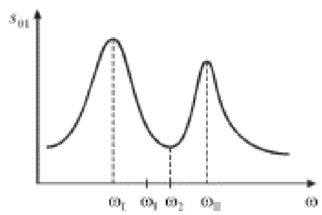

Development of Long-Term Numerical Ephemerides of Major Planets to Analytical Series

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Modern Representation of Planetary Coordinates

1. Numerical ephemerides

- DE-series (JPL NASA, the USA)
- EPM-series (IAA RAS, Russia)
- INPOP- series (l'Observatoire de Paris, France)

Advantage: high precision

Disadvantages: take up to hundreds Mb;
depend on the computer platform

2. Analytical/semi-analytical theories

- done at BDL/IMCCE, USNO, …
  the latest theory is VSOP2010 (Francou & Simon 2010)

Advantages: compactness, computer platform independence

Disadvatages: it is relatively difficult to develop them;
the theories still have to be fitted to numerical ephemerides

Modern Representation of Planetary Coordinates (cont.)

3. Frequency analysis of planetary/lunar ephemerides

- Chapront (1995): planetary ephemerides
- Kudryavtsev (2007): lunar ephemeris
- the present study : planetary ephemerides

Advantages: compactness,
computer platform independence,
the accuracy of numerical ephemeris is about reached

Disadvantages: the "close frequencies" problem,
but it can be solved (diminished) by use of long-term numerical ephemerides (over thousands of years)
-> new spectral analysis methods are required

Development of Planetary Ephemerides

Step 1: Numerical values of planetary coordinates are calculated on every day within a long-term time interval (3000BC–3000AD). The DE-406 are used as a source.

Step 2: The values series are processed by a new spectral analysis method (Kudryavtsev 2004, 2007). Expansion of a tabulated function to Poisson series is directly made where both amplitudes and arguments of the terms are high-degree time polynomials (opposed to the classical Fourier analysis where both the amplitudes are frequencies are constants).

Polynomial arguments of the series are various combinations of multipliers of planetary mean longitudes. *(In a more general case, multipliers of Delaunay arguments and those of the lunar longitude can be added, but not for planets.)*

Spectral Analysis to Poisson Series

Let $f(t)$ be sampled over an interval [-*T*, *T*] with a small step.

We shall find representation of $f(t)$ in the form

$$f(t) \approx A_0 + A_1 t + \ldots + A_p t^p + \sum_{k=1}^{N} \Big\{ \left[A_{k0}^c + A_{k1}^c t + \ldots + A_{kp}^c t^p \right] \cos\omega_k(t) + \left[A_{k0}^s + A_{k1}^s t + \ldots + A_{kp}^s t^p \right] \sin\omega_k(t) \Big\}$$

where $\omega_k(t)$ are some pre-defined polynomilal arguments:

$$\omega_k(t) = \nu_k t + \nu_{k2} t^2 + \ldots + \nu_{kq} t^q \quad \left(\omega_0(t) \equiv 0\right).$$

- At the 1st step we numerically calculate the scalar products:

$$A_{lk}^c = \frac{1}{2T} \int_{-T}^{T} f(t) t^l \cos\omega_k(t) \chi(t) dt, \quad A_{lk}^s = \frac{1}{2T} \int_{-T}^{T} f(t) t^l \sin\omega_k(t) \chi(t) dt,$$

basis functions

$\chi(t) = 1 + \cos\dfrac{\pi}{T} t$ is the weighting function (the Hanning filter)

- At the 2nd step the basis must be orthogonalized

(! $N \sim 10^3$-10^4; arguments are high-degree time polynomials).

Representation of Planetary Coordinates

Some options for planetary coordinates to be developed:

- rectangular (Cartesian) coordinates;
- spherical coordinates: distance, longitude, latitude;
- osculating Keplerian elements;
- **differences of osculating Keplerial elements from their mean values** (the latter are taken from Simon et al. 1994).

For example, the mean longitude is represented as

$$\lambda(t) = \bar{\lambda}(t) + \sum_{k=1}^{N_0} A_{k0} \sin\left[\omega_{k0}(t) + \varphi_{k0}\right] + \sum_{k=1}^{N_1} A_{k1} t \sin\left[\omega_{k1}(t) + \varphi_{k1}\right] + \\ + \sum_{k=1}^{N_2} A_{k2} t^2 \sin\left[\omega_{k2}(t) + \varphi_{k2}\right]$$

where $\bar{\lambda}(t)$ is the mean mean longitude of a planet

and $\omega_k(t) = \nu_k t + \nu_{k2} t^2 + \nu_{k3} t^3 + \nu_{k4} t^4$; $k = 1,2,\ldots$

Expansion of Mean Longitude of Telluric Planets over 3000BC-3000AD

Planet	**Maximum difference from DE-406**	**Number of terms**
Mercury	0.014"	331
Venus	0.035"	450
EMB	0.022"	586
Mars	0.068"	948

For comparison: VSOP2010 (the current version): over 4000BC - 8000AD the precision is a few 0.1" for Mercury, Venus, EMB and a few 1" for other major planets (Francou & Simon 2010: Book of Abstracts of the Journées 2010).

Conclusions and Perspectives

The new modification of the spectral analysis method is a powerful tool for expansion of planetary coordinates over long-term time intervals.

The more interval is, the better close frequencies are separated, the better approximation of numerical ephemerides by high-order Poisson series can be done.

The DE-406 long-term ephemerides are already outdated; the INPOP solution available to public is over 1000-3000.

How can I get the numerical planetary ephemerides INPOP (DE?, EPM?) over 4000BC – 8000AD (or a longer time interval)?