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Physique quantique et applications

Testing gravity law in the solar system

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Framework of metric extension of GR

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- Gravitation ↔ geometry of space-time
  - → equivalence principle preserved
  - → metric theory $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$

- Coupling of curvature to energy may be modified.

$$\mathrm{E}_{\mu\nu} \equiv \mathrm{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\mathrm{R} = \frac{8\pi\mathrm{G}}{c^4}\mathrm{T}_{\mu\nu} \quad \longrightarrow \quad \mathrm{E}_{\mu\nu}[k] = \chi_{\mu\nu}{}^{\rho\sigma}[k]\mathrm{T}_{\rho\sigma}[k]$$

  - → more general linear relation in Fourier space.

- Two main consequences :
  - → different couplings for the two independent sectors of traceless and traced $E_{\mu\nu}$.
  - → G is replaced by two “running constants” $G^{(0)}[k]$ and $G^{(1)}[k]$.

- It defines two sectors.
  - → special combinations of these two sectors can either mainly affect matter or light propagation.

Framework in a simple solar system model

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- Solution of Einstein equation in isotropic gauge $\mathrm{d}s^2 = g_{00}\,(c\mathrm{d}t)^2 + g_{rr}\,\mathbf{d}\boldsymbol{r}^2$

$$g_{00} = 1 + 2\phi + 2\phi^2 + \dots \quad , \quad -g_{rr} = 1 - 2\phi + \dots \qquad \phi = -\frac{\mathrm{GM}}{rc^2}$$

- PPN framework

$$g_{00} = 1 + 2\phi + 2\beta\phi^2 + \dots \quad , \quad -g_{rr} = 1 - 2\gamma\phi + \dots$$

→ 30 years of PPN tests have selected a vicinity of GR in the PPN framework.

$$\gamma - 1 < 2 \times 10^{-5} \quad , \quad \beta - 1 < 10^{-4}$$

- The two sectors of the new framework matches a liberty in the two functions g_{00} and g_{rr}

$$g_{00} = [g_{00}]_{\mathrm{GR}} + \delta g_{00}$$
$$g_{rr} = [g_{rr}]_{\mathrm{GR}} + \delta g_{rr}$$

Example of explicit phenomenological model

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- We can search for Taylor expansion valid inside the solar system.

$$\delta g_{00} = 2\sum_{n>0} \alpha_n r^n \quad , \quad \delta g_{rr} = 2\sum_{n>0} \chi_n r^n$$

→ equivalently, it can also be analyzed as PPN parameters depending on distance.

$$\beta(r) - 1 = \left(\frac{c^2}{\mathrm{GM}}\right)^2 \sum_{n>0} \alpha_n r^{n+2} \quad , \quad \gamma(r) - 1 = -\frac{c^2}{\mathrm{GM}} \sum_{n>0} \chi_n r^{n+1}$$

- Modifications of g_{00} are already well constrained by planetary tests.
  → in particular, a modification of g_{00} only (sector 1) accounting for the Pioneer anomaly would have been seen.

- Example of phenomenological modification of g_{rr} : $\delta g_{rr} = 2\chi_2 \, r^2$
  → it corresponds to a constant curvature (outside the sources !) $\mathrm{R}_{rr} = 8\chi$

Effects on matter

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- Geodesic equation

$$\ddot{x}^i(t) = \frac{c^2}{2}\eta^{ij}\partial_j h_{00} - c^2\eta^{ij}\partial_0 h_{0j} + \frac{c}{2}\dot{x}^i\partial_0 h_{00} - c\eta^{ik}\dot{x}^j\partial_0 h_{kj} - c\eta^{ik}\dot{x}^j\left(\partial_j h_{0k} - \partial_k h_{0j}\right)$$
$$+\dot{x}^i\dot{x}^k\partial_k h_{00} - \dot{x}^j\dot{x}^k\eta^{il}\left(\partial_j h_{kl} - \frac{1}{2}\partial_l h_{jk}\right) - \frac{\dot{x}^i\dot{x}^k\dot{x}^l}{c}\left(\frac{1}{2}\partial_0 h_{kl} - \partial_k h_{0l}\right)$$

→ anomalous (coordinate) acceleration

$$[\delta a_r]_{\chi_2} = 2\chi_2\, r(v_r^2 - v_\theta^2)$$
$$[\delta a_\theta]_{\chi_2} = 4\chi_2\, r v_r v_\theta$$

→ the radial acceleration is a negative constant (if χ_2>0) for bounded trajectories and a positive constant (if χ_2>0) for escaped trajectories.

→ the orthoradial acceleration vanishes for circular orbit and radial escape.

- The anomalous acceleration has to stay small to remain compatible with planetary ephemerides.

$$\delta a \sim \chi_2 \mathrm{GM}$$

Effects on spacecraft Doppler traking

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- Effect on light propagation

  → modification of Shapiro time delay

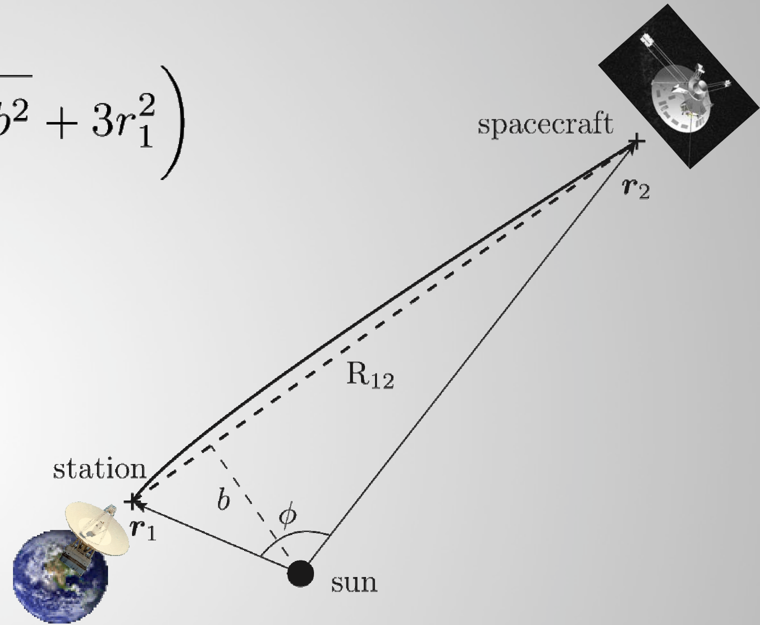
$$c\delta\mathcal{T} = -\frac{1}{3}\chi_2 \mathrm{R}_{12}\left(\mathrm{R}_{12}^2 - 3\mathrm{R}_{12}\sqrt{r_1^2 - b^2} + 3r_1^2\right)$$

  → Doppler observable and overall optical range - rate

$$y = \frac{\delta\nu}{\nu} \equiv \frac{1}{c}\frac{\mathrm{d}\ell}{\mathrm{d}t}$$

  → effective acceleration along the line of sight

$$a \sim \frac{c}{2}\dot{y}$$



- Consequences :

  → for non radial motion, the contribution coming from light is bigger than the one coming from the modified spacecraft dynamics.

  → The effect on light naturally leads to periodic anomalies modulated by the annual and diurnal motions of the Earth.

Conclusion

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- There are two sectors in an extended metric extension of GR.

- The second sector could have escaped planetary tests but could still produce observable effects in Doppler tracking of spacecrafts.

  → Continue test of gravity with spacecraft tracking !

  - The second sector produces periodic anomalies modulated by the annual and diurnal motions of the Earth :

  → through the range or Doppler observable for spacecraft tracking.

  → through anomalies of declinaison and longitude in observations of planets.

$$\delta \boldsymbol{n} = \mathbf{grad}\ c\delta\mathcal{T}$$