Relativistic aspects of the Earth's rotation

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Exact Einstein's theory of gravity (GRT)

In general: global quantities like rotation vector of a body etc. cannot be defined

future theory for EOP: numerical relativity?

fundamental problem: elastic behaviour of Earth's matter

Relativistic theory of elasticity by means of a displacement field has been worked out; problem: relation with observables or EOP unclear

Carter, B., 1973, Commun.Math.Phys. 30,261

EXACT THEORY (GRT)

- uniformely rotating ($\mathbf{\Omega} = \Omega\, \mathbf{e}_z$) axisymmetric body can be treated; the scalar quantities S, Ω and I (spin, angular-velocity and principle moment of inertia are well-defined (e.g., Komar 1959)

- to first order in Ω ‚rigid bodies' can be defined and one finds a ‚Newtonian-like' Euler-theory apart from

$$M_{ab} \neq - \mathrm{STF}(I_{ab})$$

M: mass-quadrupole tensor; I: moment of inertia tensor

Komar, A., 1959,Phys.Rev. 113, 334; Thorne, K., Gürsel, Y., 1983, M.N.R.A.S. 205, 809

Local theory working with a displacement field

has been persued by Xu, Wu, Soffel and Klioner in the 1PN approximation

1. fundamental paper an elastic deformable astronomical bodies in PN approximation using a formalism by Carter & Quintana (1972;1973), Cartesian coordinates

Xu,C., Wu,X., Soffel, M., 2001, Phys.Rev. D 63, 043002

2. Extension to spherical coordinates; junction conditions for surface and internal layers

Xu,C., Wu,X., Soffel, M., Klioner, S., 2003, Phys.Rev. D 68, 064009

3. Expansion of relevant equations in terms of generalized spherical harmonics (scalar-, vector-, tensor spherical harmonics); first for non-rotating ground state

Xu,C., Wu,X., Soffel, M., 2005, Phys.Rev. D 71, 024030

Problems with that approach, work to be done:

- expansion in terms of generalized spherical harmonics for rotating ground state; possibly only for the largest PN-terms
- estimation of orders of magnitude for the real Earth; identification and understanding of certain PN-terms
- relation with observables or Earth-rotation-parameters is unclear
- the use of that formalism in the frame of a perturbative approach (classically: rigid body + transfer functions) is unclear

Global quantities in the post-Newtonian approximation

- PN spin vector for isolated system (e.g., Fock 1959)

- PN spin vector & expression for torque
  in the gravitational N-body problem (DSX 1993)

- PN Tisserand-axes, **Ω,** and moment of inertia tensor
  (Klioner 1996)

- **Newtonian** nutation theory with PN torque for $\Delta\varepsilon$, $\Delta\psi$
  (Bizouard et al., 1992)

Fock, V., 1959, Theory of space, time and gravitation, Pergamon; Damour, T., Soffel, M., Xu, C., 1993, Phys.Rev. D47, 3124, Klioner, S., 1996, in: Ferraz-Mello, S. et al., (eds.) Dynamics, Ephemerides and Astrometry of the Solar System, IAU; Bizouard, C. et al., 1992, in: Les Journées 1992, N.Capitaine (ed.)

Largest terms in that ‚Newtonian+PN-torque' approach:

$\Delta\Psi$ = 3 x 10**(-7)'' sin **Ω** $\Delta\varepsilon$ = 4 x 10**(-7)'' cos **Ω**

Newtonian theory but with TCG and TCB:
Brumberg & Simon 2003

Brumberg, V., Simon, J.-L., 2003, in: Les Journées 2003, A.Finkelstein & N.Capitaine (eds.)

The PN rigid multipole formalism

PN generalization of the Newtonian Euler theory
for a rigid Earth: the rigid multipole moment formalism
(Klioner et al., 2001)

The PN moment of inertia tensor and all mass-multipole moments rotate rigidly with a common angular velocity (defining the rotation axis)

One position from the German Science Foundation
(within our Research unit on Earth's Rotation) with the goal:
New and improved nutation series compatible with the
1PN approximation to GRT

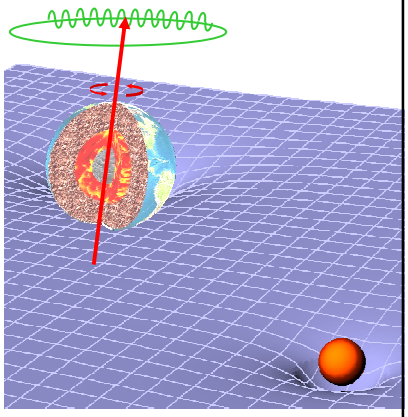
start: July 2006, researcher: Dr.Christophe Le Pocin Lafitte

Main Project Goal

Derivation of a new consistent and improved nutation series for a rigidly rotating multipole model of the Earth in the post-Newtonian approximation of general relativity

using post-Newtonian definitions of :

- potential coefficients (C_{lm}, S_{lm})
- moment of inertia tensor
- dynamical equations in the GCRS
- correct BCRS ↔ GCRS transformation



Tasks

Constants and initial conditions (6 months)
- investigate post-Newtonian relations of constants
  (dynamical ellipticity, moments of inertia, potential coefficients)
- find adequate initial conditions for post-Newtonian equations

Numerical integration (1 year)
- numerical integration of post-Newtonian equations
- identify relevant relativistic effects
- comparisons

Derivation of a post-Newtonian nutation series (1.5 years)
- semi-analytically with Poisson series processor
- numerically (FFT, non-linear least-squares fit, filtering …)

so work is still in progress .....