The new edition of the IERS Conventions: conventional reference systems and constants

G. Petit
Bureau International des Poids et Mesures
92312 Sèvres Cedex, France

Résumé

- Framework given by Unions Resolutions
- Set of "constants"
- Celestial system
- Terrestrial system
- Some future work

Framework given by Unions Resolutions (1)

- Resolutions of the International Astronomical Union (IAU)
  - 1991: First consistent representation of coordinate systems for use in the vicinity of a body (Earth, solar system) + general principles for CRS
  - 1997: Choice of ICRS/ICRF (+ other resolutions)
  - 2000: extension of the IAU'1991 framework
    - Precise definition of the BCRS and GCRS
    - Maintenance of the celestial reference
    - IAU 2000 PN model + other definitions
    - Time and frequency applications in the solar system
  - 2006:
    - B1 on "Adoption of P03 precession and definition of ecliptic"
    - B2 on "Supplement to IAU (2000) on reference systems"
    - B3 on "Re-definition of TDB"
  - 2009:
    - B2 on "IAU 2009 astronomical constants"
    - B3 on "ICRF2"

Framework set by Unions Resolutions (2)

- Resolutions of the International Union of Geodesy and Geophysics (IUGG)
  - 1991: Res 2 on "Definition of a Conventional TRS"
  - 2007: Res 2 on "GTRS and ITRS"

Set of "constants"

- See Chapter 1 of the IERS Conventions

- Recommendation of the International Astronomical Union (IAU)
  - 2009: IAU Resolution B2 "IAU(2009) system of astronomical constants"

- Updated following
  - the WG Numerical Standards in Fundamental Astronomy (see preceding talk)
    - Principles of classification
    - Values
  - CODATA

Set of "constants"

- Classes of constants following NSFA
- Change of status for L_B (and L_C)
- Notation changed to TXX-compatible
- Earth constants and tide conventions: no change

Table 1.1: IERS numerical standards.

Constant	Value	Uncertainty	Ref.	Description
	Natural defining constants			
c	$299792458\ \mathrm{ms^{-1}}$	Defining	[1]	Speed of light
	Auxiliary defining constants			
k	$1.720209895 \times 10^{-2}$	Defining	[2]	Gaussian gravitational constant
L_G	$6.969290134 \times 10^{-10}$	Defining	[3]	1−d(TT)/d(TCG)
L_B	$1.550519768 \times 10^{-8}$	Defining	[4]	1−d(TDB)/d(TCB)
TDB_0	-6.55×10^{-5} s	Defining	[4]	TDB−TCB at JD 2443144.5 TAI
θ_0	0.7790572732640 rev	Defining	[3]	Earth Rotation Angle (ERA) at J2000.0
$d\theta/dt$	1.00273781191135448 rev/UT1day	Defining	[3]	Rate of advance of ERA
	Natural measurable constant			
G	$6.67428 \times 10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}}$	$6.7 \times 10^{-15}\ \mathrm{m^3kg^{-1}s^{-2}}$	[1]	Constant of gravitation
	Body constants			
$GM_\odot$ #	$1.32712442099 \times 10^{20}\ \mathrm{m^3s^{-2}}$	$1 \times 10^{10}\ \mathrm{m^3s^{-2}}$	[5]	Heliocentric gravitational constant
$J_{2\odot}$	2.0×10^{-7}	(adopted for DE421)	[5]	Dynamical form factor of the Sun
μ	0.0123000371	4×10^{-10}	[6]	Moon-Earth mass ratio
	Earth constants			
$GM_\oplus$ †	$3.986004418 \times 10^{14}\ \mathrm{m^3s^{-2}}$	$8 \times 10^{5}\ \mathrm{m^3s^{-2}}$	[7]	Geocentric gravitational constant
a_E †‡	6378136.6 m	0.1 m	[8]	Equatorial radius of the Earth
$J_{2\oplus}$ ‡	1.0826359×10^{-3}	1×10^{-10}	[8]	Dynamical form factor of the Earth
$1/f$ ‡	298.25642	0.00001	[8]	Flattening factor of the Earth
g_E †‡	$9.7803278\ \mathrm{ms^{-2}}$	$1 \times 10^{-6}\ \mathrm{ms^{-2}}$	[8]	Mean equatorial gravity
W_0	$62636856.0\ \mathrm{m^2s^{-2}}$	$0.5\ \mathrm{m^2s^{-2}}$	[8]	Potential of the geoid
R_0 †	6363672.6 m	0.1 m	[8]	Geopotential scale factor ($GM_\oplus/W_0$)
H	3273795×10^{-9}	1×10^{-9}	[9]	Dynamical flattening
	Initial value at J2000.0			
ϵ_0	84381.406″	0.001″	[4]	Obliquity of the ecliptic at J2000.0
	Other constants			
au ††	$1.49597870700 \times 10^{11}$ m	3 m	[6]	Astronomical unit
L_C	$1.48082686741 \times 10^{-8}$	2×10^{-17}	[3]	Average value of 1−d(TCG)/d(TCB)

TCB-compatible value, computed from the TDB-compatible value in [5].

† The value for $GM_\oplus$ is TCG-compatible. For a_E, g_E and R_0 the difference between TCG-compatible and TT-compatible is not relevant with respect to the uncertainty.

‡ The values for a_E, $1/f$, $J_{2\oplus}$ and g_E are "zero tide" values (see the discussion in section 1.1 above). Values according to other conventions may be found in reference [8].

†† TDB-compatible value. An accepted definition for the TCB-compatible value of au is still under discussion.

The celestial system and frame

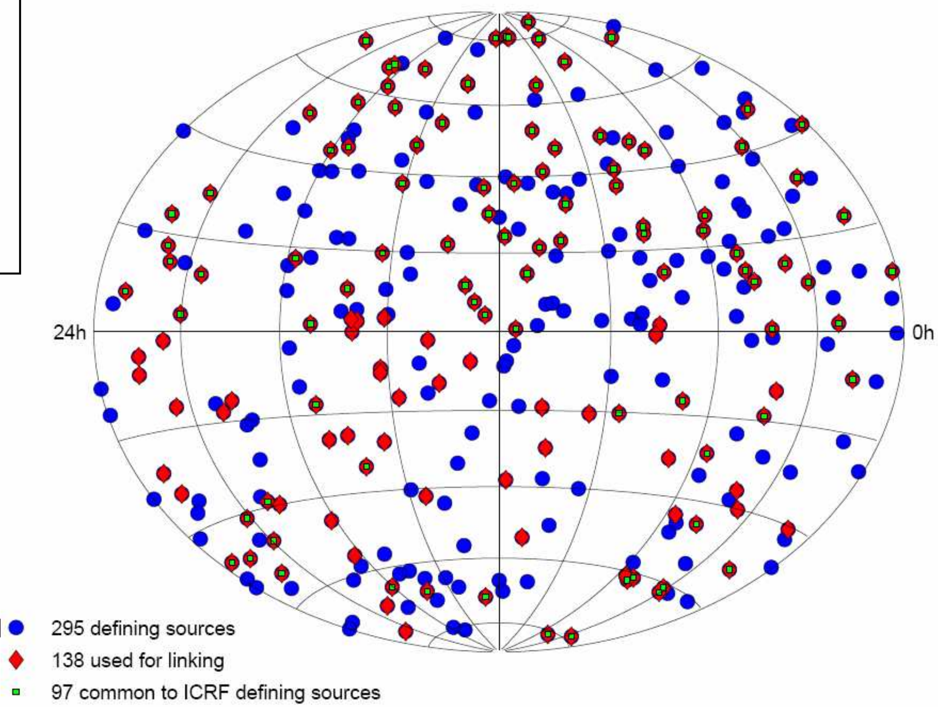
- IAU'1991 Rec VI and VII:
  - … primary sources to define the new conventional reference frame
  - … specify the choice of coordinate axes of the new frame
- IAU'1997 B2: From 1/1/1998, the IAU system shall be the ICRS as defined by the IERS … and the frame shall be the ICRF
- IAU'2000: Defines how to maintain the ICRS and ICRF
- IAU'2009 B3: From 1/1/2010, the fundamental realization shall be ICRF2 as defined by IERS/IVS WG …

- Future ….
  - Space optical observations likely to provide fundamental realization
  - Radio to provide realization from Earth
  - No need to change BCRS

The celestial frame in the IERS Conventions

- Conventions 2003: ICRF-ext1
  - 212 defining sources from ICRF1, about 0.4 mas median uncertainty.
  - 667 total number of sources
  - Noise floor, representing level of systematic effects: 250 µas
  - Axis stability: 20 µas
  - IERS Ann Rep. 1998

- Conventions 2010: ICRF-2
  - 295 defining sources (aligned to ICRF-ext2 with 138 common sources)
  - 3414 total number
  - Noise floor: 40 µas
  - Axis stability: 10 µas
  - IERS TN 35 (2009)



BIPM

The terrestrial system and frame

- IUGG'1991: Conventional TRS (CTRS)
  - defined from a geocentric non-rotating system, as defined by IAU'1991 Resolution
  - coordinate time TCG
  - origin is geocenter including oceans and atmosphere
  - no global residual rotation wrt. horizontal motions at Earth's surface
- IUGG'2007: Geocentric TRS (GTRS)
  - system of geocentric space-time coordinates, derived from GCRS following IAU'2000 B1.3 Resolution
  - ITRS is the specific GTRS…..
  - ITRF not explicitly defined

The terrestrial system and frame (2)

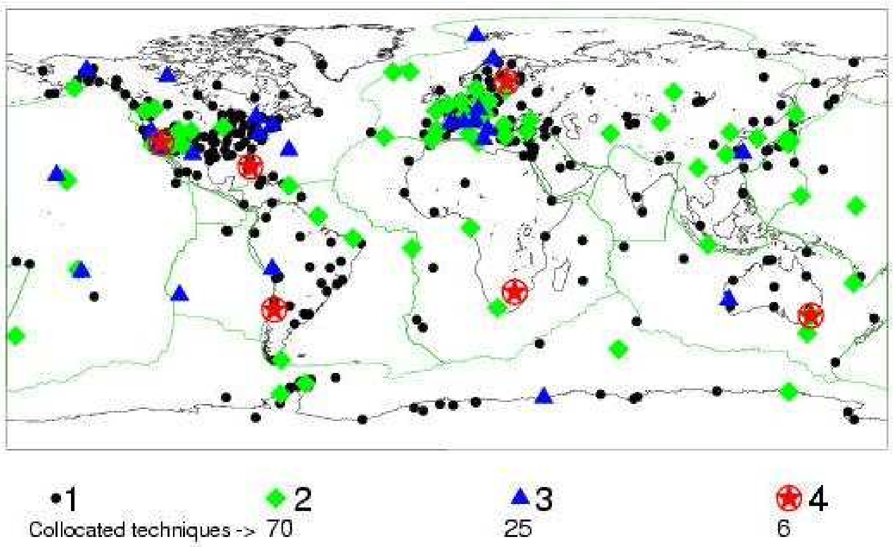
- Some questions:

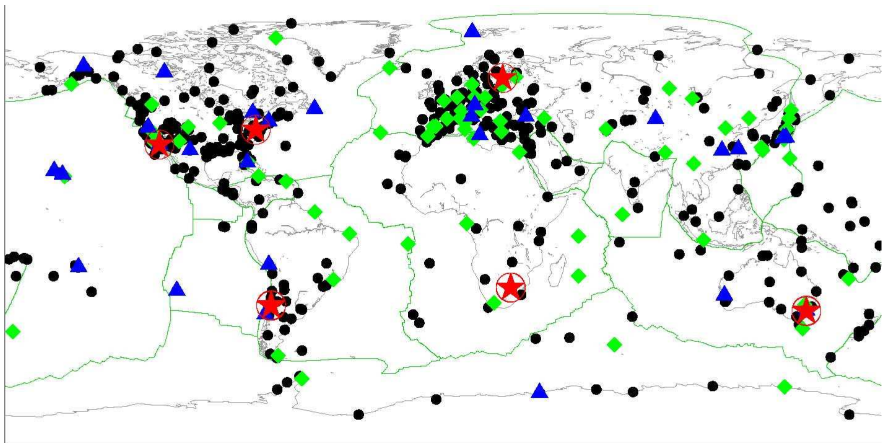
- ITRF not explicitly defined => defined by the realizations ITRFXXXX
- ITRS and ITRF have differences in this respect
  - ITRS is 4-dimensional (in principle)
  - Scale: ITRS is TCG-compatible, ITRF is TT-compatible
  - Origin: ITRS is geocentric, ITRF is "geocenter averaged over time span"
  - [Tide convention]
- Role of Inter-commission WG ICGG 1-3 "Concepts and terminology related to GRS", chaired by C. Boucher
  - unified and accepted terminology
  - implies a redefinition of ITRS

The terrestrial frame in the IERS Conventions

- Conventions 2003: ITRF2000
  - Scale from average VLBI-SLR
  - Origin from SLR
  - Orientation on ITRF97, rate on NNR-NUVEL-1A
  - 500 sites, 100 colocations
  - Altamimi et al. 2002

- Conventions 2010: ITRF2008
  - Scale from average VLBI-SLR
  - Origin from SLR
  - Orientation and rate on ITRF2005 (itself on ITRF2000)
  - 580 sites, 120 colocations
  - Systematic differences of order 1 cm (scale, T_Z and its rate)





Some future work (in relation to this talk)

- Problem of "geocenter motion"
  - ITRS defined as "geocentric"
  - ITRF origin realized as "geocenter averaged over the data span"

- Check / precise / rework some relativistic models
  - Time transformations
    - τ – TT/TCG
    - TCB – TCG
  - Models for coordinate time of propagation
    - VLBI
    - Ranging

Geocenter: Chapter 4 (terrestrial system)

- We assume the scale difference between ITRS/ITRF has no practical impact (provided that we are aware of it).
- What about the geocenter? We know the origin translation due to geocenter motion is significant but models / series are not fully reliable.

B) *Relativistic scale*

All individual centers use a scale consistent with TT. In the same manner the ITRF has also adopted this option (except ITRF94, 96 and 97, see Section 4.2.2). It should be noted that the ITRS scale is specified to be consistent with TCG. Consequently, if coordinates $\vec{X}$ consistent with TCG are needed, users need to apply the following formula: 2009/04/10

$$\vec{X} = (1 + L_G)\vec{X}_{ITRF} \qquad (15)$$

where L_G is according to Table 1.1 in Chapter 1. (IAU Resolution B1.9, 24th IAU General Assembly, Manchester 2000). Note that consistency between numerical constants should be ensured as described in Chapter 1.

C) *Geocentric positions*

The ITRF origin should be considered as the mean Earth center of mass, averaged over the time span of the SLR observations used and modeled as a secular (linear) function of time. If an instantaneous geocentric position $\vec{X}$ is required, it should be computed as 2009/04/10

$$\vec{X} = \vec{X}_{ITRF} + \Delta\vec{X}_G, \qquad (16)$$

where $\Delta\vec{X}_G$ represents the origin translation (vector from the instantaneous center of mass to the ITRF origin) due to non-secular geocenter motion. 2009/04/10

Geocenter: Chapter 5 (transformation)

- Transformation formula (1) is defined for the systems, but used with ITRF coordinates

- Are EOPs implicit in (1) affected?
- If yes, is it significant?

5.1 Introduction

The transformation to be used to relate the International Terrestrial Reference System (ITRS) to the Geocentric Celestial Reference System (GCRS) at the date t of the observation can be written as:

$$[\mathrm{GCRS}] = Q(t)R(t)W(t)\ [\mathrm{ITRS}], \tag{1}$$

where $Q(t)$, $R(t)$ and $W(t)$ are the transformation matrices arising from the motion of the celestial pole in the celestial reference system, from the rotation of the Earth around the axis associated with the pole, and from polar motion respectively.

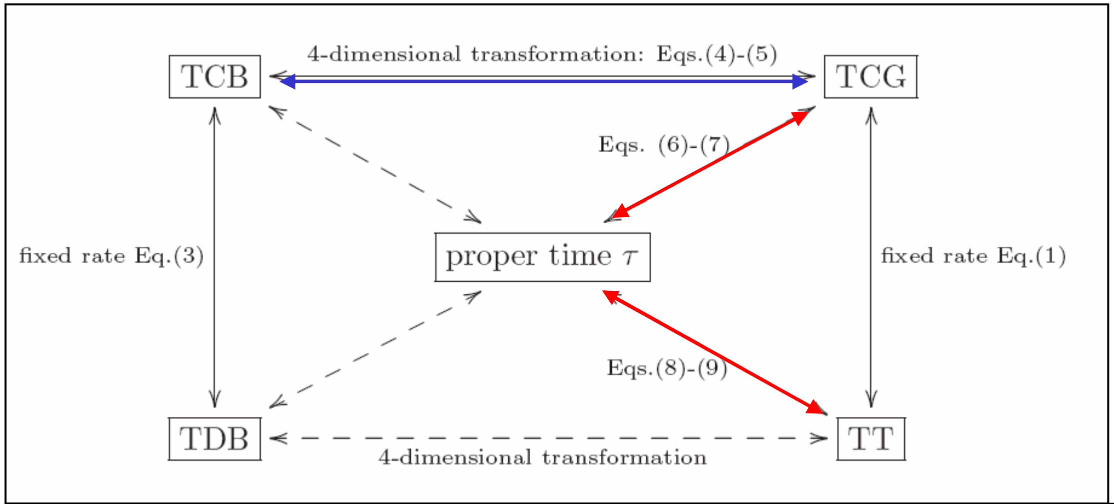
Note that Eq. (1) is valid for any choice of celestial pole and origin on the equator of that pole.

The definition of the GCRS and ITRS and the procedures for the ITRS to GCRS transformation that are provided in this chapter comply with the IAU 2000/2006 resolutions (provided in Appendices A and B). More detailed explanations about the relevant concepts, software and IERS products corresponding to the IAU 2000 resolutions can be found in IERS Technical Note 29 (Capitaine *et al.*, 2002), as well as in a number of original subsequent publications that are quoted in the following sections.

The chapter follows the recommendations on terminology associated with the IAU 2000/2006 resolutions that were made by the 2003-2006 IAU Working Group on "Nomenclature for fundamental astronomy" (NFA) (Capitaine *et al.*, 2007). We will refer to those recommendations in the following as "NFA recommendations" (see Appendix C for the list of the recommendations). This chapter also uses the definitions that were provided by this Working Group in the "IAU 2006 NFA Glossary"(available at `http://syrte.obspm.fr/iauWGnfa/`).

Eq. (1), as well as the following formulas in this chapter, are theoretical formulations that refer to reference "systems". However, it should be clear that the numerical implementation of those formulas involves the IAU/IUGG adopted realization of those reference systems, *i.e.* the International Terrestrial Reference Frame (ITRF) and the International Celestial Reference Frame (ICRF), respectively.

Time transformations (Chapter 10)



- TCB –TCG

- τ – TT/TCG

$$\mathrm{TCB} - \mathrm{TCG} = c^{-2}\left\{\int_{t_0}^{t}\left[\frac{v_e^2}{2} + U_{ext}(\vec{x}_e)\right]dt + \vec{v}_e\cdot(\vec{x}-\vec{x}_e)\right\} + O(c^{-4}), \quad (4)$$

where $\vec{x}_e$ and $\vec{v}_e$ denote the barycentric position and velocity of the Earth's center of mass, and U_{ext} is the Newtonian potential of all of the solar system bodies apart from the Earth evaluated at the geocenter. In this formula, t is TCB and t_0 is chosen to be consistent with 1977 January 1, $0^{\mathrm{h}}0^{\mathrm{m}}0^{\mathrm{s}}$ TAI, i.e. the value $T_0 = 2443144.5003725$ given above. Terms not specified in (4) are of order 10^{-16} in rate, and IAU Resolution B1.5 (2000) provides formulas to compute the $O(c^{-4})$ terms and equation (4) within given uncertainty limits up to 50000 km from the Earth.

The TCB−TCG formula (4) may be expressed as

$$\mathrm{TCB} - \mathrm{TCG} = \frac{L_C \times (TT - T_0) + P(TT) - P(T_0)}{1 - L_B} + c^{-2}\vec{v}_e\cdot(\vec{x}-\vec{x}_e) \quad (5)$$

$$\frac{d\tau_A}{dt} = 1 - 1/c^2\left[\mathbf{v}_A^2/2 + U_{\mathrm{E}}(\mathbf{x}_A)\right]. \quad (7)$$

When using TT as coordinate time, following its defining relation $dTT/dTCG = 1 - L_G$, equations (6) and (7) are rewritten with the same accuracy as

$$\frac{d\tau_A}{dTT} = 1 + L_G - 1/c^2\left[\mathbf{v}_A^2/2 + U_{\mathrm{E}}(\mathbf{x}_A) + V(X_{\mathrm{A}}) - V(X_{\mathrm{E}}) - x_A^i\partial_i V(X_{\mathrm{E}})\right] \quad (8)$$

and

$$\frac{d\tau_A}{dTT} = 1 + L_G - 1/c^2\left[\mathbf{v}_A^2/2 + U_{\mathrm{E}}(\mathbf{x}_A)\right], \quad (9)$$

Time transformations: TCB – TCG

- In the Conventions (2003) several options are presented
  - FB
  - TE405
  - HF2002

The TCB–TCG formula (4) may be expressed as

$$\mathrm{TCB} - \mathrm{TCG} = \frac{L_C \times (TT - T_0) + P(TT) - P(T_0)}{1 - L_B} + c^{-2}\vec{v}_e \cdot (\vec{x} - \vec{x}_e) \quad (5)$$

- In the Conventions (2010) one conventional transformation is proposed
  - XHF2002_IERS: analytical formula based on TE405 + linear term
- Is it necessary to update the time ephemeris?
  - Fienga et al. (2009) show 2×10^{-18} rate between TT-TDB from TE405 and from INPOP08
  - Preliminary comparisons between XHF2002_IERS and INPOP08 seem to show < 1×10^{-18} rate
  - To be confirmed

Time transformations: τ – TT/TCG for GNSS

- Work in GCRS. τ is the proper time of a GPS clock; coordinate time is TT

  $\Delta TT = (1 - L_G)\,\Delta\tau + 1/c^2 \int (U_E(X) + v^2/2)\, d\tau$

- **Central potential, elliptic orbit**

A secular rate of order -4.465×10^{-10}

included in satellite clock

a periodic term of max. amplitude 45 ns

to be taken care of

$$\mathrm{TT} = \tau_A - \Delta\tau_A^{per}, \quad \Delta\tau_A^{per} = -\frac{2}{c^2}\sqrt{a \cdot GM_\oplus} \cdot e \cdot \sin E, \qquad (10)$$

where a, e and E are the orbit semi-major axis, eccentricity and eccentric anomaly angle. Thus $\Delta\tau_A^{per}$ is the conventional GPS correction (see the GPS Interface Control Document available at <[7]>) for the periodical relativity part, which is equally due to eccentricity induced velocity and potential variations in Eq. (9). From the above equation, one can readily see that the amplitude of the periodical correction is proportional to the orbit eccentricity, *i.e.* equal to about $2.29\mu s \times e$. Since the eccentricity e for GPS orbits can reach up to 0.02, consequently the amplitude of $\Delta\tau_A^{per}$ can reach up to 46 ns. An alternative expression for the relativistic periodic correction is

$$\Delta\tau_A^{per} = -\frac{2}{c^2}\mathbf{v}_A \cdot \mathbf{x}_A, \qquad (11)$$

- **General case (Ashby 2001; Kouba 2004)**

Earth is oblate: Additional secular ($< 10^{-14}$) and **periodic (order 0.1 ns) terms**

Tidal potential: Additional secular ($< 10^{-16}$) and periodic (1 ps) terms

Terms of order higher than $1/c^2$: $< 10^{-19}$

Issue: **Additional periodic terms** **are significant vs. new clock stability => Agree on a conventional treatment**

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Models for coordinate time of propagation

- For techniques used in space-geodesy, models have been designed to ensure ~ 1 mm accuracy
- VLBI
  - Present model designed for 1 ps accuracy
  - Is sub-ps needed?

- Ranging
  - SLR
  - LLR
  - GNSS

$T_{T'} = \rho/c + 2GM_E/c^3 \, \mathrm{Ln}[(R_E+R_R+\rho)/(R_E+R_R - \rho)]$ where $R_M = |\boldsymbol{X}_M|$ and $\rho = |\boldsymbol{X}_R - \boldsymbol{X}_E|$

Models for coordinate time of propagation: VLBI

- Present model (1991) designed for 1 ps accuracy
- VLBI2010 calls for 0.3 ps (Heinkelmann and Schuh, 2009)
- Developments done 20+ years ago just to be re-examined (may be already done)

GP working on Soffel et al. 1991 paper many years ago!!

1+2+5 → A
3+7+8 → E+F
4 → D
6+9 → B+C

Using equation (8), the VLBI-delay equation (7) for (Δt) and the relation (11) for the baselines we obtain the formal expression ($\boldsymbol{B}\cdot\boldsymbol{k} = B^i k^i$ etc.):

$$\begin{aligned}\Delta T =& -\frac{1}{c}(\boldsymbol{B}\cdot\boldsymbol{k}) - \frac{1}{c^2}(\boldsymbol{B}\cdot\boldsymbol{k})\boldsymbol{k}\cdot(\boldsymbol{v}_\oplus + \boldsymbol{v}_2) + \frac{1}{c^2}(\boldsymbol{v}_\oplus\cdot\boldsymbol{B}) \\ &+ \frac{1}{c^3}(\boldsymbol{v}_\oplus\cdot\boldsymbol{v}_2)(\boldsymbol{B}\cdot\boldsymbol{k}) - \frac{1}{c^3}(\boldsymbol{B}\cdot\boldsymbol{k})[\boldsymbol{k}\cdot(\boldsymbol{v}_\oplus+\boldsymbol{v}_2)]^2 + \frac{1}{c^3}(\boldsymbol{B}\cdot\boldsymbol{k})\overline{U}(\boldsymbol{z}_\oplus) \\ &+ \frac{1}{2c^3}(\boldsymbol{v}_\oplus\cdot\boldsymbol{k})(\boldsymbol{v}_\oplus\cdot\boldsymbol{B}) + \frac{1}{c^3}(\boldsymbol{v}_\oplus\cdot\boldsymbol{B})(\boldsymbol{v}_2\cdot\boldsymbol{k}) - \frac{1}{c^2}\int_{t_1}^{t_2}\left(\overline{U}(\boldsymbol{z}_\oplus) + \frac{1}{2}v_\oplus^2\right)dt' \\ &- \frac{1}{c}k^i\Delta\xi^i + \frac{1}{2c^3}(\boldsymbol{B}\cdot\boldsymbol{k})^2\boldsymbol{k}(\boldsymbol{a}_\oplus + \dot{\boldsymbol{v}}_2) + \frac{1}{c^3}(\boldsymbol{B}\cdot\boldsymbol{k})(\boldsymbol{a}_\oplus\cdot\Delta\boldsymbol{r}_2) \\ &+ (\Delta t)_{\rm grav} + O(4). \end{aligned} \qquad (12)$$

Keeping only terms with amplitudes greater than 1 picosec for baselines of the order of 6000 km, we approximately find:

Conclusions

- IERS Conventions (2010) about completed
  - Technical chapters finished, all available this week http://tai.bipm.org/iers/convupdt/convupdt.html
  - Introduction to be finalized
  - Final version will be presented on a new web page http://tai.bipm.org/iers/conv2010 (similar at USNO)
  - Integrated document will be transmitted to IERS CB.

- This is the starting point for the future work

Thank you