Application of the spectral analysis methods for the investigation of the Moon rotation

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The aim of the research:

1) **Choice and comparison of the optimal spectral analysis schemes** for the investigation of the rotational motion of the Moon.
2) **Investigation of the discrepancies** between the high-precision numerical solutions of the Moon rotation problem and the semi-analytical theory of the Moon's rotation (SMR), which consists of Cassini relations and the semi-analytical solutions of the lunar physical libration's problem (Eckhardt D.H., 1981), (Moons M., 1982), (Moons M., 1984), (Pešek I., 1982), by the spectral analysis methods.
3) **Construction of the new high-precision Moon Rotation Series (MRS2010)** dynamically adequate to the DE200/LE200 ephemeris over 400 years.

This investigation is carried out only for the Newtonian case.

ALGORITHM:

1. **Numerical solution of the Moon rotation** is implemented with the quadruple precision of the calculations. **The initial conditions** are computed by the semi-analytical theory of the Moon rotation (SMR). **Discrepancies between the numerical solution and SMR** are obtained in Euler angles over 400 year time interval with one day spacing.
2. **Investigation of the discrepancies** is carried out by the least squares method (LSQ) and by the spectral analysis (SA) method. **The set of the frequencies of SMR theory** is used **without change** in this investigation**.** **Only the coefficients of the periodic terms** are improved. **The periodic terms** representing the new high-precision Moon rotation series **MRS2010** are determined.
3. **Numerical solution of the Moon rotation is constructed anew** with **the new initial conditions,** which are calculated by **MRS2010**. **Discrepancies of the comparison of the numerical solutions with MRS2010** do not surpass 20 mas over 400 year time interval, that means a good consistency of **MRS2010** series with the DE200/LE200 ephemeris.

Lagrange differential equations of the second kind of the Moon rotation with respect to the fixed ecliptic and equinox of epoch J2000:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\lambda}_i}-\frac{\partial L}{\partial \lambda_i}=0, \quad i=0,1,2,3.$$

where $L=T+U, \quad T=\frac{1}{2}(A\omega_1^2+B\omega_2^2+C\omega_3^2),$

$U-$ **the force function of the gravitational interaction of the Moon with the disturbing bodies (the Earth, Sun and large planets).**

Problem expressed in the Rodrigues – Hamilton parameters:

$$\lambda_0=\cos\frac{\theta}{2}\cos\frac{\psi+\phi}{2}, \lambda_1=\sin\frac{\theta}{2}\cos\frac{\psi-\phi}{2}, \lambda_2=\sin\frac{\theta}{2}\sin\frac{\psi-\phi}{2}, \lambda_3=\cos\frac{\theta}{2}\sin\frac{\psi+\phi}{2},$$

which are functions of the Euler angles ψ, θ and ϕ.

The force function *U* of the gravitational interaction of the Moon with the disturbing bodies (the Earth, Sun and large planets) is limited here to the spherical harmonics with coefficients C^*_{j0} for j=2,…,4, C^*_{22}, C^*_{3k}, S^*_{3k} for k=1,2,3 and C^*_{4k}, S^*_{4k} for k=1,…,4. All these coefficients refer to the principal inertia axes of the Moon and are calculated from the non-normalized coefficients C_{jk}, S_{jk} of the selenopotential model by Eckhardt D.H., 1981.

The orbital motions of the disturbing bodies are defined by the DE200/LE200 ephemeris.

The high-precision method of the numerical integration HIPPI (HIgh Precision Polynomial Interpolation) (Eroshkin et al., 2000) was applied.

The compilation of semi-analytical theory of the Moon rotation (SMR):

1. Cassini's relations:

a) **The Moon rotates** with a constant angular velocity around its polar axis. **The rotation period** is equal to the mean sidereal period of its orbital motion around the Earth.

b) **The inclination of the lunar equator to the ecliptic** is a constant angle (near 1^{o} 32').

c) **The ascending node** of the lunar orbit on the ecliptic coincides with **the descending node** of the lunar equator on the ecliptic.

2. The semi-analytical solutions of the lunar physical libration problem:

a) Moons M., 1982: "Physical Libration of the Moon"', *Celest. Mech.*, **26**, pp.131–142,

b) Eckhardt D.H., 1981, "Theory of Libration of the Moon", *The Moon and the planets*, **25**, pp.3–49,

c) Moons M., 1984: "Planetary Perturbations on the Libration of the Moon", *Celest. Mech.*, **34**, pp.263–273,

d) Pešek I., 1982: "An Effect of the Earth's Flattening on the Rotation of the Moon", *Bull. Astron. Inst. Czechosl.*, **33**, pp.176–179.

The compilation of semi-analytical theory of the Moon rotation (SMR):

1. Cassini's relations:

a) $\varphi_d + \psi_d = 180^\circ + L$

b) $\theta_d = I$

c) $\psi_d = \Omega$

2. The semi-analytical solutions of the lunar physical libration problem:

a) MP500 is **the 3rd degree** solution for the **force function** **(C^*_{j0} for j=2,3, C^*_{22} , C^*_{3k}, S^*_{3k} for k=1,2,3).** (Moons M., 1982),

b) Additional solution for **the 4th degree perturbations of the force function** **(C^*_{40} C^*_{4k} , S^*_{4k} for k=1,…,4).** (Eckhardt D.H., 1981),

c) Effect of planetary perturbations (Moons M., 1984),

d) Effect of the Earth's flattening (Pešek I., 1982).

The physical libration of the Moon:

Since Cassini's relations are not exact, one must consider the perturbed Euler angles:

a) $\varphi_d + \psi_d = 180^\circ + L + \tau$

b) $\theta_d = I + \rho$

c) $\psi_d = \Omega + \sigma$

where ψ_d is the longitude of the descending node of date of the lunar equator,

θ_d is the inclination of the lunar equator to the ecliptic of date,

φ_d is the proper rotation angle between the descending node of date and the principal axis A (with the minimum moment of inertia);

L is the mean longitude of the Moon and

Ω is the mean longitude of the ascending node of its orbit;

τ, ρ and σ are the perturbing terms of the physical librations in the longitude, in the inclination and in the node longitude, respectively.

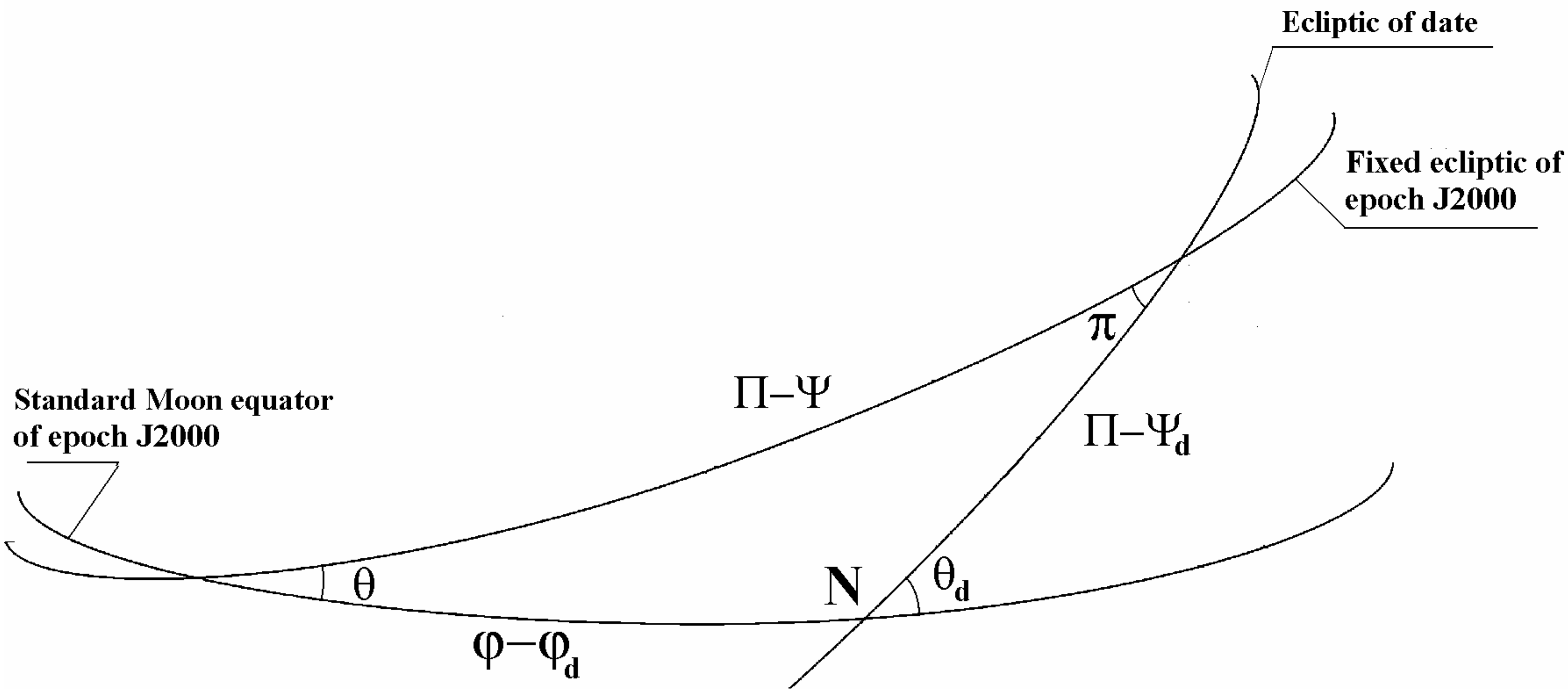


Fig.1. Triangle defines the reduction of Euler angles from ecliptic of date to the fixed ecliptic of epoch J2000 for the Moon rotation.

Reduction formulae of Euler angles from the ecliptic of date to the fixed ecliptic of epoch J2000 for the Moon rotation.

$$\cos\theta = \cos\theta_d \cos\pi + \sin\theta_d \sin\pi \cos(\Pi - \psi_d)$$

$$\sin(\varphi - \varphi_d) = \frac{\sin\pi \sin(\Pi - \psi_d)}{\sin\theta}$$

$$\sin(\psi - \psi_d) = -\frac{\cos\theta + \cos\theta_d \sin(\varphi - \varphi_d)}{1 + \cos\pi}$$

$$\dot{\theta} = \dot{\theta}_d \cos(\varphi - \varphi_d) - \dot{\pi}\cos(\Pi - \psi) + (\dot{\Pi} - \dot{\psi}_d)\sin\pi \sin(\Pi - \psi)$$

$$\dot{\varphi} - \dot{\varphi}_d = \frac{(\dot{\Pi} - \dot{\psi}_d)\sin\pi\cos(\Pi - \psi) + \dot{\pi}\sin(\Pi - \psi) - \dot{\theta}_d \sin(\varphi - \varphi_d)\cos\theta}{\sin\theta}$$

$$\dot{\Pi} - \dot{\psi} = \frac{(\dot{\Pi} - \dot{\psi}_d)\sin\theta_d \cos(\varphi - \varphi_d) + \dot{\pi}\sin(\Pi - \psi)\cos\theta - \dot{\theta}_d \sin(\varphi - \varphi_d)}{\sin\theta}$$

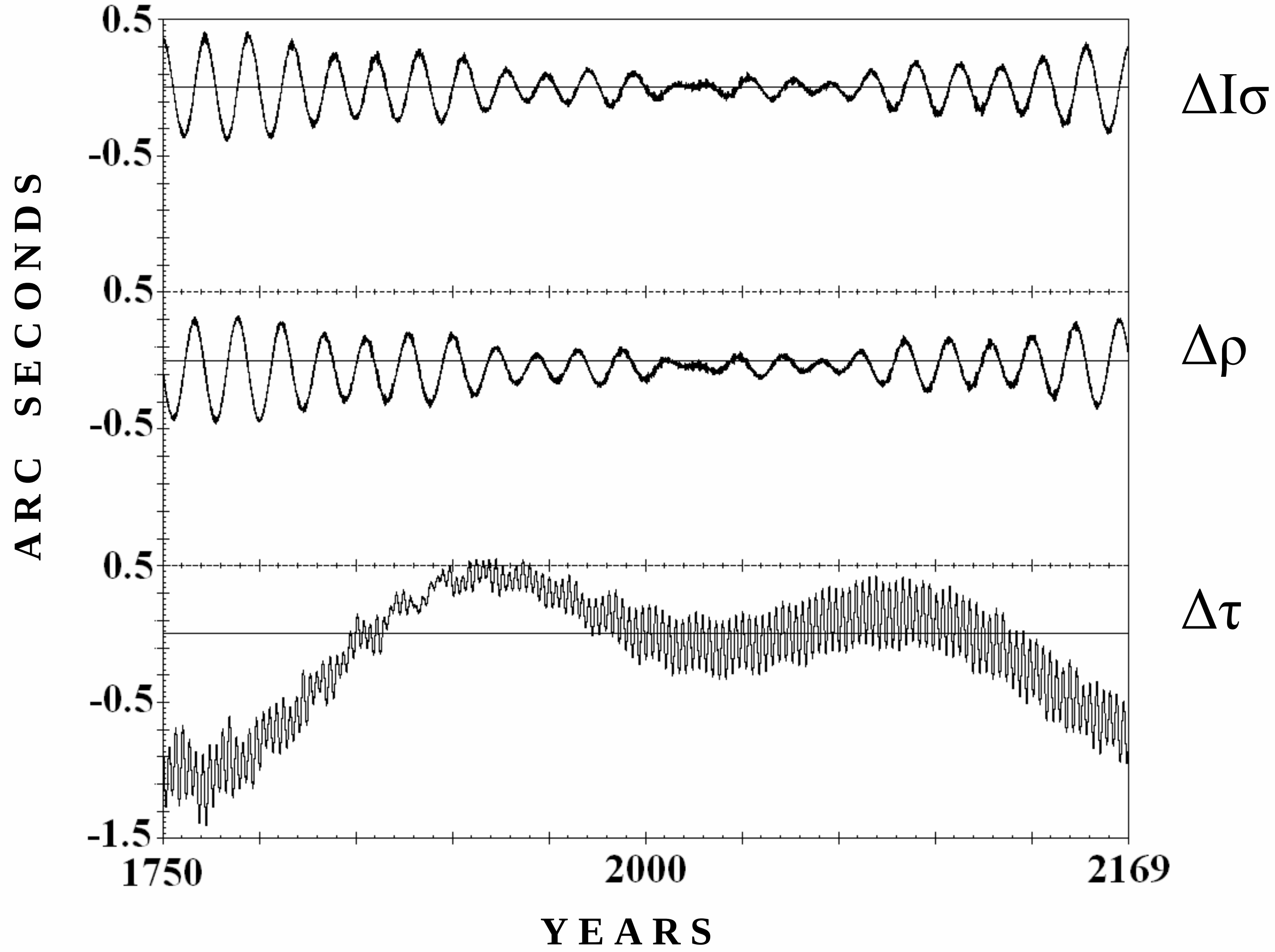
θ – the inclination of the lunar equator to the fixed ecliptic J2000;

ψ – the longitude of the descending node of epoch J2000 of the lunar equator;

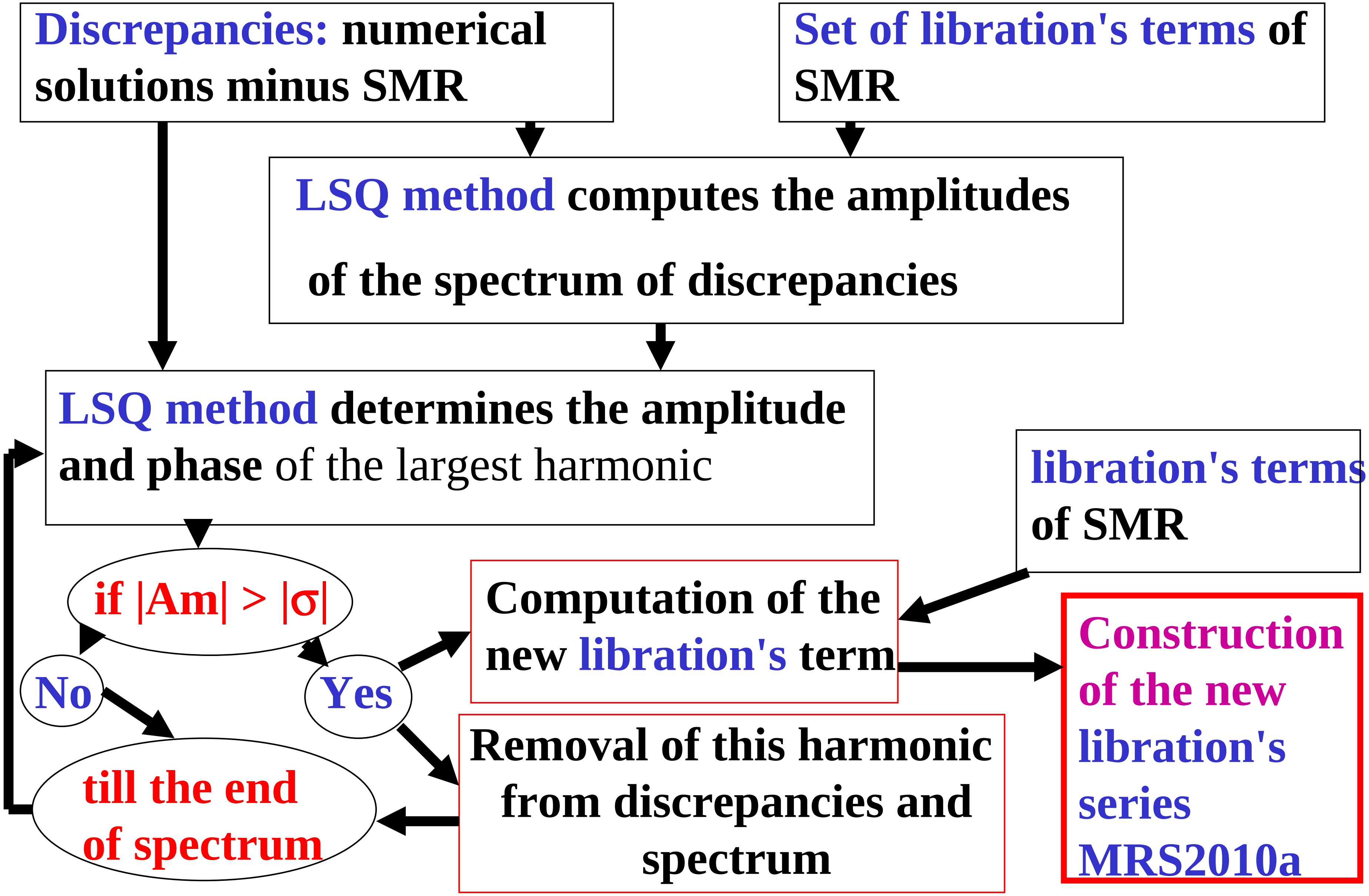
φ – the proper rotation angle between the descending node of epoch J2000 and the principal axis A (with the minimum moment of inertia).

Fig.2. **Numerical solution *minus* SMR solution**

for the perturbing terms of the Lunar physical librations



Spectral analysis (SA) method (A L G O R I T H M –A)
for removal of the periodical terms from the discrepancies



Spectral analysis (SA) method (A L G O R I T H M –A)
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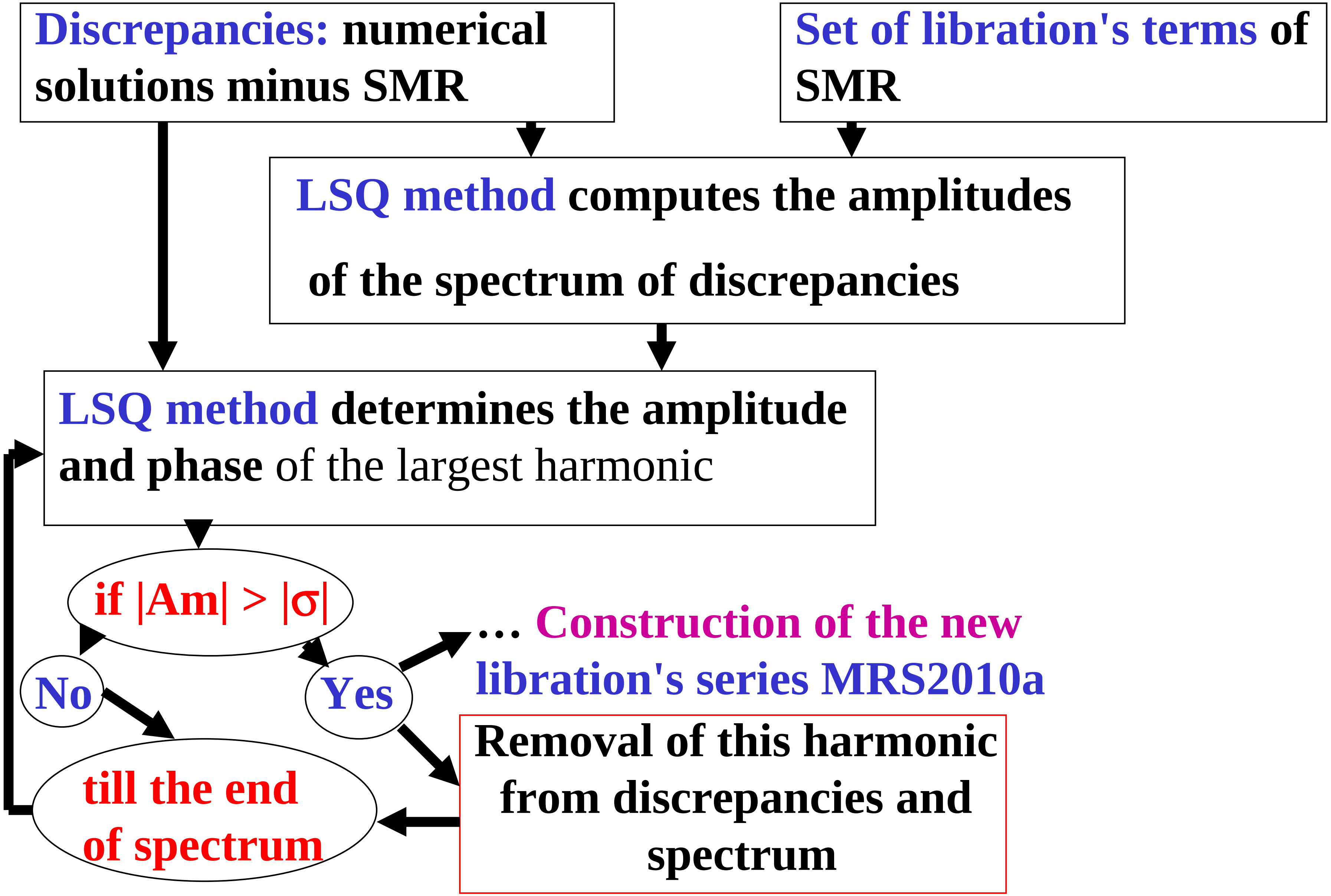
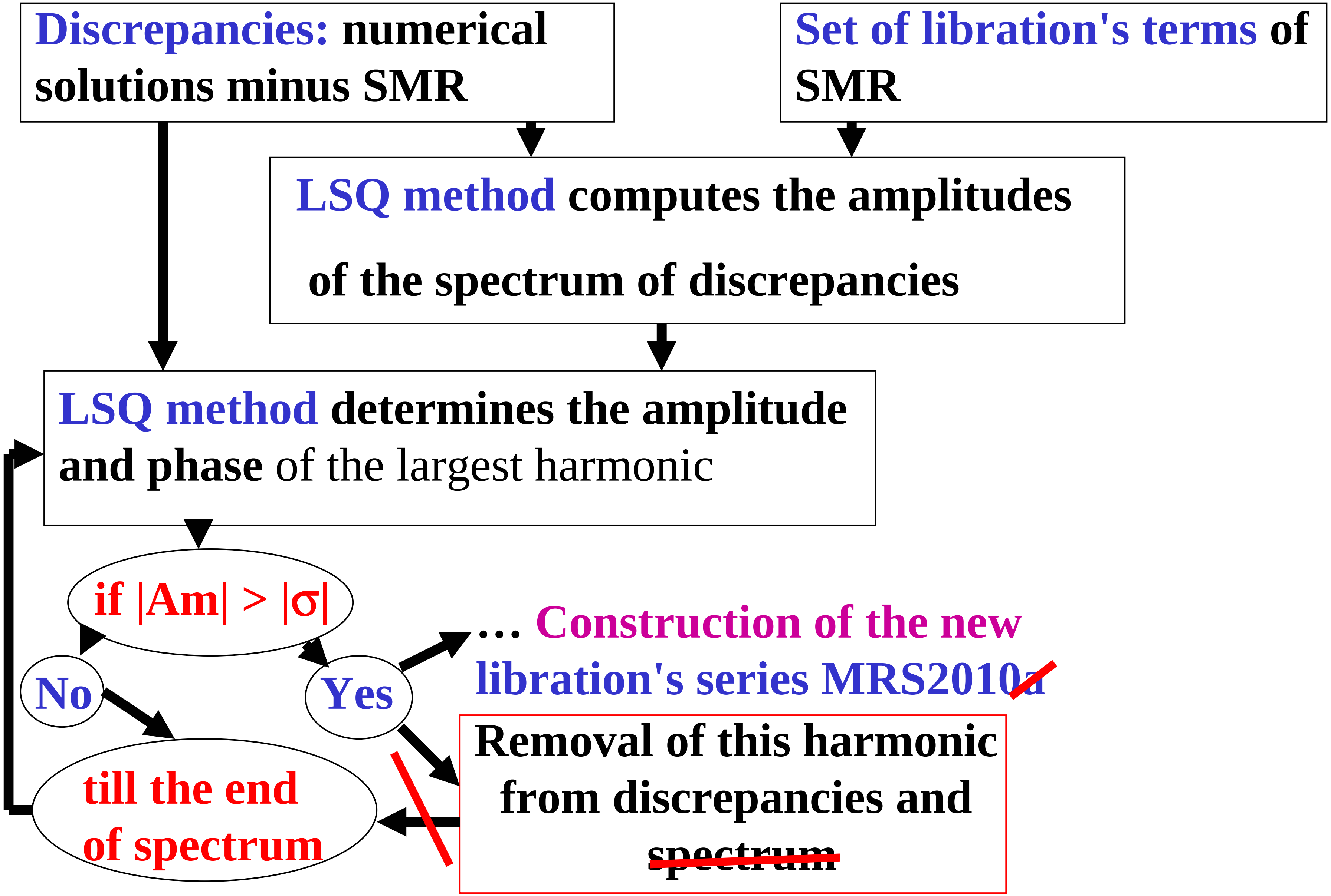


Spectral analysis (SA) method ~~(A L G O R I T H M A)~~
for removal of the periodical terms from the discrepancies



Spectral analysis (SA) method (A L G O R I T H M –B)
for removal of the periodical terms from the discrepancies

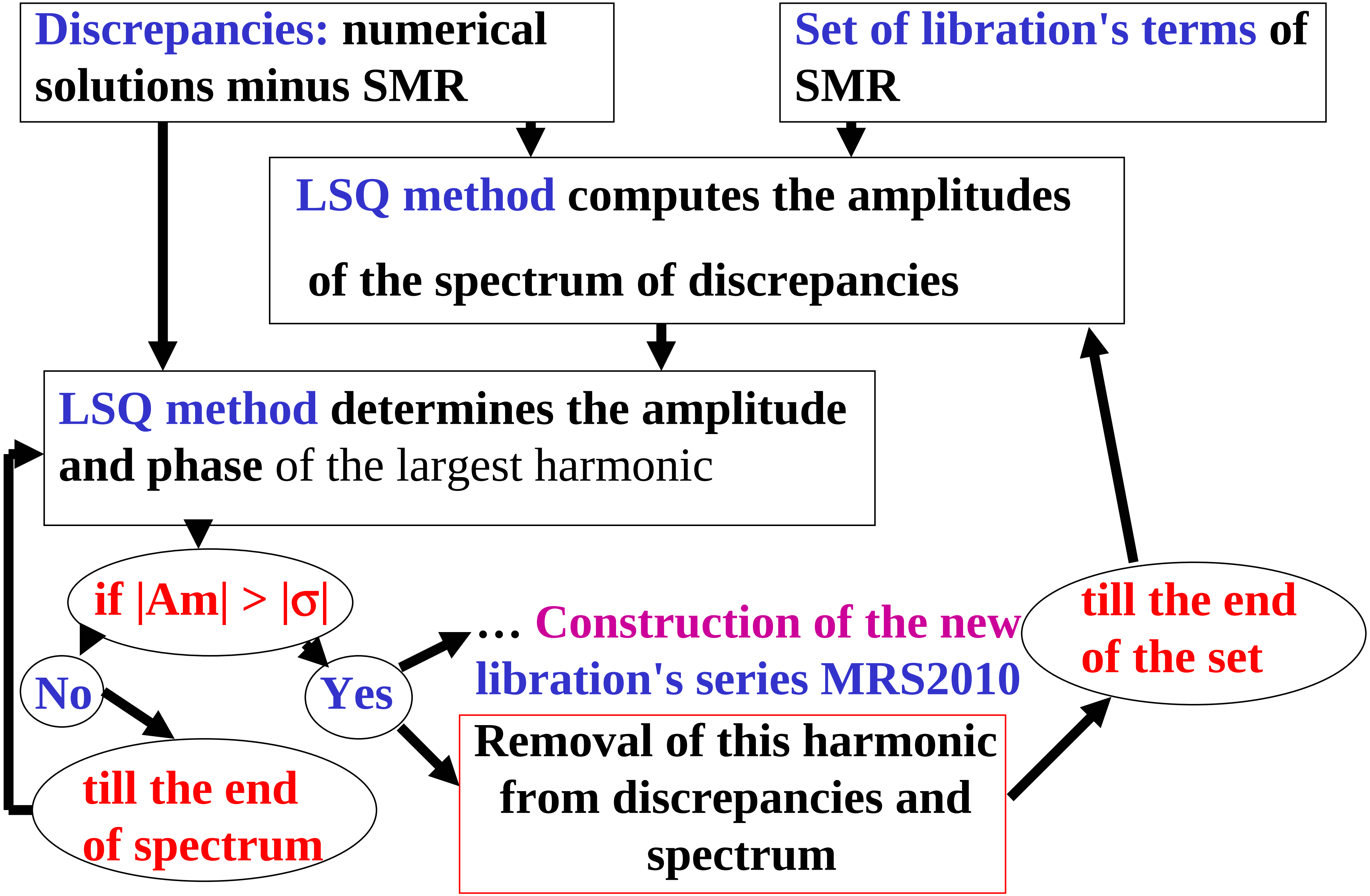


Fig.3. **Spectrum of the discrepancies** between **numerical and semi-analytical (SMR) solutions** for **τ.**

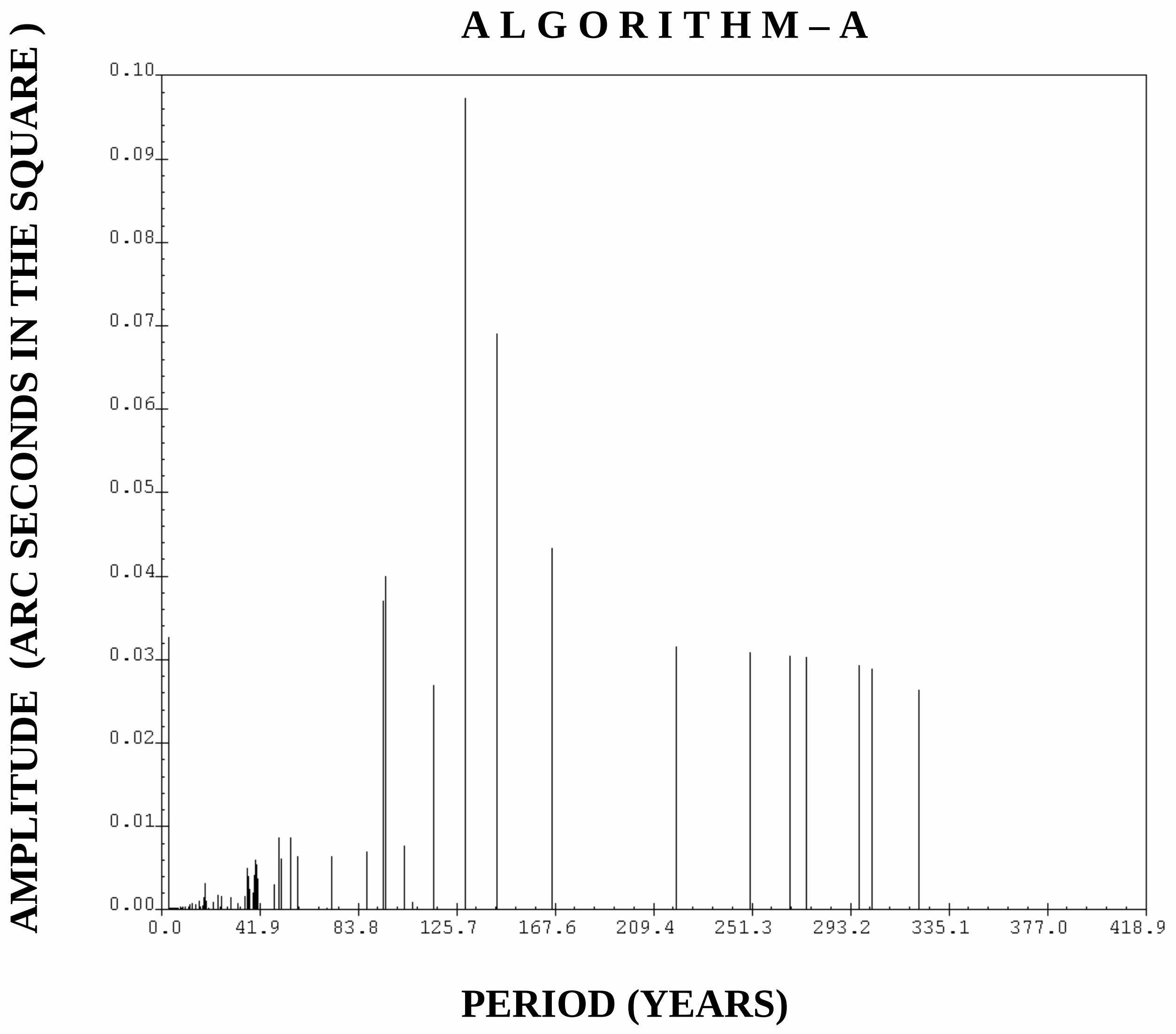


Fig.4. **Spectrum of the discrepancies** between

numerical and semi-analytical **(SMR) solutions** for **τ.**

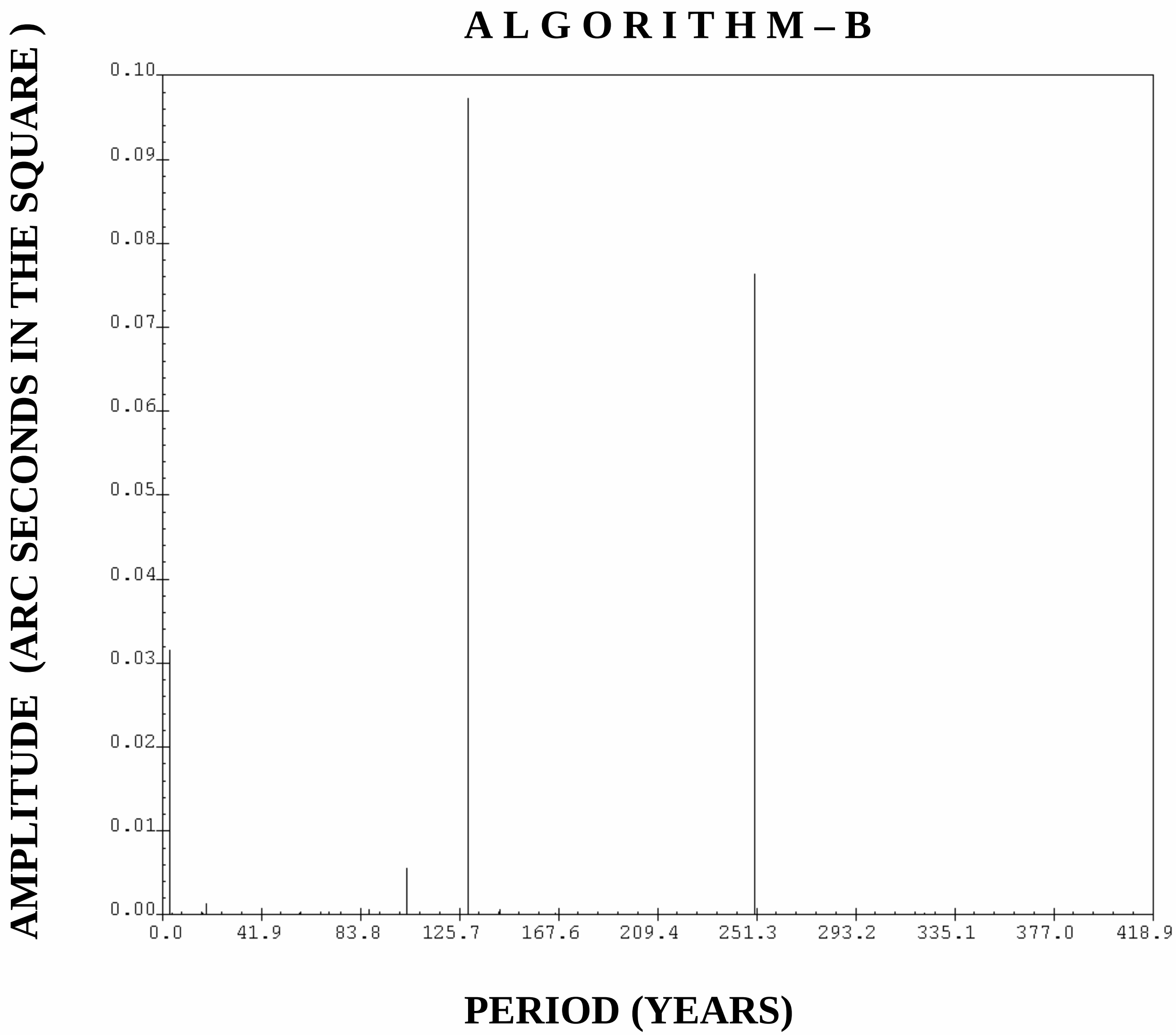


Fig.5. Residuals after the formal removal of the 351 periodical terms from the discrepancies

between **numerical and semi-analytical (SMR) solutions**

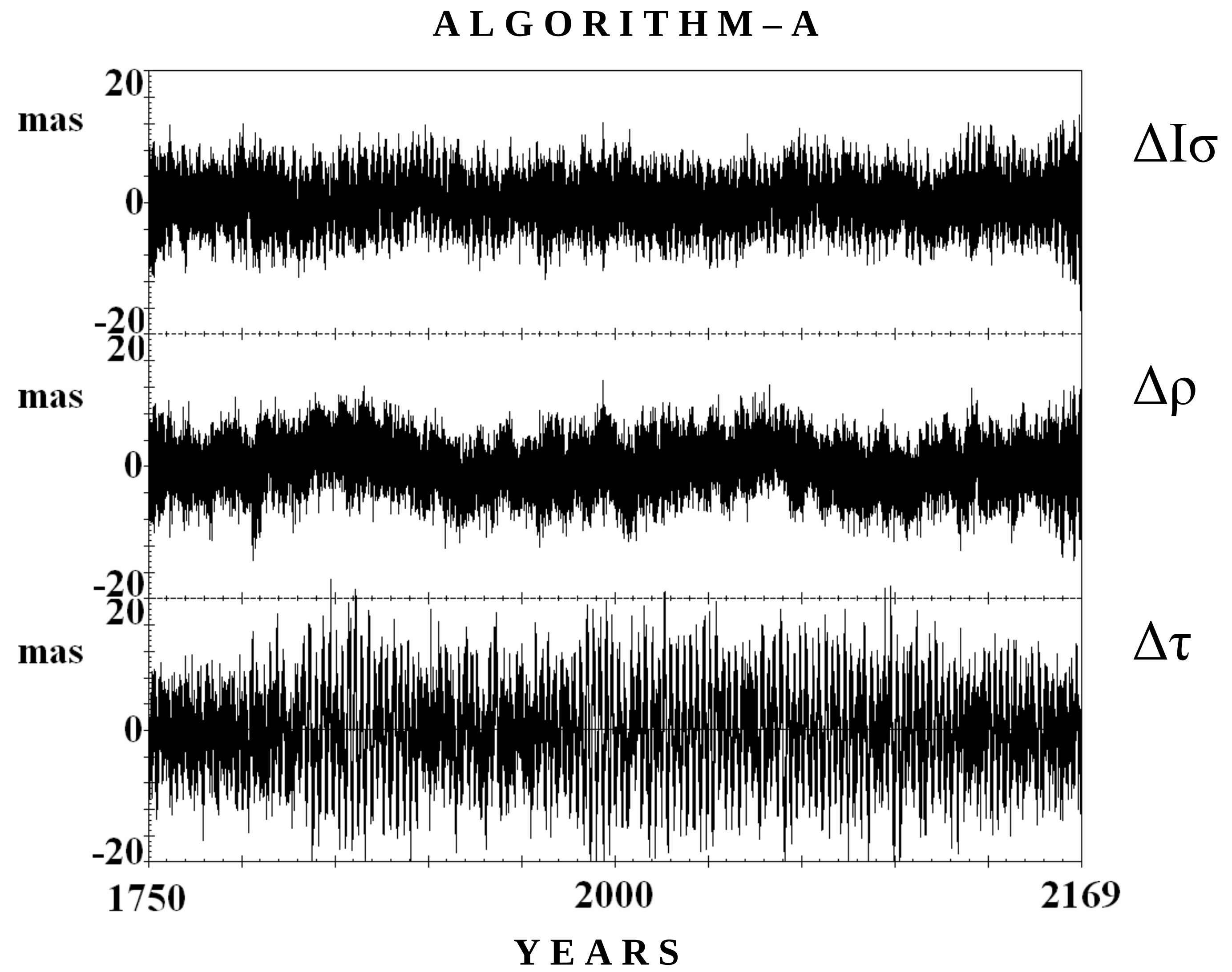


Fig.6. Residuals after the formal removal of the 351 periodical terms from the discrepancies

between **numerical and** **semi-analytical** **(SMR) solutions**

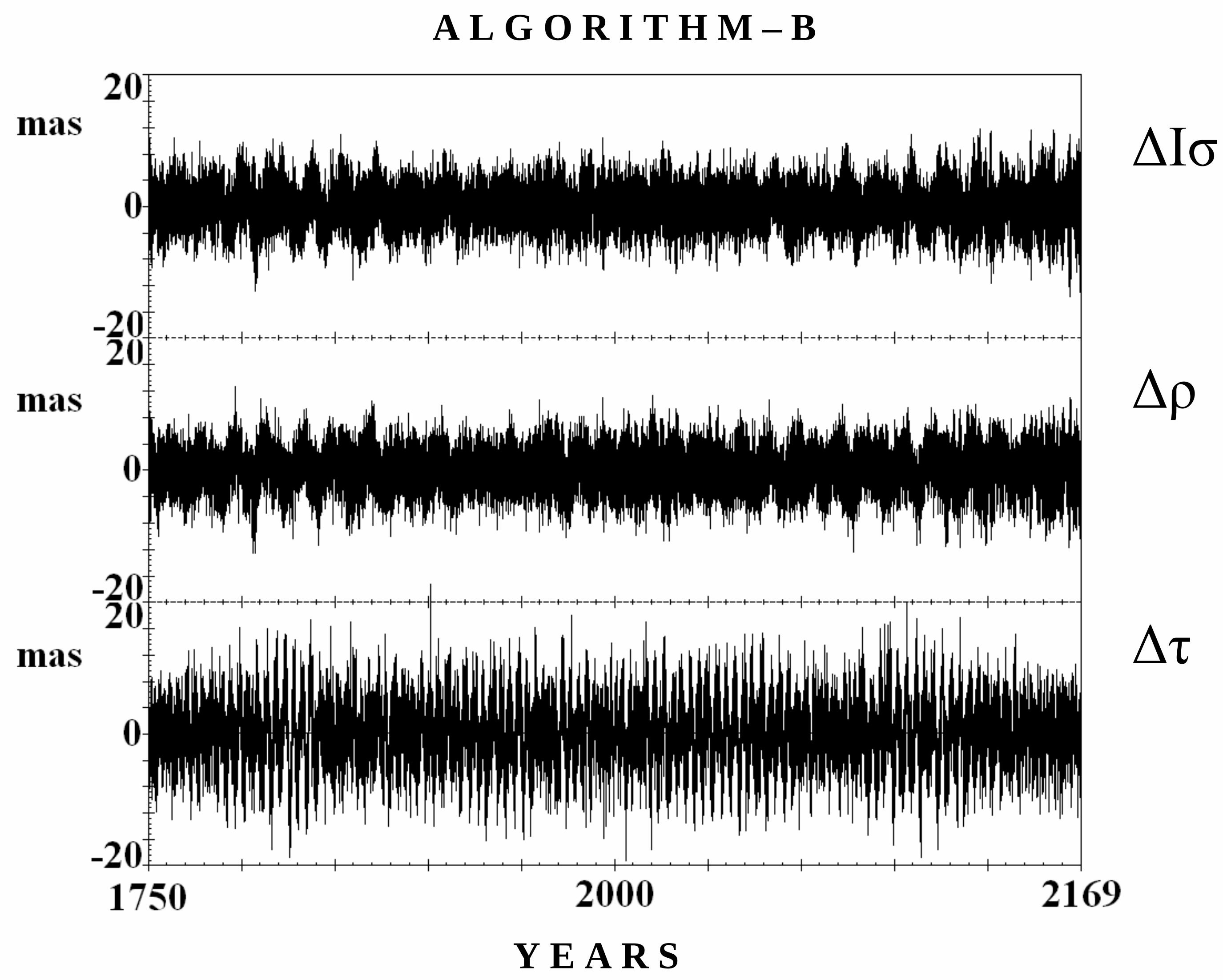


Fig.7. **Numerical solution** 2 *minus* **MRS2010 solution**

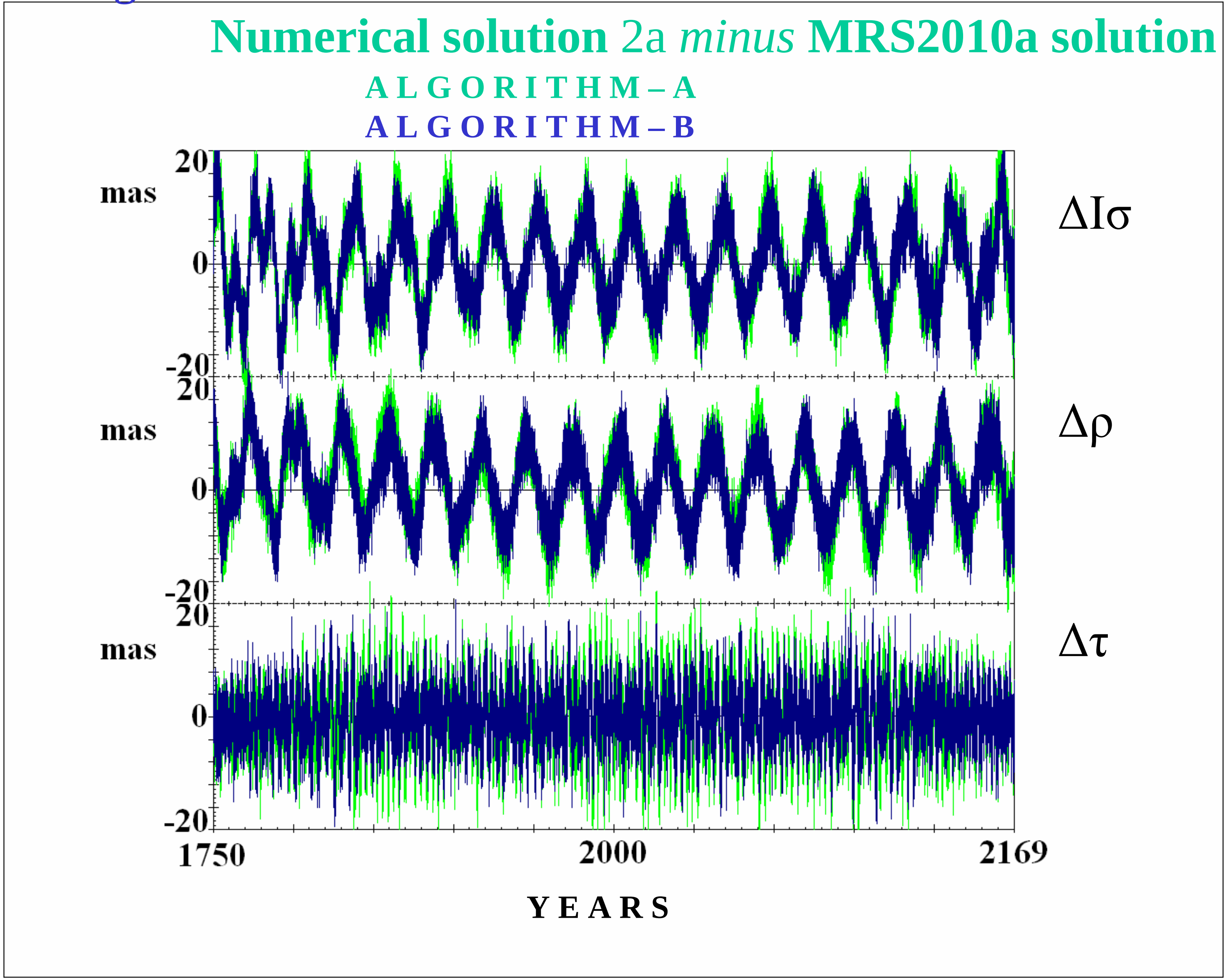


Fig.8. **Numerical solution** *minus* **SMR solution**

Numerical solution 2 *minus* **MRS2010 solution**

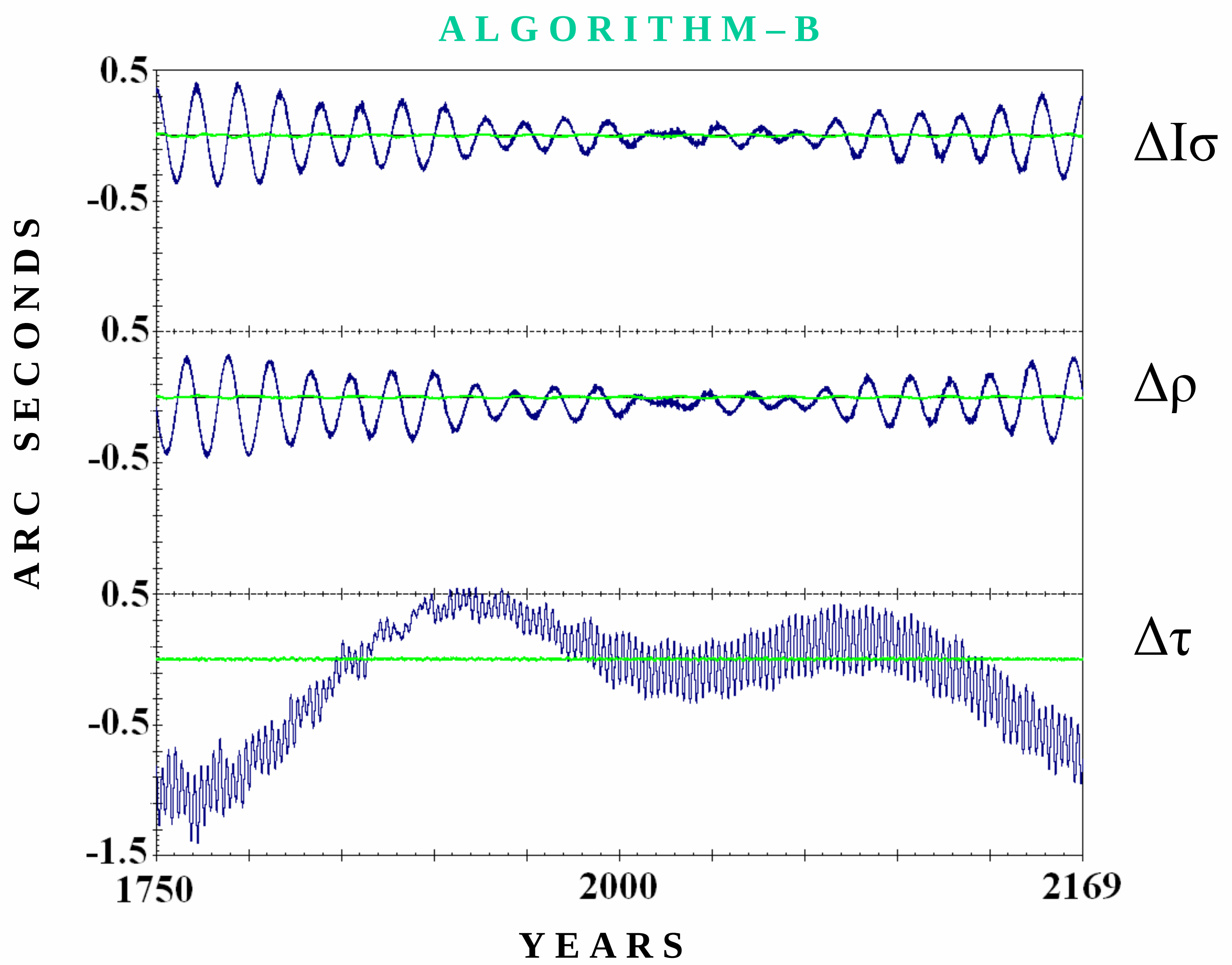
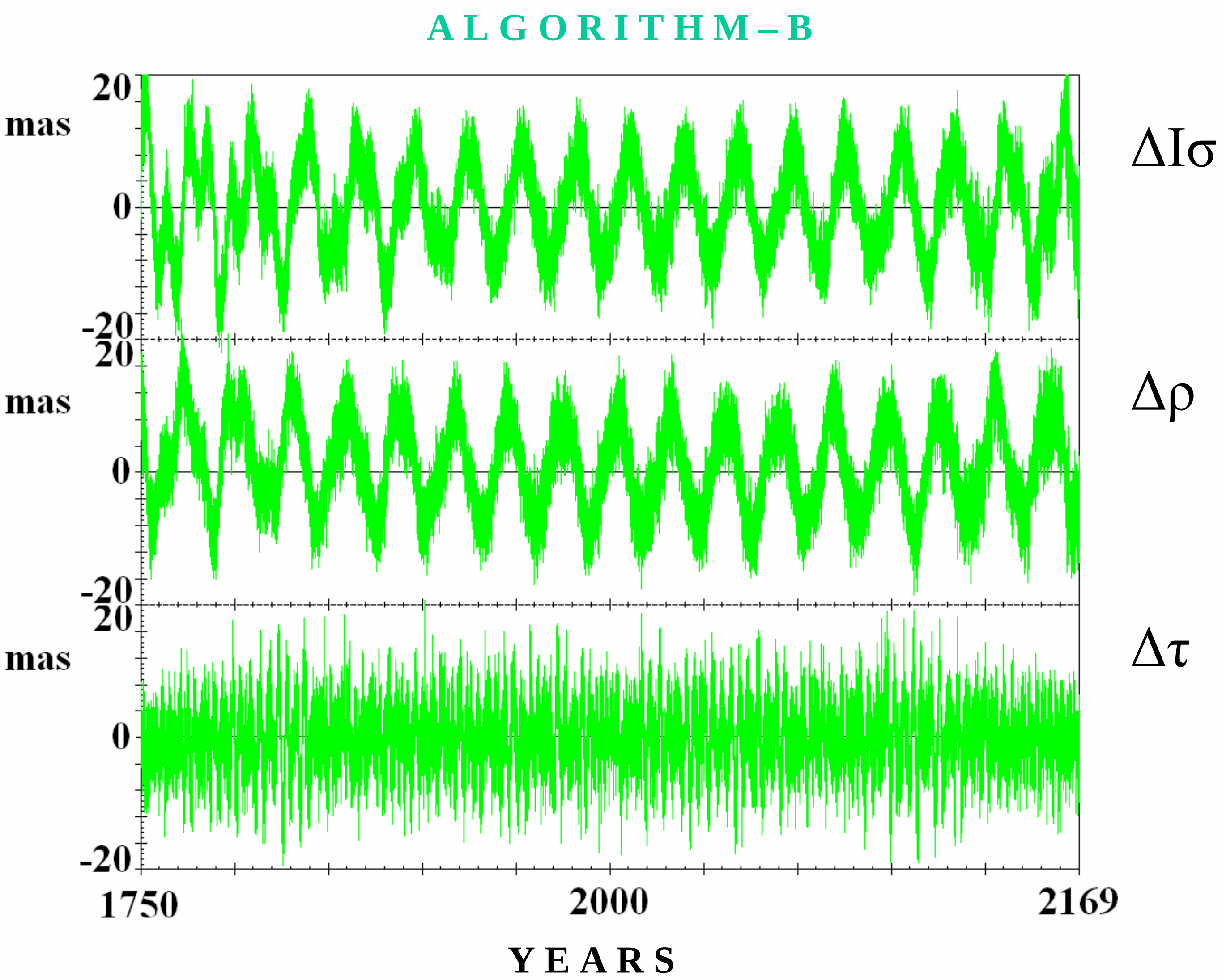


Fig.8. **Numerical solution** 2 *minus* **MRS2010 solution** (zoom)



Summary

1. The **appearance of the harmonics of the fictive free physical librations** in the numerical solution **is explained by errors of the initial conditions,** which are defined by not enough precise semi-analytical solutions. **The periods of the harmonics of the free librations** are equal 2.9 years for $\Delta\tau$, and 27.2 days, 24 years for $\Delta\rho$ and $\Delta I\sigma$.
2. We demonstrated that **the ALGORITHM –B (where the spectrum is constructed anew after every removal of the largest of the residual harmonic from the discrepancies) is more accurate than the ALGORITHM –A (where the spectrum is constructed only one time).**
3. The **ALGORITHM –B, is very time-consuming hence not suitable for the investigation of the long time series** (for example, our previous investigation the Earth rotation (V.V.Pashkevich and G.I.Eroshkin, Proceedings "Journees 2005", 2005)), **but** the **ALGORITHM –B is very well suitable for the investigation of the small time series (the Moon rotation).**

CONCLUSION

- **The ALGORITHM –B is the optimal scheme** for the research and elaboration of the numerical model of the Moon rotation.
- **The new high-precision the Moon rotation series (MRS2010),** dynamically adequate to the DE200/LE200 ephemeris over 400 year time interval, **was constructed. MRS2010 include** about 1520 periodical terms.
- **Discrepancies between the numerical solution and MRS2010** do not surpass 20 mas over 400 year time interval.

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