \frac{(2\gamma+2-\lambda_j)}{c^2} \vec{v}_i \times \left(\vec{v}_j \times \frac{GM_j}{r_{ij}^3} \vec{r}_{ji} \right)$$

depends on the choice of coordinates λ_j.

Motivations

- Comprehensive theory of PN EoM
  - Einstein-Infeld-Hoffman: *N*-point particle
  - Barker & O'Connell: spin (point particle)
  - Brumberg-Kopeikin: *N*-body (Newtonian multipoles+spins)
  - Damour-Soffel-Xu: *N*-body (PN multipoles+spins in GR)
  - Kopeikin & Vlasov: *N*-body (PN multipole+spins in STT)
  - Racine & Flanagan: extend BK+DSX (flux integral method)
  - Xie & Kopeikin: Earth-Moon reference frames (offprints available)
- Our goals:

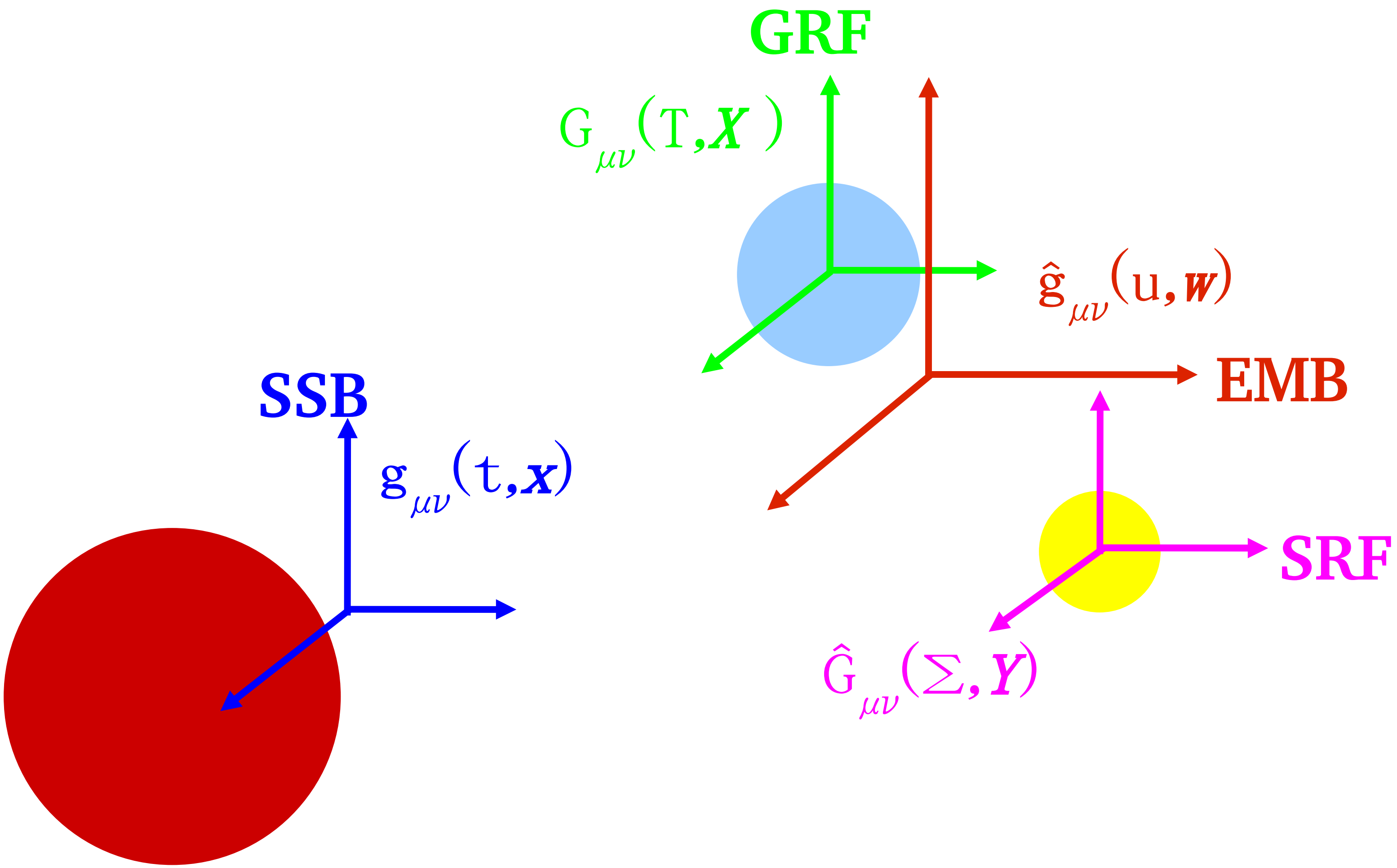
PN EoM with multipoles expressed explicitly with coordinates and velocities of bodies

$$M_B \boldsymbol{a}_B = \boldsymbol{F}\ (\boldsymbol{x}_C, \boldsymbol{v}_C; \boldsymbol{I}_C^{<L>}, \boldsymbol{S}_C^{<L>}; \gamma, \beta)$$

Motivations

- Current and future high-precision experiments:
  - Lunar Laser Ranging
    relativisitic parameters
    selenophysics: liquid core?
  - Pulsar timing
  - Gravitational wave astronomy:
    Earth-Moon system as a detector of GWs

PN Reference Frames



PN Reference Frames

- A global frame

  Solar System Barycentric Frame
  orbital motions

- *N* local frames

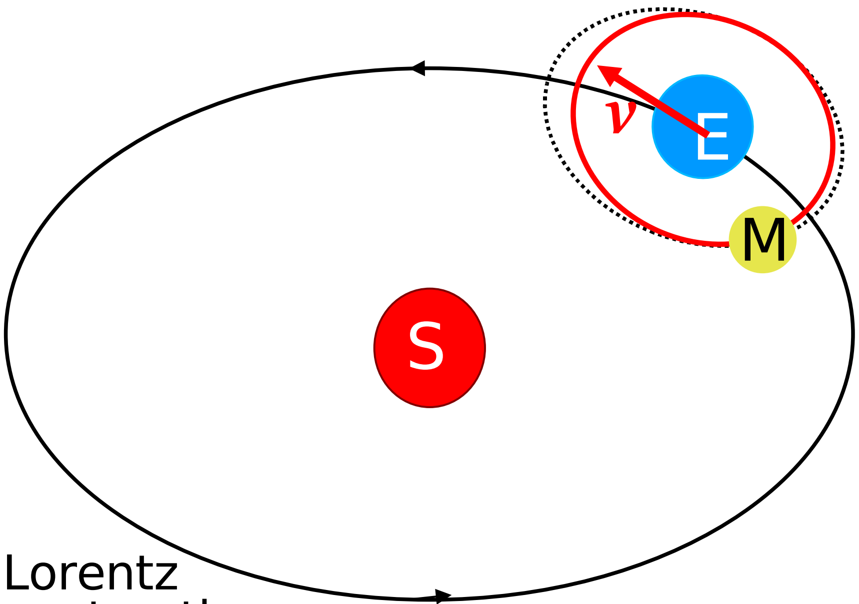
  eg. Geocentric Frame
  internal motions of matter, time evolution of multipoles

- *Some* local frames for subsystems

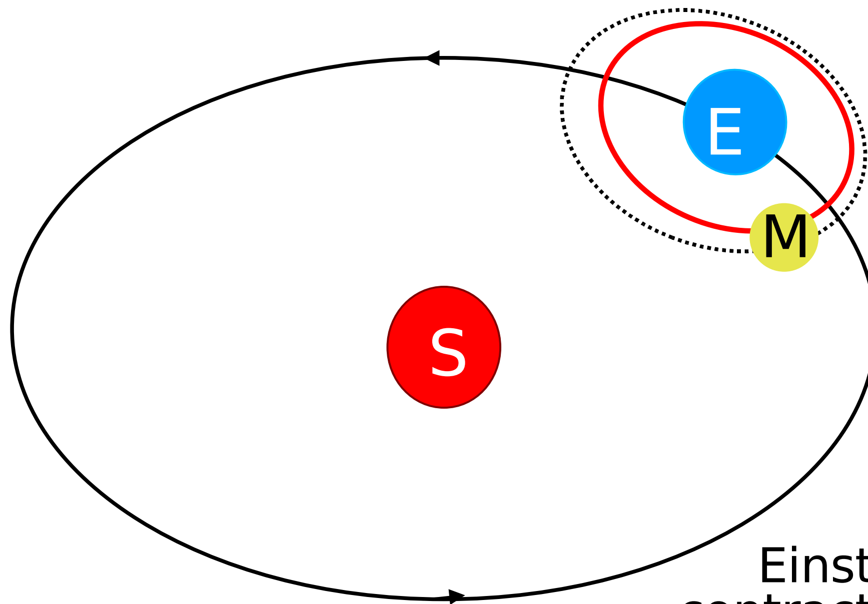
  eg. the Earth-Moon Barycentric Frame
  relative motions for LLR

Why local frames?

- Multipole moments



Lorentz contraction
~2 m
(Lunar Orbit)



Einstein contraction
~4 m
(Lunar Orbit)

Earth+Moon as a composite body

Why to use the local frame for subsystems?

- Hierarchical systems: extension of IAU2000 resolutions

  Universe -> superclusters -> clusters -> groups

  -> galaxies -> stellar clusters -> stellar systems

  -> planetary systems

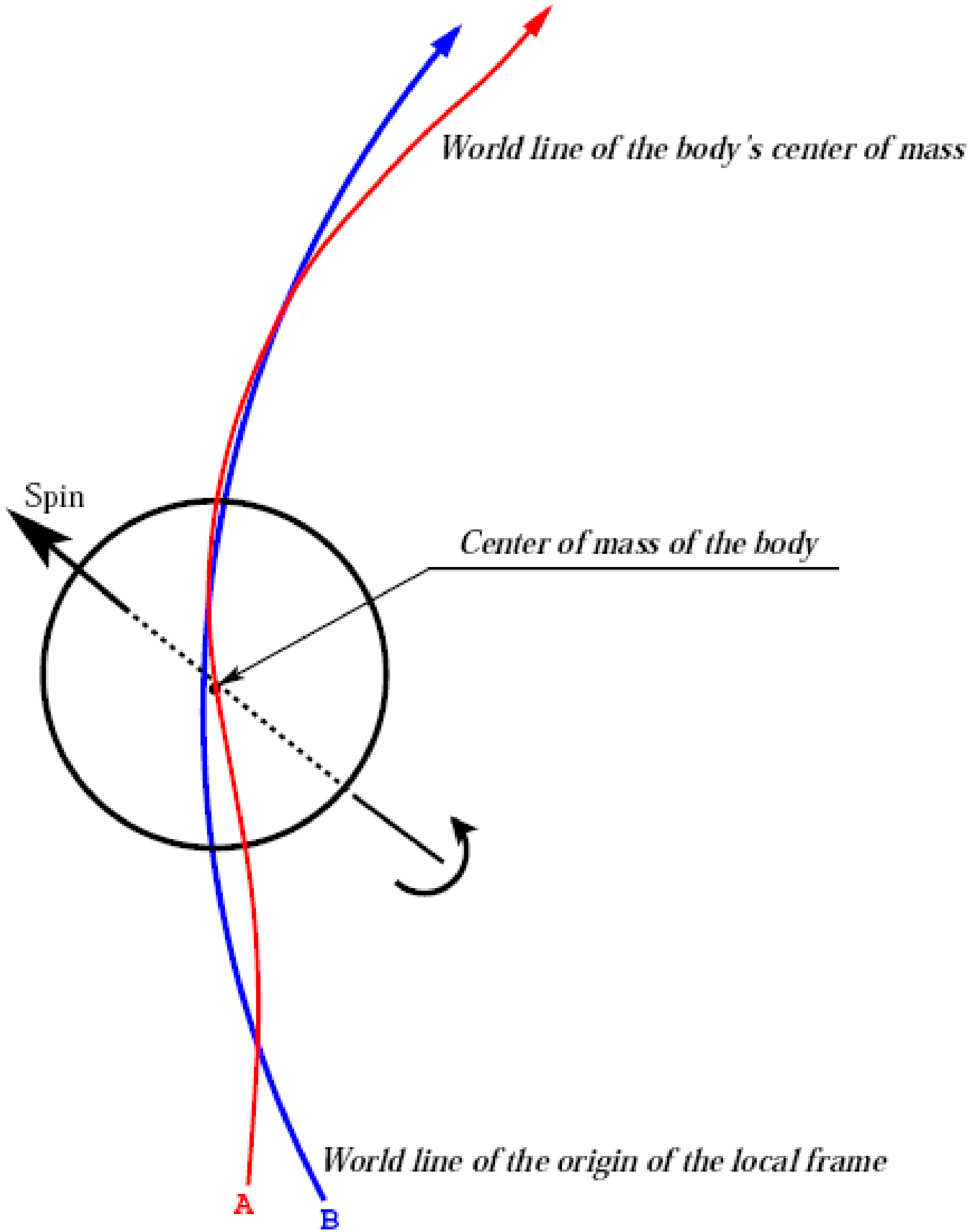
- Lunar mass is *not* negligible: M/E~1/80

  The Geocentric Frame can *not* cover the Moon.

- LLR is a *local frame* experiment

  EMB frame helps to understand the gauge freedom

  of equations of motion.



Momentum centered

A coincides with B
(a constraint on dipole)

$d\boldsymbol{I}/du = 0$

$\boldsymbol{I} = \boldsymbol{P}u + \boldsymbol{I}_0$

$\boldsymbol{I} = 0$

PN Mechanics

- PN orbital motion (external problem)

  the body B's motion in the SSB frame

- PN relative motion (internal problem)

  Moon with respect to Earth in the EMB frame

PN orbital motion

- Conservation of dipole moment: $d^2\boldsymbol{I}/du^2=0$

$$\dot{\mathscr{P}}^i = \mathscr{M} Q_i(u) + \sum_{l=1}^{\infty} \frac{1}{l!} Q_{iL}(u) \mathscr{I}^L(u) + \epsilon^2 \Delta \dot{\mathscr{P}}^i \quad \dots \dots$$

- Matching procedure: coordinate transformation, the law of motion of the origin of the local frame

$$Q_i = \bar{U}_{,i}(\boldsymbol{x}_B) - a_B^i + O(\epsilon^2)$$

⇒ The PPN translational equations of motion of extended bodies in the global frame

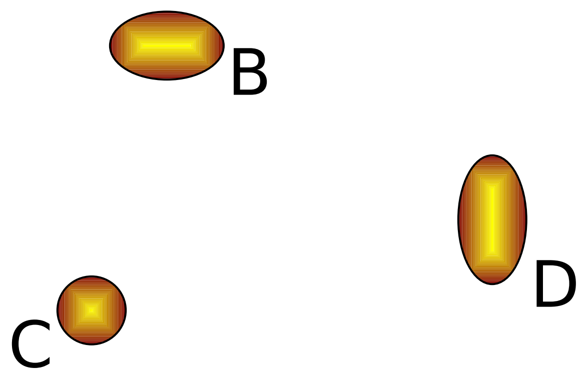
PN orbital motion

$$\mathrm{M}_B a_B^i = F_N^i + \epsilon^2 (F_{\mathcal{I}^2}^i + F_{\mathcal{I}^3}^i + F_{\mathcal{S}}^i) + \mathcal{O}(\epsilon^4)$$

2-body interactions: R_{BC}

3-body interactions: $R_{\mathrm{BC}} R_{\mathrm{BD}}$, $R_{\mathrm{BC}} R_{\mathrm{CD}}$

spin effects

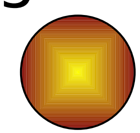


Definition of Mass (Blanchet-Damour)

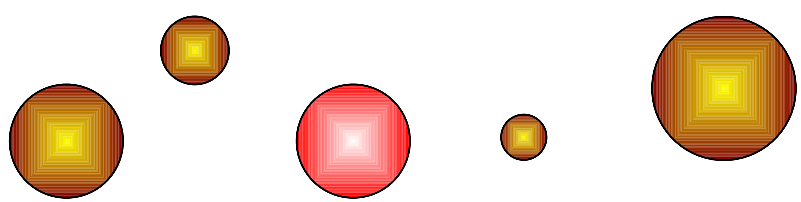
$$\mathrm{M}_B = \int_B \underbrace{\rho^* \left[1\right.}_{\text{rest mass}} + \epsilon^2 \underbrace{\left(\frac{1}{2}\nu^2 + \Pi - \frac{1}{2}\hat{U}^{(B)}\right)}_{\text{internal energy}} \left.\right] d^3w \; \underbrace{- \, \epsilon^2 \sum_{j=1}^{\infty} \frac{j+1}{j!} Q_J \mathcal{I}_B^{<J>}}_{\text{external effects}} + \mathcal{O}(\epsilon^4)$$

1. a single particle: rest mass

2. a single body: rest mass + internal energy



3. a body in *N*-body system:
   rest mass + internal energy + external effects



Violation of Strong Equivalence Principle

$$\mathrm{M}_B a^i_B = F^i_N + \epsilon^2 (F^i_{\mathcal{I}^2} + F^i_{\mathcal{I}^3} + F^i_{\mathcal{S}}) + \mathcal{O}(\epsilon^4)$$

$$F^i_N = -\sum_{C \neq B} \sum_{j,l=0}^{\infty} \frac{(-1)^j (2j+2l+1)!!}{j!l!} \frac{G \mathcal{I}_B^{<J>} \mathcal{I}_C^{<L>}}{R_{BC}^{2j+2l+3}} R_{BC}^{<iJL>}$$

$M_B \neq I_B$ → inertial mass ≠ gravitational mass

violation of Strong Equivalence Principle in Scalar-Tensor theory

PN relative Motion

- PN orbital motion of Moon in EMB
- PN orbital motion of Earth in EMB
- Moon-Earth in the EMB frame

Outlook: Selenophysics

- Fluid core
  more convincing evidences, radius
- Elastic properties
  Love numbers
- Spin-orbit coupling
- Harmonics in the multi-layer Moon
- Dissipation and convection
- Free libration modes
- Topography and mascons

Outlook: Fundamental Physics

- Stronger limitations
  nonlinear parameter β, dark energy, scalar fields
- New relativistic effects
  relativistic precession of lunar orbit w.r.t. ICRF
  relativistic quadrupole moment of the Earth
  tidal gravitomagnetic effects
  violation of SEP
- Stochastic GW background

Thank you!