An extension of the IAU framework for reference systems

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Content

- Scaling Laws
- Matching to cosmology
- Experimental Gravity Issues
- Parameter’s manifold and gauge freedom

Reference Systems in a Few Words

Theory
- Lagrangian
- Hamiltonian
- Field Equations and Variables

Gauge Conditions :
- Field Variables
- Configuration Parameters
- Harmonic, Standard, Painleve, Others
- Osculating, Canonical, Computation-induced

Boundary Conditions on the Field Variables
- External Problem of Motion (orbit)
- Internal Problem of Motion (rotation)

Solution of the Field Equations
- Global Coordinates (BCRS -> galaxy -> cosmology)
- Local Coordinates (GCRS -> Earth-Moon ->observer)
- Field Variables

Solution of the dynamic equations of motion
- Initial conditions
- Configuration Parameters
- Method of integration

IAU scaling rules

- Current time scales:
  - Barycentric   t = TCB
  - Geocentric   T=TCG
- Scaling of time coordinates:
  - TT = F_G(TCG) $F_G = 1 - L_G$ $L_G = 6.969\ldots \times 10^{-10}$
  - TDB = F_B(TCB) $F_B = 1 - L_B$ $L_B = 1.550\ldots \times 10^{-8}$
- Scaling of time entails the scaling of space

Scaling the metric or coordinates?

$$g_{00}c^2dt^2 + 2g_{0i}c\ dt\ dx^i + g_{ij}dx^i dx^j = 0$$

$$g_{00}c^2\ d(\underbrace{F_B t}_{TDB})^2 + 2g_{0i}c\ d(\underbrace{F_B t}_{TDB})dx^i + g_{ij}dx^i dx^j \neq 0$$

$$g_{00}c^2\ d(F_B t)^2 + 2g_{0i}c\ d(F_B t)dx^i + g_{ij}d(F_B x^i)d(F_B x^j) = 0$$

$g_{\alpha\beta} \Rightarrow F_B^2\ g_{\alpha\beta}$ conformal transformation of the metric

(time and space coordinates are not scaled !)

The metric tensor

$$ds^2 = \underbrace{\left(-1 + \frac{2U}{c^2} + \dots\right)}_{g_{00}} c^2 dt^2 + \underbrace{\frac{4U^i}{c^2}}_{2g_{0i}} dt dx^i + \underbrace{\left[\left(1 + \frac{2U}{c^2}\right)\delta_{ij} + \dots\right]}_{g_{ij}} dx^i dx^j$$

$$U = U_{\oplus} + \bar{U}; \qquad U^i = U^i_{\oplus} + \bar{U}^i$$

$$\Delta U_{\oplus} = -4\pi G \rho_{\oplus} \qquad \Delta U^i_{\oplus} = -4\pi G \rho_{\oplus} v^i_{\oplus}$$

$$\underbrace{\Delta \bar{U} = 0} \qquad \underbrace{\Delta \bar{U}^i = 0}$$

Solution of these equations determine the type of the coordinates: BCRS or GCRS

Time scaling and the metric

$$\Delta\bar{U} = 0$$

$\Downarrow$ solution for external field, the inertial term, and the scaling term

$$\bar{U} = Q + Q_i X^i + \frac{1}{2} Q_{ij} X^i X^j + \frac{1}{3} Q_{ijk} X^i X^j X^k + \ldots$$

Time transformation with $Q \neq 0$

$$\frac{dT}{dt} = 1 + \underbrace{\frac{1}{2}\frac{v_{\oplus}^2}{c^2} + \sum_{a\neq\oplus}\frac{GM_a}{c^2 r_a}}_{\text{secular term} = \mathrm{L_B}} - \frac{Q}{c^2} + O\left(\frac{1}{c^4}\right)$$

The scaling function Q in the metric allows to keep the time unit the same in all reference frames. This is SI second.

Scaling of the spatial coordinates

$$\Delta \bar{U} = 0$$

$\Downarrow$ solution for the space-space component of the metric

$$\bar{U} = P + P_i X^i + \frac{1}{2} P_{ij} X^i X^j + \frac{1}{3} P_{ijk} X^i X^j X^k + \ldots$$

P and P_i are arbitrary, $\quad P_{ij} = Q_{ij}, \quad P_{ijk} = Q_{ijk}, \ldots$

IAU resolutions rescale the space coordinates to keep the speed of light constant: c=dx/dt=dX/dT with $P = 0$.

Another way around is to introduce $P \neq 0$ to the metric:

$$X^i = \left(1 + \frac{1}{2c^2} v^i v^j + \delta^{ij} \sum_{a \neq \oplus} \frac{GM_a}{c^2 r_a} - \underbrace{\frac{P}{c^2}}_{=:\ L_B} \right) r_\oplus^i + \ldots + O\left(\frac{1}{c^4}\right)$$

Scaling of mass

Mass is re-scaled to keep the equations of motion scale invariant. Blanchet-Damour's definition works perfect:

$$M = m\left(1-\frac{P}{c^2}\right)-\frac{1}{c^2}\left(2P_i I^i+\frac{3}{2}P_{ij}I^{ij}+\frac{4}{3}P_{ijk}I^{ijk}+\ldots\right)$$

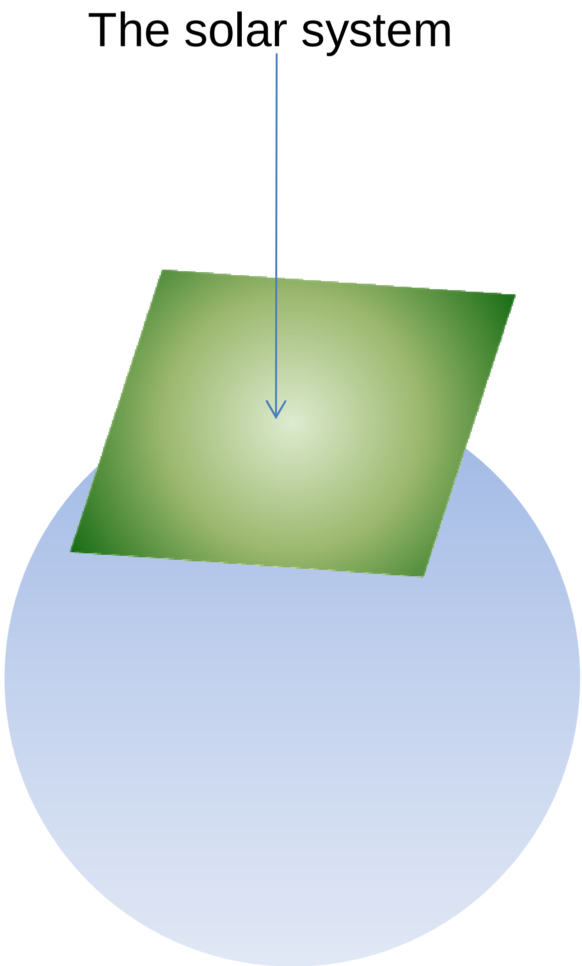
where

$$m=\int_{V_\oplus}\rho^*\left[1+\frac{1}{c^2}\left(\frac{1}{2}v^2+\Pi-\frac{1}{2}U_\oplus\right)\right]d^3X+O\left(\frac{1}{c^4}\right)$$

IAU Resolutions and Cosmology

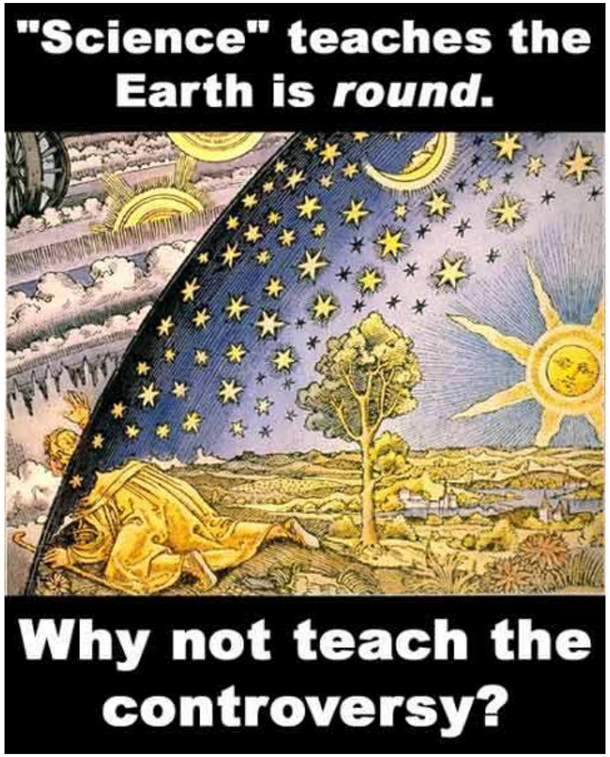
The standard IAU resolutions assume that the space-time is asymptotically flat at infinity.

However, we live in expanding universe, which is described by the cosmological metric of Friedmann-Robertson-Walker



IAU resolutions on reference frames are formulated in the “tangent plane”.

This is similar to the ancient teaching of the flat Earth (which is not dead)



Extending the IAU Resolutions to Cosmology

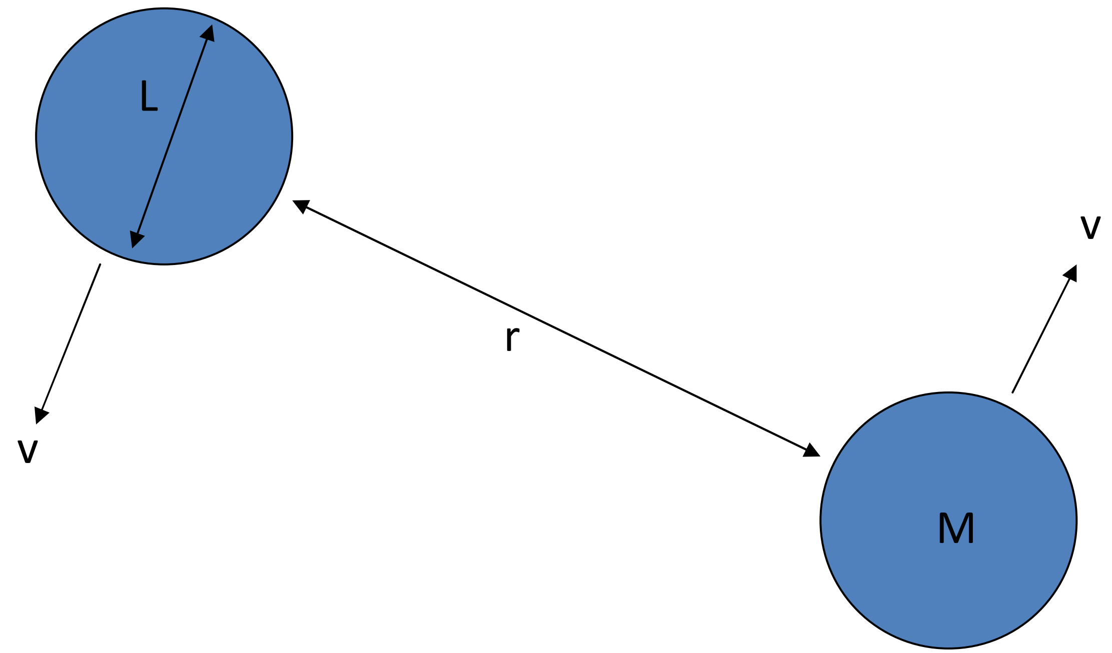
- Gravitational-wave astronomy of cosmological sources in early universe
- Anisotropy and polarization of cosmic microwave background radiation
- Cosmological gravitational lenses
- Dynamics of interacting galaxies
- Dark matter and dark energy
- Cosmological effects inside the solar system (Pioneer anomaly ? Secular increase of AU?)
- Cosmological tests of gravity theories

Mathematics to solve

- Interaction between gravity of the solar system and cosmological expansion (Do the orbits of the solar system objects take part in the cosmological expansion?)
- Unambiguous separation of matter and metric tensor to the background and perturbed values
- Back-reaction of the perturbations on the background expansion (non-linear effects)

$$\eta \approx \frac{GM}{c^2 L}$$

characterizes the strength of gravity inside the body



$$\varepsilon \approx \frac{\mathrm{v}}{c} \cong \left(\frac{GM}{c^2 r} \right)^{1/2} \cong \frac{r}{\lambda}$$

Characterizes:

(1) the speed of relative motion between the bodies

(2) the strength of gravity between the bodies

(3) a wavelength of gravitational waves

Post-Newtonian approximations in asymptotically-flat space-time

The Minkowski metric

The gravitational constant

$$g_{\alpha\beta} = f_{\alpha\beta} + G h^{(1)}_{\alpha\beta} + G^2 h^{(2)}_{\alpha\beta} + \ldots$$

The metric tensor

The metric tensor perturbations

Two basic parameters:

$$\eta \approx \frac{GM}{c^2 L} \qquad\qquad \varepsilon \approx \frac{\mathrm{v}}{c}$$

Post-Newtonian approximations in cosmology

‘radius’ of the universe

The metric tensor of the standard IAU resolutions

$$g_{\alpha\beta}(t,\vec{x}) = R^2(t)\left[f_{\alpha\beta} + G h^{(1)}_{\alpha\beta}\big(t,\vec{x},R(t)\big) + G^2 h^{(2)}_{\alpha\beta}\big(t,\vec{x},R(t)\big) + \ldots \right]$$

The metric tensor

The PN metric tensor perturbations

Three basic parameters:

$$\eta \approx \frac{GM}{c^2 L} \qquad \varepsilon \approx \frac{\mathrm{v}}{c} \qquad \sigma \approx \frac{r}{R} = \varepsilon\delta \qquad \delta \approx \frac{\lambda}{R}$$

Typical values of the post-Newtonian parameters

	Binary pulsar	Solar system	Galaxy	Cluster of galaxies	Super-cluster
η	10^{-1}	2×10^{-6}	10^{-6}	10^{-8}	10^{-8}
ε	10^{-3}	10^{-4}	10^{-3}	10^{-4}	10^{-4}
σ	10^{-17}	10^{-13}	10^{-7}	10^{-5}	10^{-3}

Extension of the IAU metric to the Universe

$$\gamma^{\alpha\beta} \equiv h^{\alpha\beta} - \frac{1}{2} f^{\alpha\beta} h$$

$$\gamma^{0\beta}{}_{|\beta} = 2H\varphi; \qquad \gamma^{i\beta}{}_{|\beta} = 0$$

$$\gamma^{00} = \frac{w}{4c^2}; \quad \gamma^{0i} = -\frac{w^i}{4c^2}; \quad \gamma^{ij} = \frac{w^{ij}}{4c^2}; \quad \chi = w - \frac{\varphi}{2}$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\partial_\tau^2\right)\chi - 2H\partial_\tau\chi + \frac{5}{2}H^2\chi = -4\pi G\left(T_{00}^{(m)} + T_{ii}^{(m)}\right)$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\partial_\tau^2\right)w - 2H\partial_\tau w = -4\pi G\left(T_{(m)}^{00} + T_{(m)}^{ii}\right) - 4H^2\chi$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\partial_\tau^2\right)w^i - 2H\partial_\tau w^i + H^2 w^i = -4\pi G T_{(m)}^{oi}$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\partial_\tau^2\right)w^{ij} - 2H\partial_\tau w^{ij} = -4\pi G T_{(m)}^{ij}$$

IAU resolutions and Experimental Gravity

Standard IAU resolutions are formulated in GR framework.

PPN formalism is a generalized framework for testing alternative theories of gravity.

PPN is not covariant. It was formulated in a single coordinate system BCRS (PPN reference frame)

PPN in local frames - ? Good progress (talk of Y.Xie)

The standard PPN:

*1) Operates with a single (global) coordinate system (t,**x**)*

2) Mixes up the scalar, vector and tensor modes

$$h_{00} = A^0 = \varphi \equiv U$$ "scalar" potential

$$h_{0i} = A^i \equiv U^i$$ "vector" potential

$$h_{ij} = \gamma U \delta_{ij} + \xi U_{ij}$$ "tensor" potential

PPN metric tensor

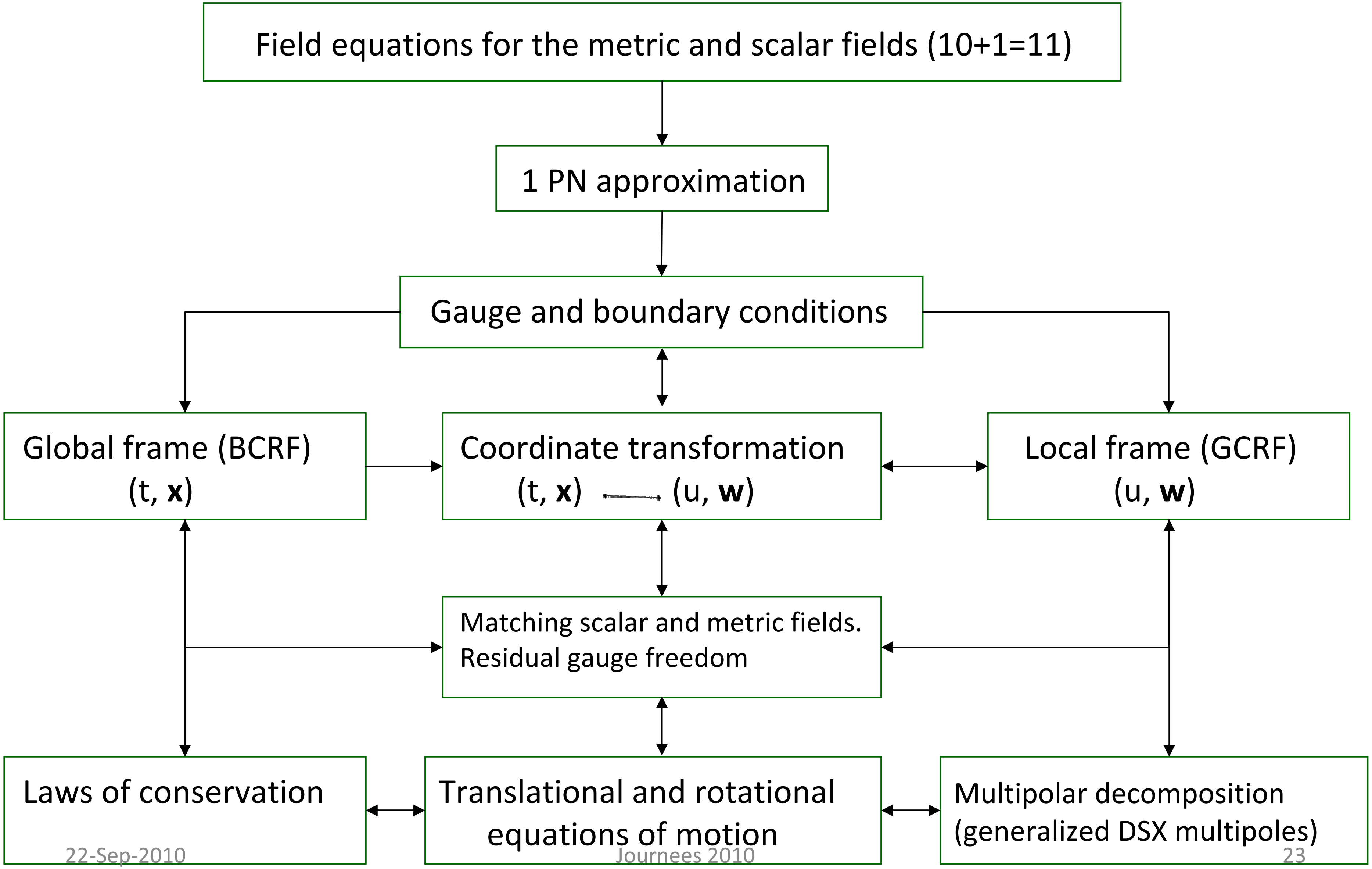
$$g_{\alpha\beta}(t,x) = F_{\alpha\beta}\left[U(t,x), U^{i}(t,x), U^{ij}(t,x); \alpha_1, \alpha_2, \alpha_3, \beta, \gamma, \zeta_1, \zeta_2, \zeta_3, \zeta_4, \xi\right]$$

Problems to fix:

1. The gauge freedom of the field variables
2. Transformations from the global to local coordinates
3. Gauge-independent definition of observables

Scalar-tensor PPN theory of reference frames

Kopeikin & Vlasov(2004); Xie & Kopeikin (2010)



PPN Transformations from GCRS to BCRS

$$u = t - \epsilon^2(\mathscr{A} + v_B^k R_B^k)$$
$$+ \epsilon^4 \left[\mathscr{B} + \left(\frac{1}{3} v_B^k a_B^k - \frac{1}{6} \dot{\bar{U}}(x_B) \right) R_B^2 - \frac{1}{10} \dot{a}_B^k R_B^k R_B^2 + \sum_{l=1}^{\infty} \frac{1}{l!} \mathscr{B}_{\langle L \rangle} R_B^L \right] + O(\epsilon^5) \,, \quad (8.6.1)$$
$$w^i = R_B^i + \epsilon^2 \left[\left(\frac{1}{2} v_B^i v_B^k + \gamma \delta^{ik} \bar{U}(x_B) + F^{ik} \right) R_B^k + a_B^k R_B^i R_B^k - \frac{1}{2} a_B^i R_B^2 \right] + O(\epsilon^4) \,. \quad (8.6.2)$$

Here functions $\mathscr{A}$ and $\mathscr{B}$ are solutions of the ordinary differential equations

$$\dot{\mathscr{A}} = \frac{1}{2} v_B^2 + \bar{U}(x_B) - Q \,,$$
$$\dot{\mathscr{B}} = -\frac{1}{8} v_B^4 - \left(\gamma + \frac{1}{2} \right) v_B^2 \bar{U}(x_B) + Q \left[\frac{1}{2} v_B^2 + \frac{3}{2} Q - \bar{U}(x_B) \right]$$
$$+ \left(\beta - \frac{1}{2} \right) \bar{U}^2(x_B) + 2(1+\gamma) v_B^k \bar{U}^k(x_B) - \bar{\Phi}(x_B) + \frac{1}{2} \bar{\chi}_{,tt}(x_B) \,,$$

while the other functions are defined as follows:

$$\mathscr{B}_i = 2(1+\gamma) \bar{U}^i(x_B) - (1+2\gamma) v_B^i \bar{U}(x_B) - \frac{1}{2} v_B^i v_B^2 - v_B^i Q \,, \quad (8.6.5)$$
$$\mathscr{B}_{\langle ik \rangle} = Z_{ik} + 2(1+\gamma) \bar{U}^{\langle i,k \rangle}(x_B) - 2(1+\gamma) v_B^{\langle i} \bar{U}^{,k\rangle}(x_B) + 2 v_B^{\langle i} a_B^{k\rangle} \,, \quad (8.6.6)$$
$$\mathscr{B}_{\langle iL \rangle} = Z_{iL} + 2(1+\gamma) \bar{U}^{\langle i,L \rangle}(x_B) - 2(1+\gamma) v_B^{\langle i} \bar{U}^{,L\rangle}(x_B) \,, \quad (l \geqslant 2), \quad (8.6.7)$$
$$\dot{F}^{ik} = (1+2\gamma) v_B^{[i} \bar{U}^{,k]}(x_B) - 2(1+\gamma) \bar{U}^{[i,k]}(x_B) + v_B^{[i} Q^{k]} \,. \quad (8.6.8)$$

Kopeikin & Vlasov,
Physics Reports,
Vol. 400, 209-318 (2004)

Xie & Kopeikin,
Acta Physica Slovaca,
Vol. 60, No. 4 (2010)

PPN β and γ parameters as astronomical constants

$$S = \frac{c^3}{16\pi}\int_{R^4}\left[\underbrace{\phi R}_{\text{gravity}} + \underbrace{\theta(\phi)\frac{\phi^{,\alpha}\phi_{,\alpha}}{\phi}}_{\text{scalar}} + \underbrace{\Lambda(\psi)}_{\text{matter}}\right]\sqrt{-g}\,d^4x$$

$$\phi = \phi_0\left(1+\zeta\right) \qquad \theta(\phi) = \omega + \omega'\zeta + \frac{1}{2}\omega''\zeta^2 + \ldots$$

$$G \equiv \frac{1}{\phi_0}$$

$$\gamma \equiv 1 - \frac{1}{\omega+2}$$

$$\beta \equiv 1 + \frac{\omega'}{4(2\omega+3)(\omega+2)^2}$$

$$U = \frac{GM}{r}$$

$$g_{ij} = \delta_{ij}\left[1 + \left(1+\gamma\right)\frac{U}{c^2}\right]$$

$$g_{00} = -1 + \frac{2U}{c^2} + \beta\frac{2U^2}{c^4}$$

Two facets of the fundamental speed

Speed of light

- Maxwell's electrodynamics
- No gravity, flat (Minkowskian) spacetime
- Speed of light is a fundamental constant, $\mathbf{c}_0$, in the Maxwell equations

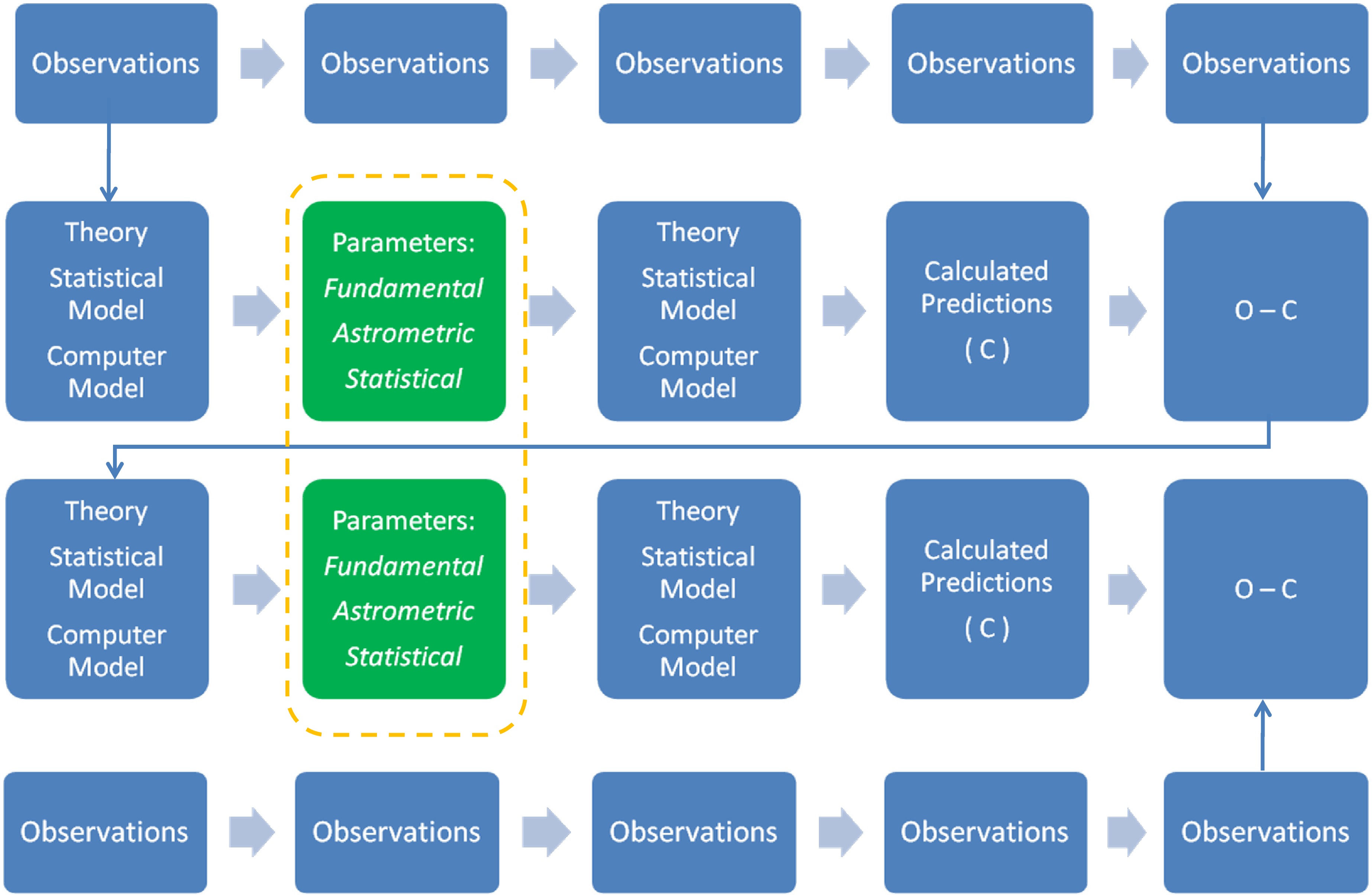
Speed of gravity

- Einstein's gravitodynamics
- Space-time is curved, has non-trivial affine connection
- Speed of gravity is a fundamental constant, $\mathbf{c}_g$, in the Einstein equations

Einstein's fundamental speed postulate (not a law!) $\boxed{\mathbf{c}_g = \mathbf{c}_0}$

The postulate was tested for the first time in 2002 VLBI experiment with Jupiter
Pulsar timing is not able to make it since it measures the combination GM / c_g^3

Manifolds, the gauge freedom, and fitting parameters


Observations
Observations
Observations
Observations
Observations
Theory
Statistical Model
Computer Model
Parameters:
Fundamental
Astrometric
Statistical
Theory
Statistical Model
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Calculated Predictions
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O – C
Theory
Statistical Model
Computer Model
Parameters:
Fundamental
Astrometric
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Theory
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Calculated Predictions
(C)
O – C
Observations
Observations
Observations
Observations
Observations

Fundamental parameters

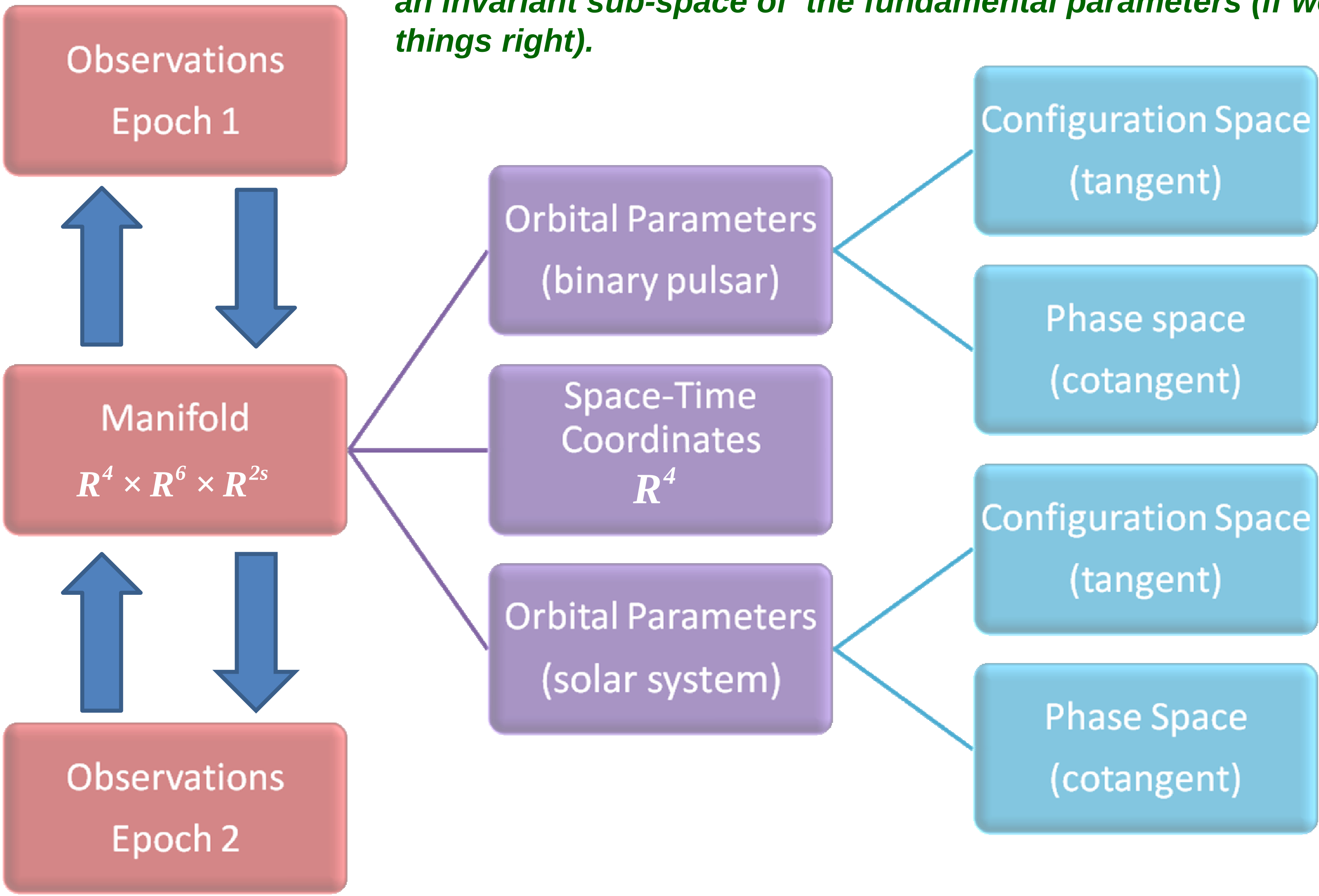
- Fundamental parameters stay invariant (= keep the same numerical value) under the change of computational algorithm, coordinates, gauge conditions

- Measured value converges to a unique limit as observations progress.
- Examples:
  - *c* *electrodynamics;*
  - *G, c* *general relativity*;
  - β, γ *scalar-tensor theory;*
  - *Gauge-invariant combinations of the post-Newtonian (post-Keplerian in pulsar timing) parameters usually made of the integrals of motion and/or adiabatic invariants.*

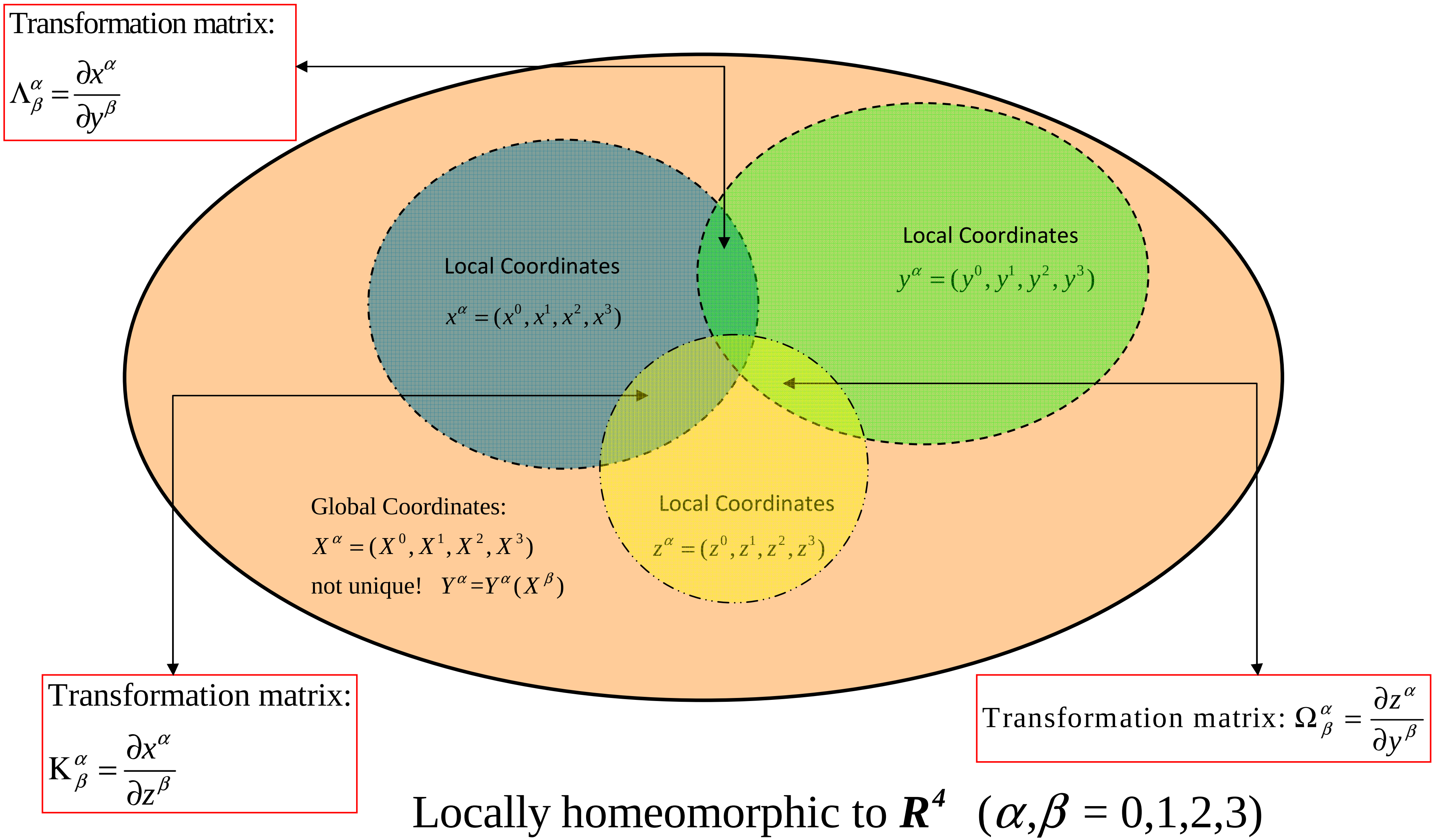
Theoretical model

- We study physical motion of celestial bodies by making use of light rays.
- Physical motion of bodies and light are described by solutions of the equations of motion which, in their own turn, depend on the solutions of equations of a field theory (= the metric, etc.)
- The physical motion is mapped onto the coordinate-parameter manifold to make a mathematical model of motion

Manifold is an intermediate mathematical device to handle observations. It has no absolute value. However, it must have an invariant sub-space of the fundamental parameters (if we do things right).

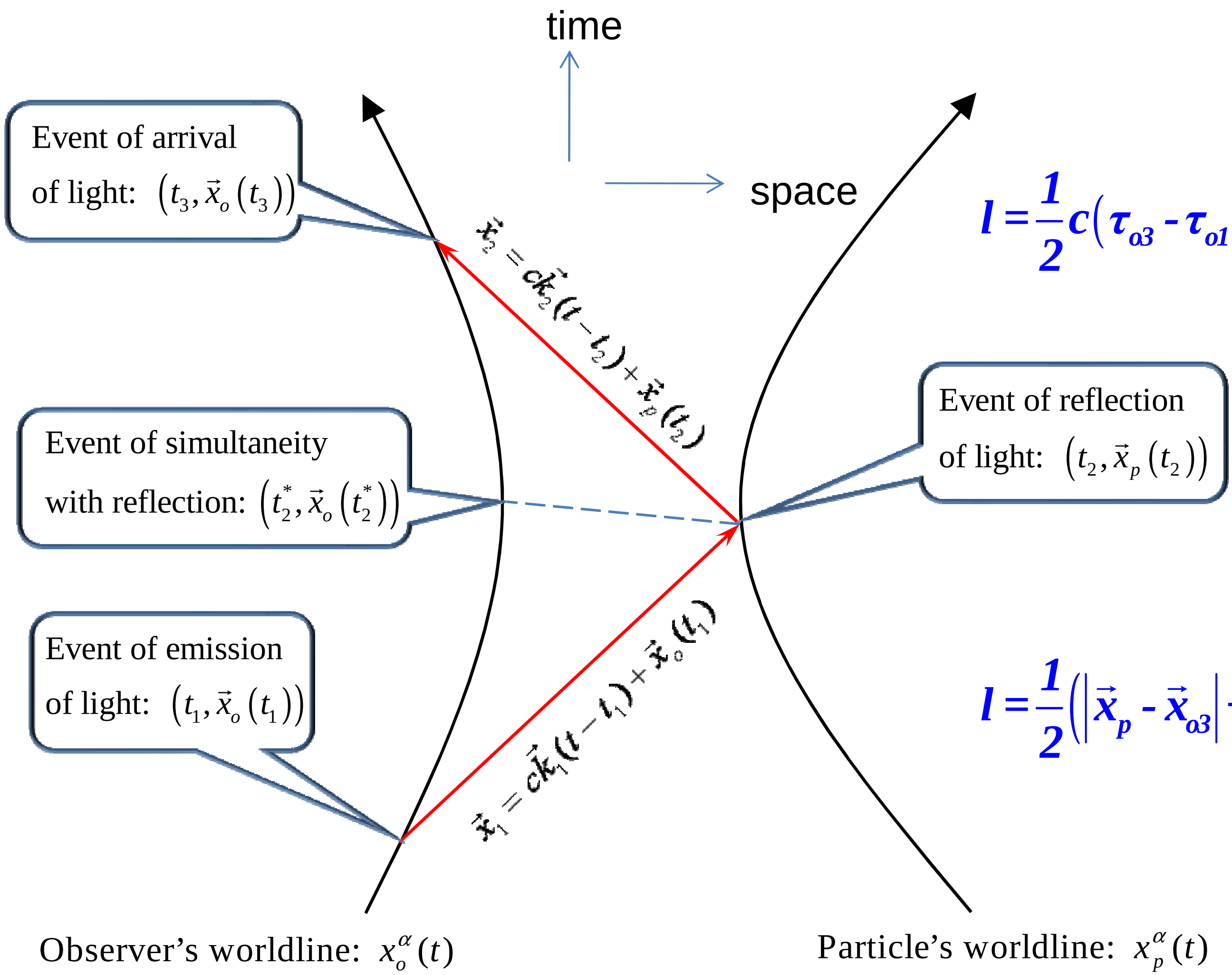


Coordinates are not the fundamental parameters



Are space-time coordinates observable!

Radar distance *does not depend on the choice of coordinates*



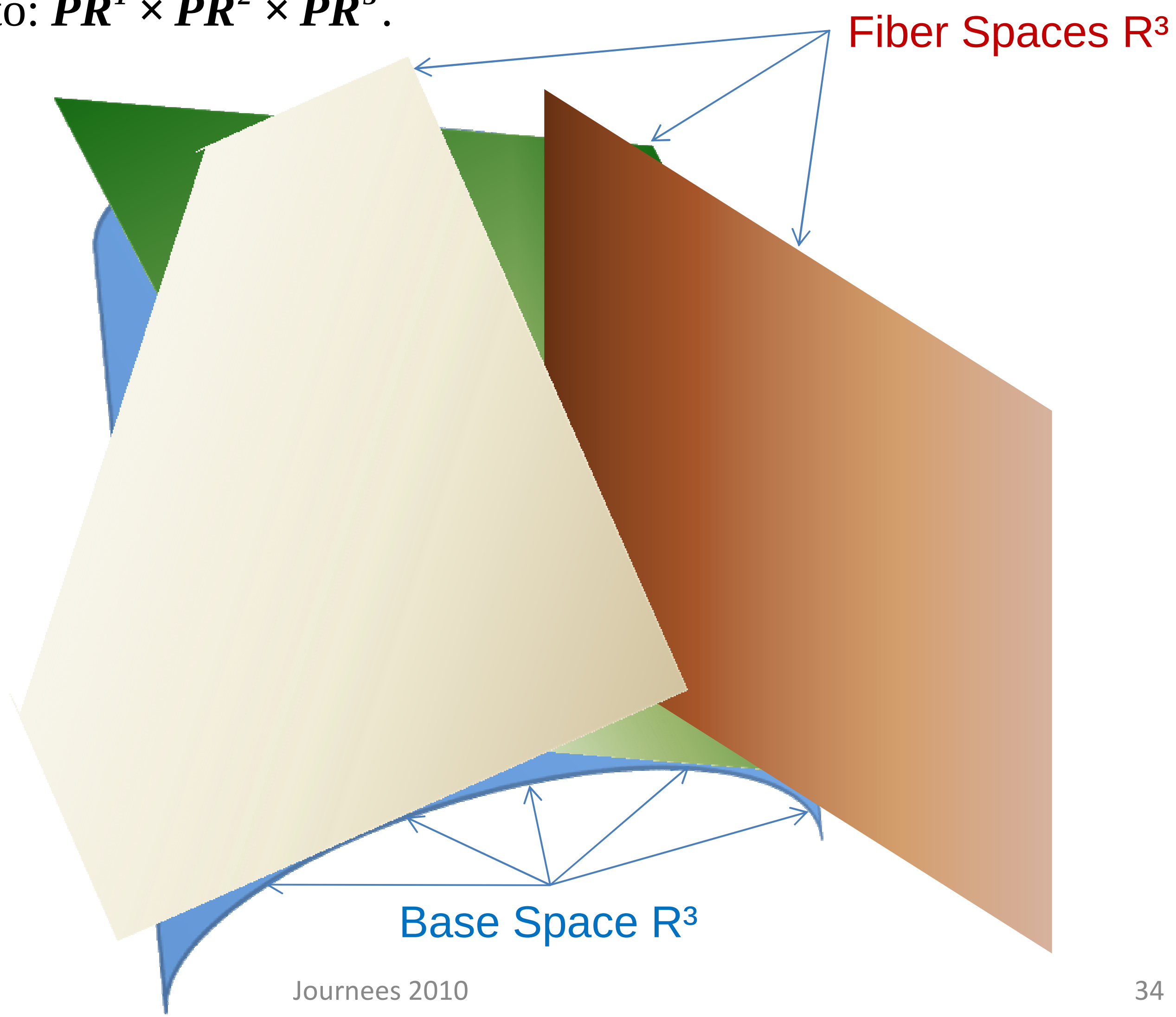
$$l = \frac{1}{2} c\left(\tau_{o3} - \tau_{o1}\right) \quad \textit{where } \tau \textit{ is the proper time}$$

$$l = \frac{1}{2}\left(\left|\vec{x}_p - \vec{x}_{o3}\right| + \left|\vec{x}_p - \vec{x}_{o1}\right|\right) + \Delta_{grav.delay} + \Delta_{Shapiro\,delay}$$

Configuration space of orbital parameters is a fiber bundle manifold

Locally homeomorphic to: $\boldsymbol{R}^3 \times \boldsymbol{R}^3 = \boldsymbol{R}^6$.

Globally homeomorphic to: $\boldsymbol{PR}^1 \times \boldsymbol{PR}^2 \times \boldsymbol{PR}^3$.



Gauge Dependence in the Configuration Space

$$\left.\begin{aligned}
&\boldsymbol{r} = \boldsymbol{r}\left(C_1, C_2, \ldots, C_6; t\right) = \boldsymbol{r}\left(a, e, M, i, \omega, \Omega; t\right)\\
&\boldsymbol{v} = \boldsymbol{v}\left(C_1, C_2, \ldots, C_6; t\right) = \frac{\partial \boldsymbol{r}\left(a, e, M, i, \omega, \Omega; t\right)}{\partial t}\\
&\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{GM}{r^3}\boldsymbol{r}
\end{aligned}\right\} \quad \text{the unperturbed problem}$$

$$\left.\begin{aligned}
&\boldsymbol{r} = \boldsymbol{r}\left[C_1(t), C_2(t), \ldots, C_6(t); t\right]\\
&\dot{\boldsymbol{r}} = \dot{\boldsymbol{r}}\left[C_1(t), C_2(t), \ldots, C_6(t); t\right] \neq \boldsymbol{v}\\
&\dot{\boldsymbol{r}} = \frac{\partial \boldsymbol{r}}{\partial t} + \sum_{j=1}^{6} \frac{\partial \boldsymbol{r}}{\partial C_j}\frac{dC_j}{dt} = \boldsymbol{v} + \boldsymbol{\Phi}\left[C_1(t), C_2(t), \ldots, C_6(t); t\right]\\
&\ddot{\boldsymbol{r}} = -\frac{GM}{r^3}\boldsymbol{r} + \boldsymbol{F}
\end{aligned}\right\} \quad \text{the perturbed problem}$$

$$\left\{\begin{aligned}
&\sum_{j=1}^{6} \frac{\partial \boldsymbol{r}}{\partial C_j}\frac{dC_j}{dt} = \boldsymbol{\Phi}\\
&\sum_{j=1}^{6} \frac{\partial \boldsymbol{v}}{\partial C_j}\frac{dC_j}{dt} = \boldsymbol{F} - \dot{\boldsymbol{\Phi}}
\end{aligned}\right. \quad \Rightarrow \quad \frac{dC_i}{dt} = \sum_{j=1}^{6}\left\{C_i \;\; C_j\right\}\left[\frac{\partial \boldsymbol{r}}{\partial C_j}\bullet\left(\boldsymbol{F} - \dot{\boldsymbol{\Phi}}\right) - \frac{\partial \boldsymbol{v}}{\partial C_j}\bullet \boldsymbol{\Phi}\right]$$

Gauge Transformations in the Configuration Space

$$\left.\begin{array}{l} \displaystyle\sum_{j=1}^{6} \frac{\partial \boldsymbol{r}}{\partial C_j} \frac{dC_j}{dt} = \boldsymbol{\Phi}\left[C_1(t), C_2(t), \ldots, C_6(t); t\right] \\ \displaystyle\sum_{j=1}^{6} \frac{\partial \boldsymbol{r}}{\partial \tilde{C}_j} \frac{d\tilde{C}_j}{dt} = \tilde{\boldsymbol{\Phi}}\left[\tilde{C}_1(t), \tilde{C}_2(t), \ldots, \tilde{C}_6(t); t\right] \end{array}\right\} \Rightarrow \begin{cases} \boldsymbol{\Phi} = 0 & \text{Lagrange} \\ \tilde{\boldsymbol{\Phi}} = -\dfrac{\partial R}{\partial \dot{\boldsymbol{r}}} & \text{Hamilton} \\ \tilde{\boldsymbol{\Phi}} = & \text{a function defined by} \\ & \text{the computational rule} \end{cases}$$

$$\boldsymbol{r}\left[C_1(t), C_2(t), \ldots, C_6(t); t\right] = \boldsymbol{r}\left[\tilde{C}_1(t), \tilde{C}_2(t), \ldots, \tilde{C}_6(t); t\right]$$

$$\dot{\boldsymbol{r}}\left[C_1(t), C_2(t), \ldots, C_6(t); t\right] \neq \dot{\boldsymbol{r}}\left[\tilde{C}_1(t), \tilde{C}_2(t), \ldots, \tilde{C}_6(t); t\right]$$

$$\tilde{C}_i = C_i + \alpha_i\left[C_1(t), C_2(t), \ldots, C_6(t); t\right] \qquad \Rightarrow \qquad \alpha_i = \sum_{j=1}^{6} \left\{C_i \ \ C_j\right\} \frac{\partial \boldsymbol{r}}{\partial C_j} \bullet \left(\tilde{\boldsymbol{\Phi}} - \boldsymbol{\Phi}\right)$$

Pulsar Timing Example

Klioner & Kopeikin, ApJ, 427, 951-5 (1994)

- **Osculating Orbital Elements**

$$r(t) = a(t)\,[1 - e(t)\cos E]$$

$$E - e(t)\sin E = n(t - T_0) + \mathcal{M}(t)$$

$$\tan\frac{E}{2} = \sqrt{\frac{1-e(t)}{1+e(t)}}\tan\frac{f}{2}$$

$$\theta = f + \omega(t)$$

$$a(t) = a_0[1 + \alpha_1\cos f + \alpha_2\cos(2f) + \alpha_3\cos(3f)]$$

$$e(t) = e_0\left[1 + \beta_1\cos f + \frac{\alpha_2}{2}\cos(2f) + \frac{\alpha_3}{2}\cos(3f)\right]$$

$$\omega(t) = \omega_0 + k\,f + \gamma_1\sin f + \frac{\alpha_2}{2}\sin(2f) + \frac{\alpha_3}{2}\sin(3f)$$

$$\mathcal{M}(t) = \delta_1\sin E + \delta_2\sin f - \frac{\alpha_2}{2}\sin(2f) - \frac{\alpha_3}{2}\sin(3f)$$

- **Damour-Deruelle Orbital Elements**

$$r(t) = a_R\,[1 - e_R\cos U]$$

$$U - e_T\sin U = n(t - T_0)$$

$$\tan\frac{U}{2} = \sqrt{\frac{1-e_\theta}{1+e_\theta}}\tan\frac{A_\theta}{2}$$

$$\theta = \theta_0 + (1+k)A_\theta$$

$$a_R = a_0 + \Delta a \qquad e_T = e_0 + \Delta e_T$$

$$e_R = e_0 + \Delta e_R \qquad \delta_R = \frac{e_R - e_T}{e_T}$$

$$e_\theta = e_0 + \Delta e_\theta \qquad \delta_\theta = \frac{e_\theta - e_T}{e_T}$$

Conclusions

- Rescaling the metric tensor seems to be more preferable than the IAU rescaling of coordinates;
- A consistent extension of the resolutions to PPN formalism is desirable to prevent the bias in measuring the PPN parameters;
- Cosmological framework for IAU resolutions has a solid theoretical platform;
- Gauge freedom of the parameter space should be carefully studied before “measuring” the “relativistic effects”

Merci!

Dziekuje!

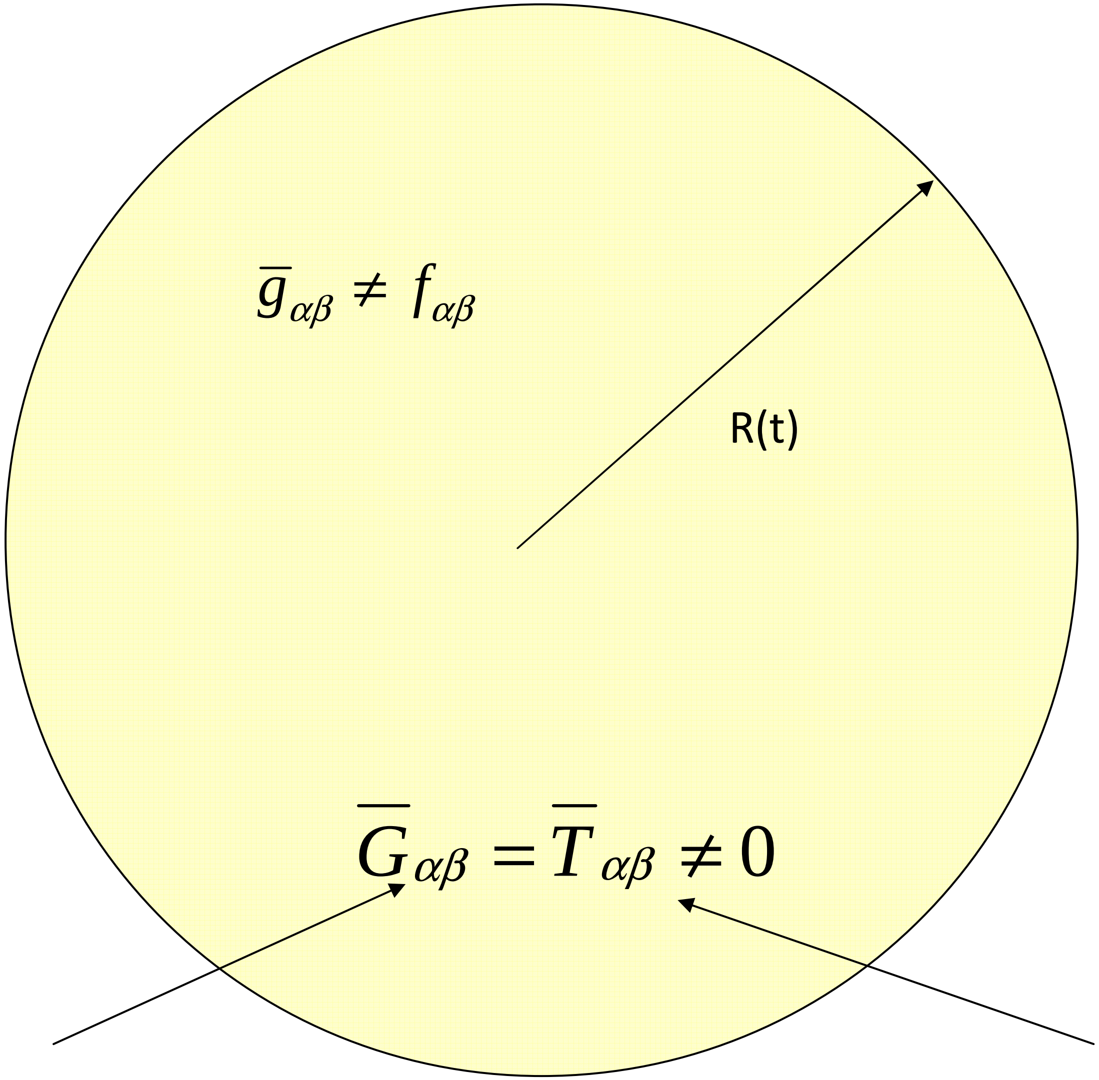
Спасибо!

Thank You!

Dankeschön!

谢谢你!

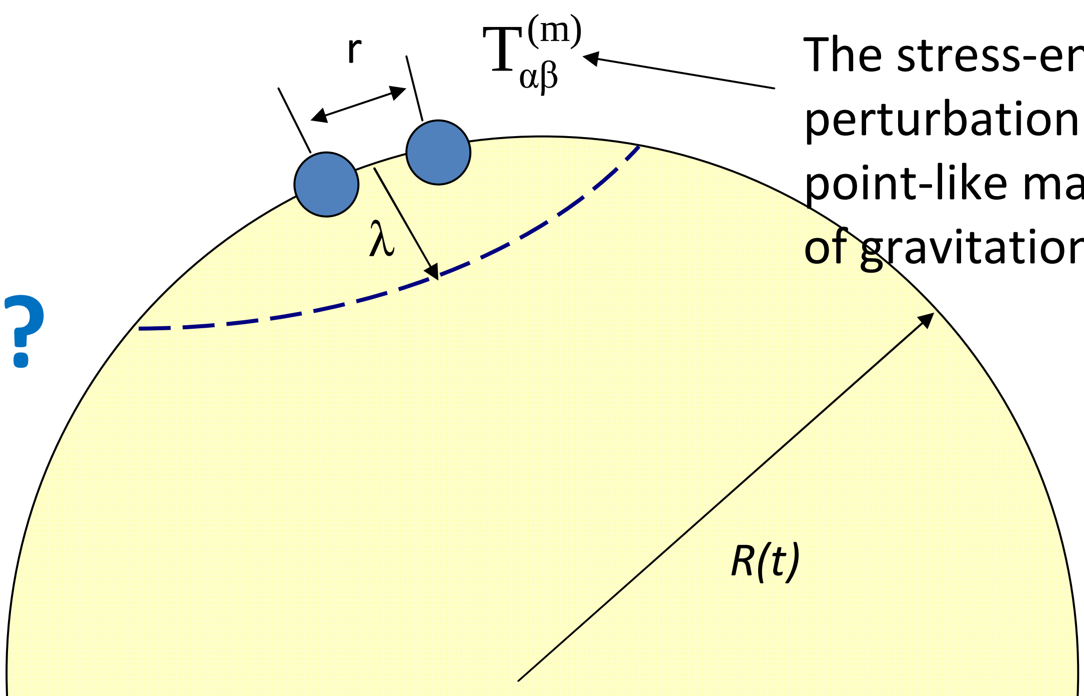
Post-Newtonian approximations in cosmology



The Einstein tensor of the Background geometry $\bar{g}_{\alpha\beta}$

The stress-energy tensor of the background matter

How to perturb?



The stress-energy tensor of a bare perturbation (isolated system, point-like mass, localized package of gravitational waves, etc.)

$$\bar{G}_{\alpha\beta} + \delta G_{\alpha\beta} = \bar{T}_{\alpha\beta} + T^{(m)}_{\alpha\beta} \quad \Rightarrow \quad \delta G_{\alpha\beta} = T^{(m)}_{\alpha\beta}$$

NO ☹

$$\bar{G}_{\alpha\beta} + \delta G_{\alpha\beta} = \underbrace{\bar{T}_{\alpha\beta} + \delta T_{\alpha\beta}} + T^{(m)}_{\alpha\beta} \quad \Rightarrow \quad \delta G_{\alpha\beta} - \delta T_{\alpha\beta} = T^{(m)}_{\alpha\beta}$$

YES ☺

PPN Formalism

Gravity is not a pure geometric phenomenon. Other long range fields (scalar φ, vector A^{α}, etc.) are present.

PPN Field Equations:

$$\Box g_{\alpha\beta} = \kappa T_{\alpha\beta}^{(matter)}(\rho, \vec{v}, g_{\mu\nu}) + \zeta_1 T_{\alpha\beta}^{(scalar)}(\varphi, g_{\mu\nu}) + \zeta_2 T_{\alpha\beta}^{(vector)}(A^{\gamma}, g_{\mu\nu})$$

$$\Box A_{\alpha} = \zeta_3 T_{\alpha}^{(matter)}(\rho, \vec{v}, g_{\mu\nu}) + \zeta_4 T_{\alpha}^{(scalar)}(\varphi, g_{\mu\nu})$$

$$\Box \varphi = \zeta_5 T^{(matter)}(\rho, \vec{v}, g_{\mu\nu}) + \zeta_6 T^{(vector)}(A^{\gamma}, g_{\mu\nu})$$