Influence of the inner core geopotential variations on the rotation of the Earth

A. Escapa[1], J. Getino[2], D. Miguel[2] & J. M. Ferrándiz[1]

[1]Dept. Applied Mathematics. University of Alicante. Spain
[2]Dept. Applied Mathematics. University of Valladolid. Spain

Alberto.Escapa@ua.es

Contents

Three layer Earth model

Accurate modeling of the Earth rotation requires the consideration of a three layer Earth model composed of

- A solid mantle
- A fluid outer core
- A solid inner core

The presence of the inner core forces to consider its differential rotation with respect to the mantle in the dynamics

This induces new internal interactions with the mantle and the fluid. Among others, we have

- Hydrodynamical
- Gravitational
- Electromagnetic

Standard influence on the Earth rotation

The gravitational perturbations originated by the external bodies on the non–spherical Earth modify its rotational dynamics

The main contributions are due to the second degree harmonic. This gravitational potential energy is commonly assumed to be

$$V = G\frac{m}{r^3}\left(C - A\right) C_{20}\left(\sin\eta\right)$$

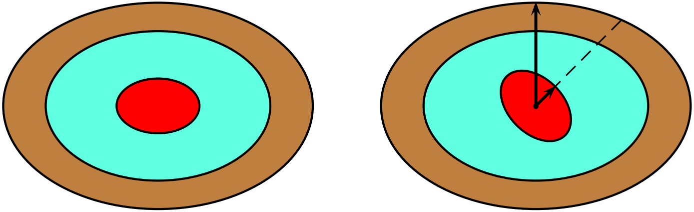
The effects induced by the inner core are a modification in the amplitudes of the nutation terms

$$\Delta\psi = \sum_i A_i\left(n_i;\overline{p}\right)\sin\Theta_i,\ \Delta\varepsilon = \sum_i B_i\left(n_i;\overline{p}\right)\cos\Theta_i$$

This modification is due to the existence of two new normal modes: the Prograde Free Core Nutation and Inner Core Wobble

New influence on the Earth rotation

The differential rotation of the inner core, however, induces a change in the mass distribution inside the Earth



Hence, the presence of the inner core also changes the external gravitational potential

This variation is small, but it could affect to the Earth rotation motion at the level of the microarcsecond

Our objective is to estimate this new contribution to the motion of the Earth figure axis

Earth model

We will consider an Earth model composed of three nearly spherical, elliptical layers

1. An axial–symmetrical rigid mantle
2. An stratified fluid outer core
3. An axial–symmetrical rigid inner core

The interactions of the system are

1. Hydrodynamical interaction of the fluid with the solids (internal)
2. Gravitational interaction among the mantle, the fluid and the inner core (internal)
3. Gravitational perturbations of the Moon and the Sun, whose orbital motion is assumed to be a known function of time (external)

VARIATIONAL FORMULATION

Within the framework of Analytical Mechanics we need to obtain the kinetic and potential energies of the system

The kinetic energy, $\mathcal{T}$, is the sum of the kinetic energy of each layer

- It is assumed a rigid field of velocities
- It accounts for the hydrodynamical interaction

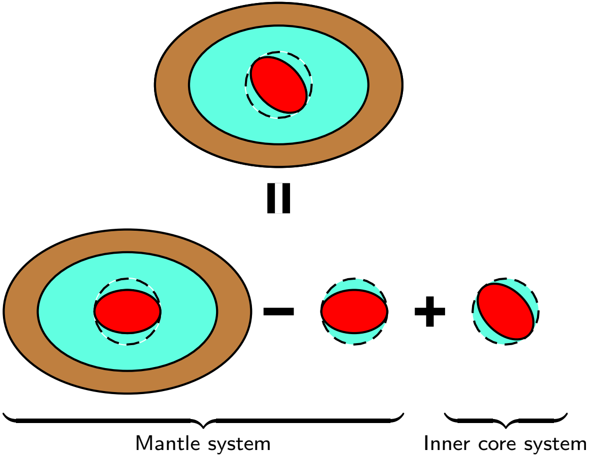
The gravitational potential energy has a twofold origin

- The internal contribution, $\mathcal{V}_i$, is due to the variation of the ellipticity inside the Earth (Ramsey 1940, Jeffreys 1976)
- The external contribution, $\mathcal{V}_e$, is due to the Moon and the Sun. Its main part will be derived from MacCullagh's formula

We will consider these expressions at the first order in the ellipticities

Construction of the matrices of inertia

Matrices of inertia play a key role for constructing the kinetic energy and the external gravitational potential



Matrices of inertia of the the Earth

In the mantle system the matrix of inertia of the total Earth

$$\Pi = \underbrace{A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+e \end{pmatrix}}_{\Pi_0} \underbrace{+ A_s\,(e_s - \delta) \begin{pmatrix} k_1^2 & k_1 k_2 & k_1 k_3 \\ k_1 k_2 & k_2^2 & k_2 k_3 \\ k_1 k_3 & k_2 k_3 & k_3^2 - 1 \end{pmatrix}}_{\Delta\Pi}$$

We have introduced the components of the figure axis of the inner core $\vec{e}_{3s}$ in the mantle system

$$\begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = R_s^t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

and the ellipticities are defined as $e = \dfrac{C - A}{A}$, $e_s = \dfrac{C_s - A_s}{A_s}, \ldots$

External gravitational potential

MacCullagh's formula can be written in the form

$$\mathcal{V}_e = G\frac{m}{2r^5}\left[3\begin{pmatrix} x \\ y \\ z \end{pmatrix}^t \Pi \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \operatorname{trace}(\Pi)\ r^2\right]$$

With the former expression of Π we have $\mathcal{V}_e = \mathcal{V}_2 + \Delta\mathcal{V}_2$,where

$$\mathcal{V}_e = G\frac{1}{2}\frac{m}{r^5}\left[3\begin{pmatrix} x \\ y \\ z \end{pmatrix}^t \Pi_0 \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \operatorname{trace}(\Pi_0)\ r^2\right]$$

$$\Delta\mathcal{V}_2 = \frac{3}{2}G\frac{m}{r^5}\begin{pmatrix} x \\ y \\ z \end{pmatrix}^t \Delta\Pi \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

CANONICAL SOLUTION

The equations of motion are constructed and solved by means of the Hamiltonian formalism (Getino & Ferrándiz 2001) starting from

$$\mathcal{H} = \mathcal{T} + \mathcal{V}_i + \mathcal{V}_2 + \Delta\mathcal{V}_2$$

The Hamiltonian is formulated with an Andoyer–like set of variables (Escapa et al. 2001) and the solution is obtained by perturbation methods (Hori 1966) at the first order

$$\mathcal{H}_0 = \mathcal{T} + \mathcal{V}_i \rightarrow \text{Unperturbed part}$$

$$\mathcal{H}_1 = \mathcal{V}_2 + \Delta\mathcal{V}_2 \rightarrow \text{Perturbation}$$

This procedure allows us to isolate the new contributions arising from $\Delta\mathcal{V}_2$ and to obtain their analytical expression (see Escapa et al. 2011)

PRELIMINARY NUMERICAL ESTIMATION

We have obtained the order of magnitude of the new contributions

Argument					Period	Figure axis (μas)	
l_M	l_S	F	D	$\overline{\Omega}$	Days	$\Delta\psi$	$\Delta\varepsilon$
+0	+0	+0	+0	+1	-6793.48	2.79	-0.31
+0	+0	+0	+0	+2	-3396.74	0.00	-0.01
+0	+1	+0	+0	+0	365.26	14.95	9.29
+0	-1	+2	-2	+2	365.25	-1.78	0.48
+0	+0	+2	-2	+2	182.63	44.61	-19.92
+0	+1	+2	-2	+2	121.75	1.64	-0.72
+1	+0	+0	+0	+0	27.55	-2.22	0.02
+0	+0	+2	+0	+2	13.66	7.17	-3.08
+0	+0	+2	+0	+1	13.63	1.22	-0.63
+1	+0	+2	+0	+2	9.13	0.96	-0.41

Summary

- The presence of the inner core changes the nutational response to the J_2 term, but also induces a variation in the Earth geopotential
- By means of a Hamiltonian approach we have obtained the contributions of this variation to the rotation of the Earth
- The motion of the figure axis is affected only through a change in the Oppolzer terms
- The amplitudes of the new contributions are of the order of tens (μas) for some nutational arguments
- As far as we know, these contributions are not taken into account currently. In view of its magnitude they should be incorporated to the actual standards and models

Acknowledgements

AE's contribution was carried out thanks to a sabbatical leave from the University of Alicante at the National Astronomical Observatory of Japan (NAOJ), supported by the Spanish Ministerio de Educación, project PR2009-0379, within the Programa Nacional de Movilidad de Recursos Humanos I-D+i 2008-2011. The generous hospitality of the NAOJ staff is gratefully acknowledged.

This work has been partially supported by the Spanish projects I+D+I AYA2010-22039-C02-01, AYA2010-22039-C02-02, and by the International Teams program, project 172, of the International Space Science Institute (ISSI) at Bern, Switzerland.