LLR residuals of INPOP10a and constraints on post-newtonian parameters

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Journées systèmes de référence spatio-temporels
20, 21, 22 september 2010

Fitting parameters to LLR observations

Planetary/lunar solution depends on:

- a dynamical model
- a model of data reduction
- a set of parameters ← least square fit to observations

Parameters involved in LLR measurements (188)

- positions of reflectors
- positions and velocities of stations
- Moon's initial conditions (position, velocity and librations)
- EMB's initial conditions (position and velocity)
- Stokes coefficients (up to 4th degree)
- time delays, Love numbers (Earth, Moon)
- post-newtonian parameters
- offsets applied on some observations (40x2)

But some of them are:

- not independent (transmission and reception stations of Haleakala)
- better determined with planetary observations (M_E/M_M, EMB's initial conditions)
- better determined with an another technique (VLBI → motion of stations)
- are badly determined: $S_{43} = (-2.0 \pm 13.5) \times 10^{-6}$

Selection of fitted parameters

Iterations with elimination of the parameter having the greatest ratio error/value
→ increase of residuals (but weak)
→ decrease of formal errors on other parameters

Solution:		S074	…	S065	…	S059	…	S055	…	S051
Maximum ratio		750%	…	9%	…	3.6%	…	1.2%	…	0.3%
Station	Period	σ (cm)	…	σ (cm)	…	σ (cm)	…	σ (cm)	…	σ (cm)
Grasse (1)	1984-1986	15,9	…	15,9	…	16,0	…	15,6	…	16,2
Grasse (2)	1987-1995	6,3	…	6,3	…	6,4	…	6,0	…	8,2
Grasse (3)	1995-2010	3,7	…	3,7	…	4,0	…	5,4	…	6,9
Mc Donald	1969-1985	31,2	…	31,4	…	31,8	…	36,1	…	50,0
MLRS1 (1)	1982-1985	73,3	…	73,0	…	73,3	…	72,5	…	71,7
MLRS1 (2)	1986-1988	8,0	…	7,5	…	7,3	…	7,4	…	9,8
MLRS2 (1)	1988-1999	4,3	…	4,3	…	4,3	…	4,3	…	6,5
MLRS2 (2)	1999-2008	4,6	…	4,6	…	4,8	…	4,9	…	6,5
Haleakala	1984-1992	8,1	…	8,2	…	8,1	…	8,4	…	11,6
Apollo	2006-2009	4,8	…	4,9	…	4,9	…	5,3	…	7,1

formal error (1-σ) on C_{33M} : $6.8\times10^{-7} \rightarrow 3.3\times10^{-8} \rightarrow 6.3\times10^{-9} \rightarrow 5.2\times10^{-9} \rightarrow 4.6\times10^{-9}$

Choice of maximum ratio <5% leads to 59 parameters fitted

Residuals comparison
INPOP10a / DE423

		INPOP10a	*DE423*
Station	Period	σ (cm)	σ (cm)
Grasse (1)	1984-1986	16,0	*14,7*
Grasse (2)	1987-1995	6,4	*5,9*
Grasse (3)	1995-2010	4,0	*3,9*
Mc Donald	1969-1985	31,8	*29,8*
MLRS1 (1)	1982-1985	73,3	*70,3*
MLRS1 (2)	1986-1988	7,3	*6,1*
MLRS2 (1)	1988-1999	4,3	*4,7*
MLRS2 (2)	1999-2008	4,8	*4,6*
Haleakala	1984-1992	8,1	*8,1*
Apollo	2006-2009	4,9	*4,7*

DE423: fit of all parameters only involved in reduction of observations
Residuals DE423 better than INPOP10a ← lunar core ?

Tests of post-newtonian parameters β and γ

PN parameters are involved in:

- the dynamical part of the solution (equations of motion)
  - EIH acceleration vector (β,γ)
  - geodesic additional torque upon the Earth and the Moon (γ)
- the reduction of observations
  - time scale transformation TT-TDB (β,γ)
  - time delay due to relativistic light deflection (γ)

Fit of β and/or γ together with the same 59 parameters as in INPOP10a:
→ confidence limits at 99.7% (~9.8σ):

- (59)+β: β-1=(-0.2±1.4)x10^{-3}
- (59)+γ: γ-1=(-1.1±2.5)x10^{-3}

→ ~ Müller et al., 2008

- (59)+β+γ: β-1=(5.1±5.1)x10^{-3} and γ-1=(-9.7±9.0)x10^{-3}

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- (59)+β: β-1=(-0.2±1.4)x10^{-3} cor(β;X)<0.35
- (59)+γ: γ-1=(-1.1±2.5)x10^{-3} cor(γ;X)<0.33
- (59)+β+γ: β-1=(5.1±5.1)x10^{-3} and γ-1=(-9.7±9.0)x10^{-3} cor(β;γ)=-0.96

Strong correlation → biased values ?

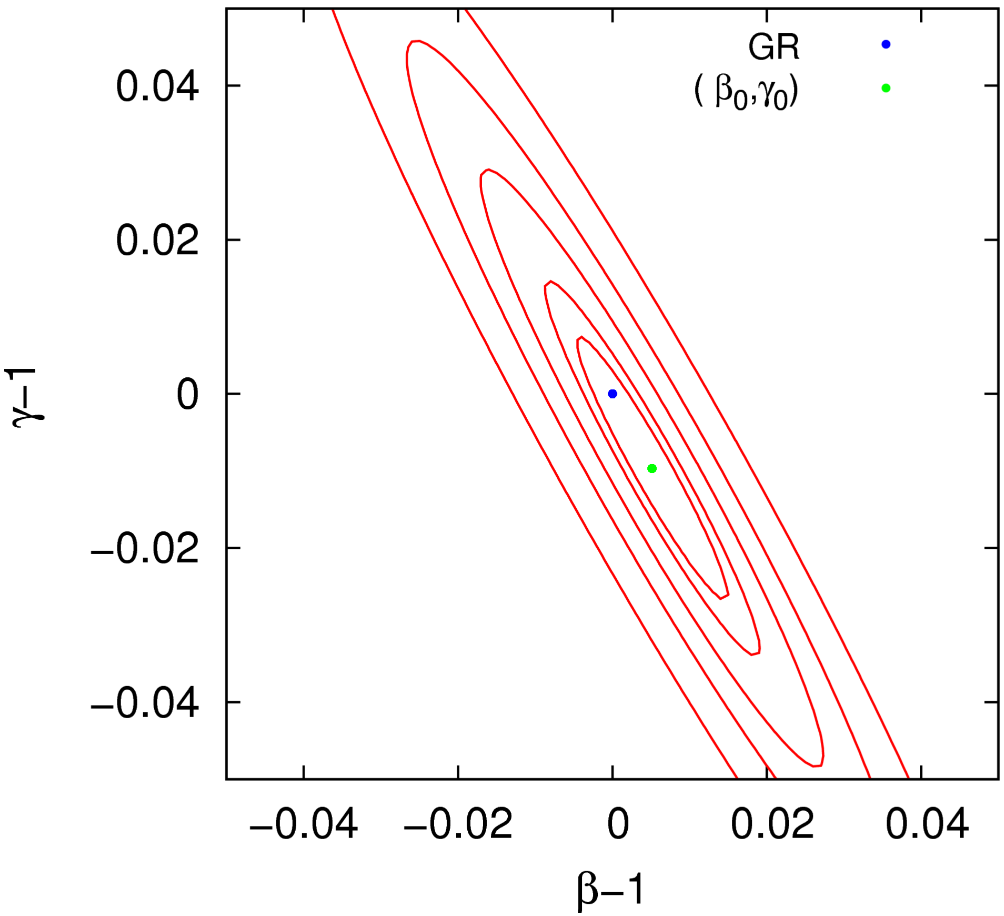
Map of (β,γ)

Method:

1600 couples of (β,γ) values fixed in $[-0.05,0.05]^2$
for each set, fit of the same 59 parameters as in INPOP10a

then computation of $\chi^2(\beta,\gamma)=\sum_i \rho_i^2 (O-C)_i^2$ and $R(\beta,\gamma)=\sqrt{\chi^2/\chi_0^2}-1$

Contour lines for R= 1%, 2%, 5%, 10% and 20%



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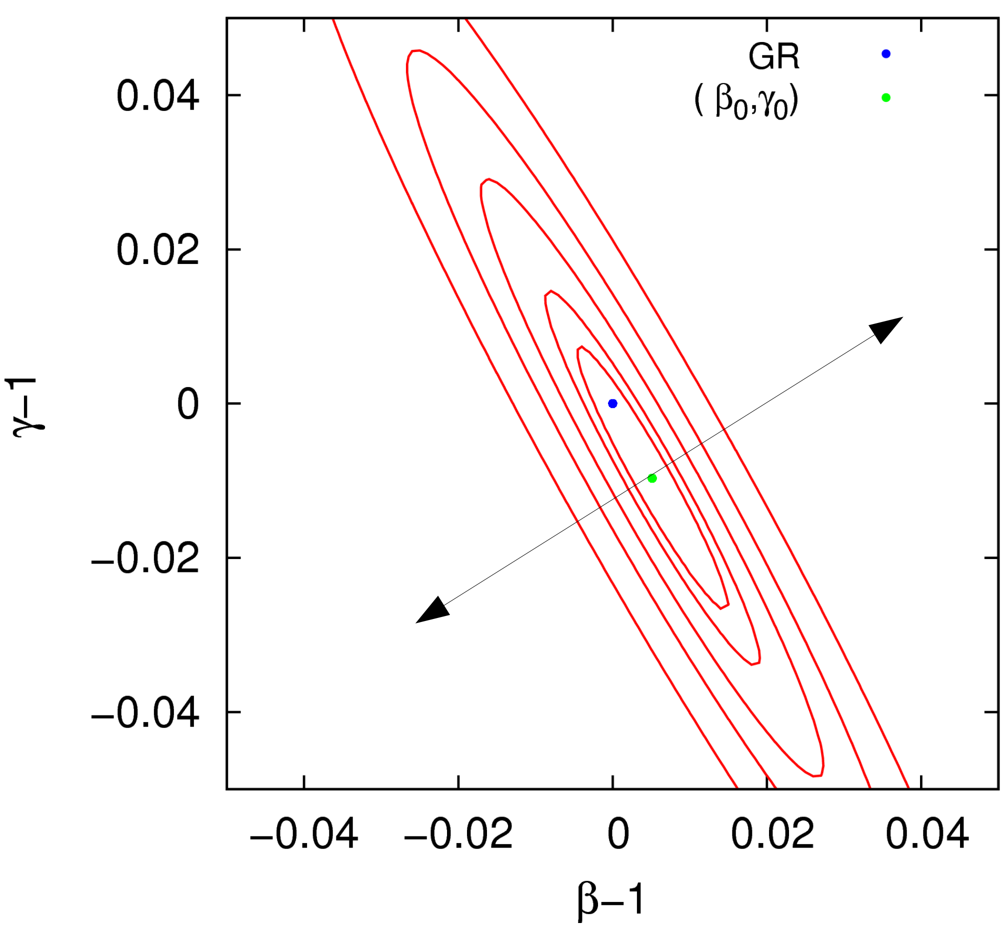

better determined combination:

2β-11γ ≠4β-γ (η+3)

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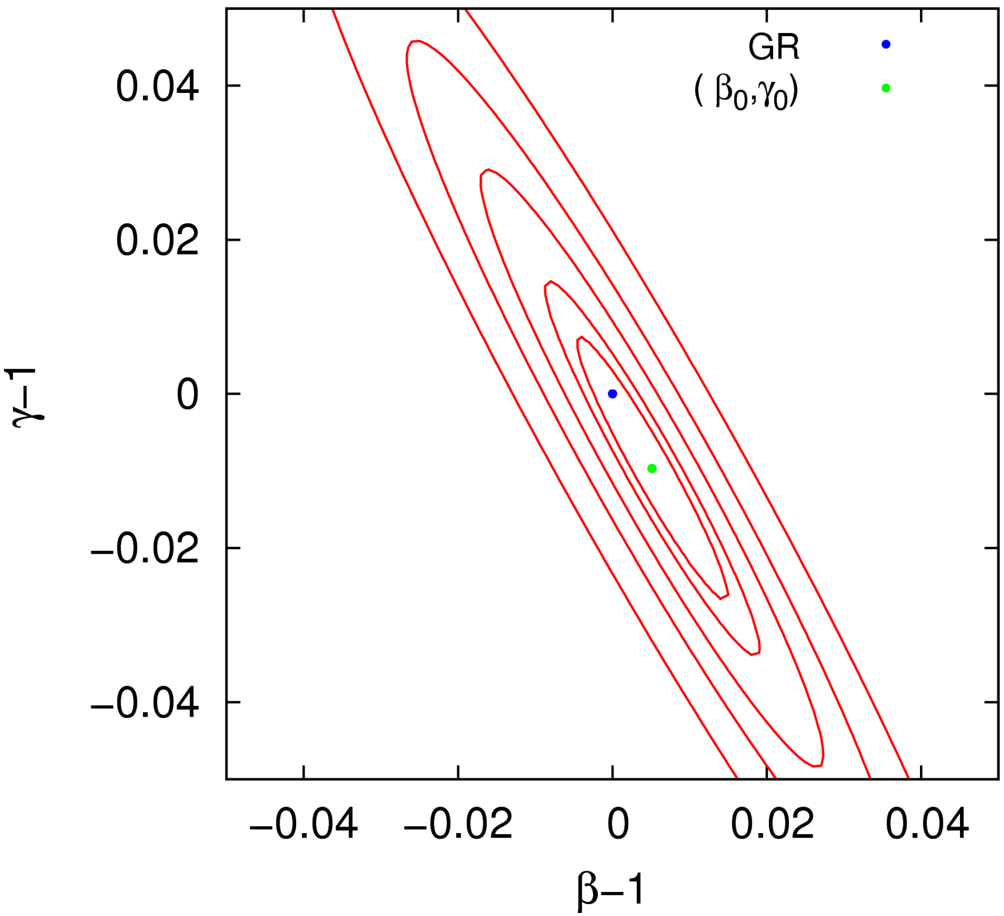

better determined combination:
$2\beta-11\gamma \neq 4\beta-\gamma \quad (\eta+3)$

$R(1,1) < 1.005 \times R(\beta_0,\gamma_0)$
Grasse LLR residuals 4 cm → 4.02 cm
→ not significant

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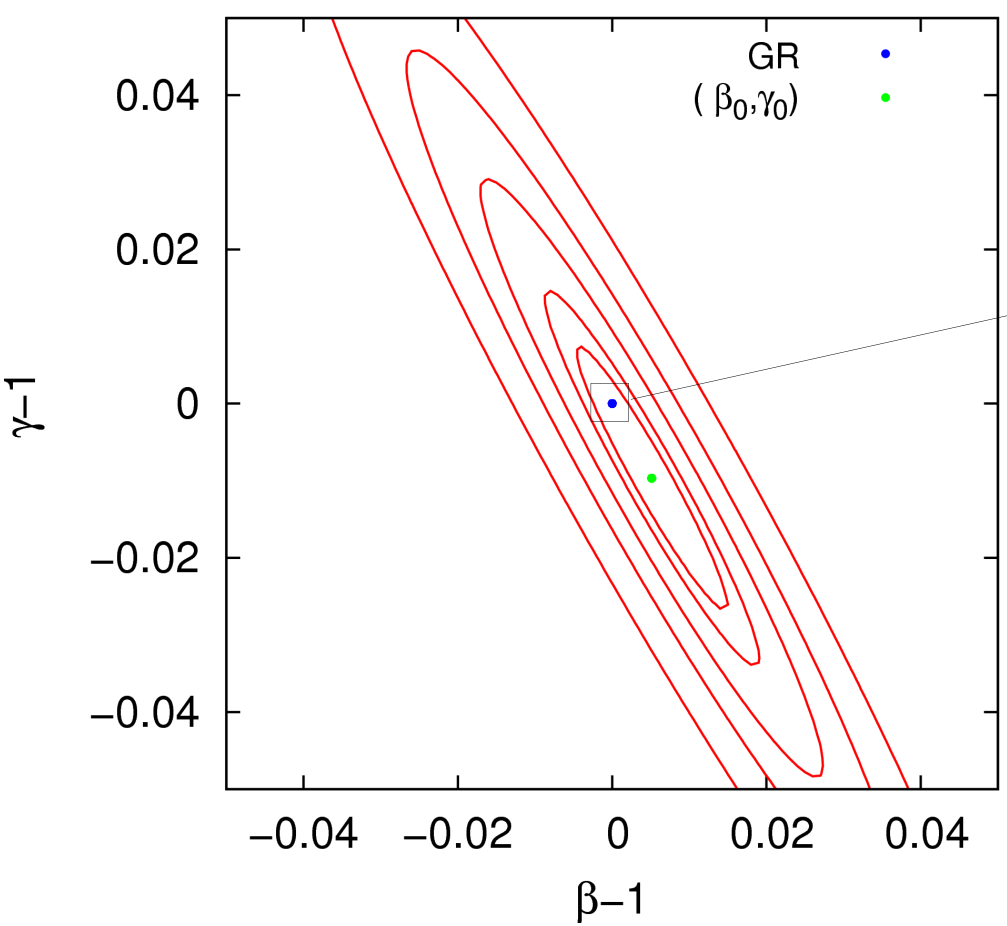

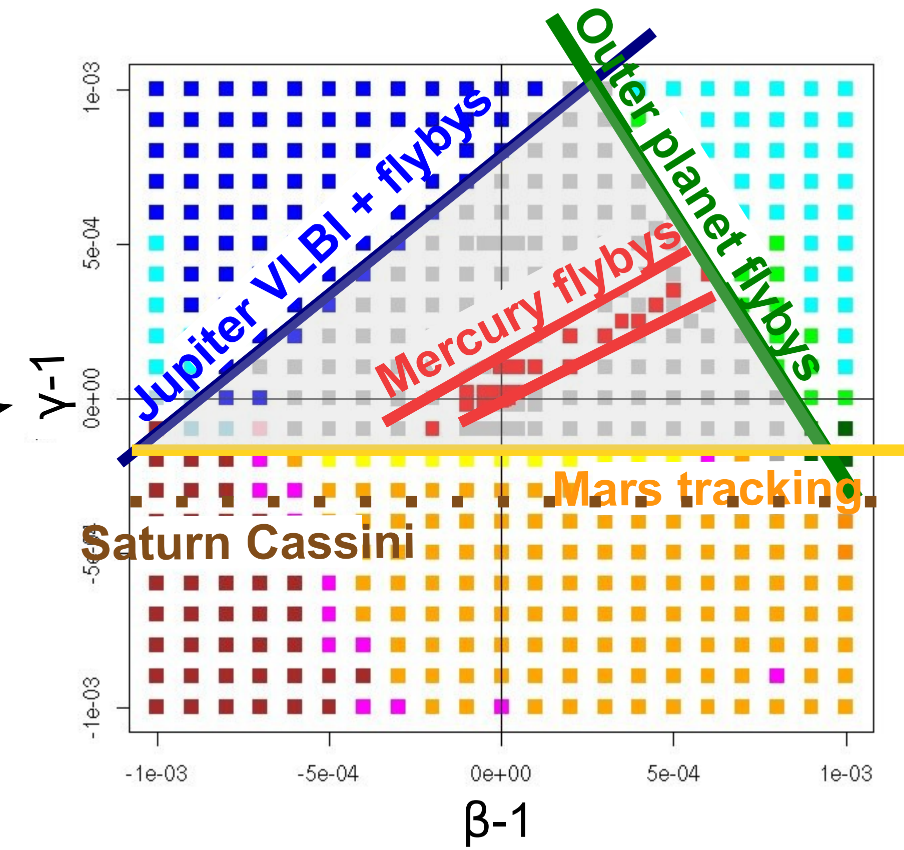

better constraints with planetary observations

Conclusion

- INPOP10a built with 59 fitted parameters to LLR observations
- residuals close but not as good as DE423 ones
- uncertainties on β or γ are consistent with Müller et al., 2008
- fitted value of (β,γ) together might be not significant
- better constraints with planetary observations than with LLR