

# 3D NORMAL MODE THEORY OF A ROTATING EARTH MODEL USING A LAGRANGIAN PERTURBATION OF A SPHERICAL MODEL OF REFERENCE

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**ABSTRACT.** The normal mode theory of a rotating Earth model is based on the superposition of two perturbations. The first one is the perturbation of a spherically-averaged model of reference by rotation; it provides the rotating Earth model. The second one is a perturbation of the rotating model; it is a normal mode. In both cases, we consider Lagrangian perturbations. This implies that we define first a new coordinate system in the spherical configuration of reference. These coordinates, which are non-orthogonal, are such that the parameters of the spherical model depend on one of the coordinates only. The relation between the physical spherical coordinates in the rotating configuration and the new coordinates involves the radial discrepancy  $h$  between the spherical model of reference and the rotating model. We assume that, prior to being perturbed, the rotating model is in hydrostatic equilibrium. We determine the shape of the rotating configuration to the second order in  $h$ , using the theory of hydrostatic equilibrium figures. Next, we write the equations of motion of the rotating model in the new coordinate system. We suppose that the stress-strain relation is linearly elastic and isotropic. By inserting the analytical solution for the tilt-over mode in the equations of motion, we show that the terms containing the initial equilibrium gravity must be computed to the second order in  $h$ . Finally, we separate the variables in the equations of motion by expanding the unknown functions on the basis of surface spherical harmonics. We obtain an infinite set of coupled first-order ordinary differential equations that, if truncated, is suitable for numerical integration.

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