THE DYNAMICAL MODEL OF THE PLANET MOTIONS AND EPM EPHEMERIDES

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Model of the planet part of the EPM2006 ephemerides

The Ephemerides of Planets and the Moon ephemerides (EPM2006) has been constructed by a simultaneous numerical integration of the equations of motion of the nine major planets, the Sun, asteroids, the Moon and the lunar physical libration performed in the Parameterized Post-Newtonian metric for the harmonic coordinates $\alpha = 0$ and General Relativity values $\beta = \gamma = 1$, taking into account perturbations due to the solar oblateness, many big asteroids and perturbation from a massive ring of small asteroids.

The equations of motion of bodies (considering them as point masses) in a nonrotating solar-system-barycentric Cartesian coordinate frame is:

$$\ddot{\mathbf{r}}_i = A + B + C + D, \text{where}$$

A — the newtonian gravitation accelerations,
B — the relativistic terms (Newhall et al., 1983),
C — the terms produced by the solar oblateness,
D — the terms caused by the asteroid ring (Krasinsky et al., 2002).

The dynamical model of EPM2006 ephemerides includes the following:

- mutual perturbations from major planets, the Sun, the Moon and 5 more massive asteroids;
- perturbations from the other 296 asteroids chosen due to their strong perturbations on Mars and the Earth;
- perturbation from a massive ring with constant mass distribution in the ecliptic plane;
- perturbations from the largest trans-Neptunian objects — TESTS;
- perturbations due to the solar oblateness.

Ephemerides of the planets and the Moon have been produced by numerical integration over a 170-year time interval (1880-2050) carried out of the Everhard method.

Table 1. Radiometric observations

station, object	type	time interval	number of obs.	normal points	*a priori* accuracy
			MERCURY		
Millstone	τ	1964	5	—	7.5–75 km
Haystack	τ	1966–1971	217	—	3 km
Arecibo	τ	1964–1982	341	323	3–30 km
Goldstone	τ	1971–1997	259	138	1.5–3 km
Goldstone cl.p.	τ	1990–1997	40	—	0.15–2.5 km
Crimea	τ	1980–1995	75	23	1.2–4.8 km
Mariner-10	τ	1974–1975	—	2	0.1 km
			VENUS		
Millstone	τ	1961–1967	135	—	1.5–120 km
Haystack	τ	1966–1971	219	—	1.5 km
Arecibo	τ	1964–1970	319	—	3–15 km
Goldstone	τ	1964–1990	512	—	1.5–6 km
Crimea	τ	1962–1995	1139	170	0.15–22.5 km
Magellan	$\alpha\delta$	1990–1994	—	18	$0.''001$–$0.''004$
Magellan	dr	1992–1994	220	—	0.002—0.1 mm/sec
			MARS		
Haystack	τ	1967–1973	3801	133	0.075–12 km
Arecibo	τ	1965–1973	1680	43	0.075–45 km
Goldstone	τ	1969–1994	48989	149	0.075–0.6 km
Crimea	τ	1971–1995	381	78	0.15–4.8 km
Mariner-9	τ	1971–1972	643	—	15–270 m
Viking-1	τ	1976–1982	1161	—	7–12 m
Viking-1	$d\tau$	1976–1978	14980	—	0.16–3.2 m
Viking-2	τ	1976–1977	80	—	7–10 m
Pathfinder	τ	1997	90	—	10–22 m
Pathfinder	$d\tau$	1997	7576	—	0.012 m
Phobos	τ	1989	—	1	0.2 km
MGS	τ	1998–2005	138341	4930	2–7.5 m
Odyssey	τ	2002–2005	142408	3442	2-3 m
spacecraft	$\alpha\delta$	1984–2003	—	44	$0.''0003$–$0.''006$
			JUPITER		
spacecraft, VLA	α	1979–1995	—	4	$0.''003$–$0.''046$
spacecraft, VLA	δ	1979–1995	—	4	$0.''005$–$0.''2$
spacecraft	τ	1973–1995	—	6	1–6 km
spacecraft	$\alpha\delta$	1996–1997	—	24	$0.''007$–$0.''012$
Arecibo s 3,4	τ	1992	—	4	3–14 km

All required reductions of ranging are:

- the relativistic correction — the tine delay of the propagation of radio-signals in the gravitational field of the Sun (the Shapiro effect) and the reduction of observations from the coordinate time of the ephemerides to the proper time of the observer;
- the delay from the solar corona: the following model was used for the solar corona reduction:

  $$N_e(r) = \frac{A}{r^6} + \frac{B + \dot{B}t}{r^2},$$

  where $N_e(r)$ is the electron density, the parameters B and $\dot{B}$ determined from observations for different solar conjunctions;
- the delay from the Earth's troposphere (the model of Niell, 1996 has been used for the global mapping function);
- the correction of observations of Venus and Mars for their topography has been carried out with the help of modern hypsometric maps of surfaces of these planets and using the representation of the global topography by an expansion of spherical functions of 16 – 18 degrees;

  the topography of Mercury has been represented by Legendre functions to the second order with estimating the expansion coefficients from radar observations.

Table 2. Optical and VLA observations of the outer planets

station, object	planet satellite	type	time interval	number of obs.	a priori accuracy
		JUPITER			
USNO	p	transit	1913–1994	4388	0.″5
Tokyo	p	ph-e transit	1963–1988	568	0.″5–0.″8
La Palma	s 3,4	ph-e transit	1986–1997	1316	0.″25
Nikolaev	s 1,2,3,4	photo	1962–1998	2628	0.″2
Flagstaff	s 1,2,3,4	CCD	1998–2005	2860	0.″2
Mountain	s 1,2,3,4	CCD	2002–2002	16	0.″5
		SATURN			
USNO	p	transit	1913–1982	3054	0.″5
Tokyo	p	ph-e transit	1963–1988	506	0.″5–0.″8
Bordeaux	s 6,8	ph-e transit	1987–1993	238	0.″25
La Palma	s 5,6,7,8	ph-e transit	1987–1997	1460	0.″25
Nikolaev	s 3,4,5,6,8	photo	1973–1997	1264	0.″2
Flagstaff	s 3,4,5,6,7,8,9	CCD	1998–2005	4886	0.″2
Mountain	s 3,4,5,6,7,8,9	CCD	2002–2003	628	0.″15
VLA	p	radio	1984	8	0.″03–0.″06
		URANUS			
USNO	p	transit	1913–1993	4244	0.″5
Tokyo	p	ph-e transit	1963–1988	366	0.″5–0.″8
Bordeaux	p	ph-e transit	1985–1992	330	0.″25
Bordeaux	p	CCD	1997	34	0.″2
La Palma	p, s 4	ph-e transit	1984–1997	2072	0.″25
Nikolaev	p	photo	1961–1998	440	0.″2
Flagstaff	p, s 3,4	CCD	1995–2004	2730	0.″2
Mountain	p, s 3,4	CCD	1998–2003	174	0.″15
VLA,ring occ.	p	radio	1977–1985	16	0.″03–0.″2
		NEPTUNE			
USNO	p	transit	1913–1993	3804	0.″5
Tokyo	p	ph-e transit	1963–1988	320	0.″5–0.″8
Bordeaux	p	ph-e transit	1985–1993	366	0.″25
Bordeaux	p	CCD	1997	28	0.″2
La Palma	p	ph-e transit	1984–1998	2212	0.″25
Nikolaev	p	photo	1961–1998	436	0.″2
Flagstaff	p, s 1	CCD	1995–2004	2480	0.″2
Mountain	p, s 1	CCD	1998–2003	120	0.″15
VLA,ring occ.	p	radio	1981–1997	22	0.″03–0.″2
		PLUTO			
station, object	planet satellite	type	time interval	number of obs.	a priori accuracy
Different stat.	p	photo	1914–1967	1164	0.″5–1″
Different stat.	p	photo	1969–1988	674	0.″5–1″
Different stat.	p	photo	1989–1995	82	0.″5–1″
Pulkovo	p	photo	1930–1993	416	0.″5
Tokyo	p	photo	1994	24	0.″3
Bordeaux	p	ph-e transit	1996	12	0.″3
Bordeaux	p	CCD	1995–1997	64	0.″2
La Palma	p	ph-e transit	1986-1998	760	0.″25
Flagstaff	p	CCD	1995–2005	14294	0.″2
Mountain	p	CCD	2000–2003	68	0.″15

The main reductions of optical observations

The values of coefficients B_k for additional phase corrections have been determined from optical observations as in the paper by Standish (1990):

$$\Delta\alpha = B_k \sin\Theta \sin 2i, \quad \Delta\delta = B_k \cos\Theta \sin 2i,$$

where i is the angle between the Earth and the Sun as seen from the planets;
Θ is the position angle of the mid-point of the illuminated edge.

Table 3. The values of the B_k coefficient for the phase effect correction

planet	Jupiter	Saturn	Uranus	Neptune
B_k	$0.''845$ $\pm0.''036$	$0.''425$ $\pm0.''090$	$-0.''046$ $\pm0.''087$	$-0.''228$ $\pm0.''125$

Let k be the coephicient such that kX_1, kY_1, kZ_1 — the shift light center from the barycenter of the system Pluto-Charon, then

$$k = 0.011 \pm 0.006.$$

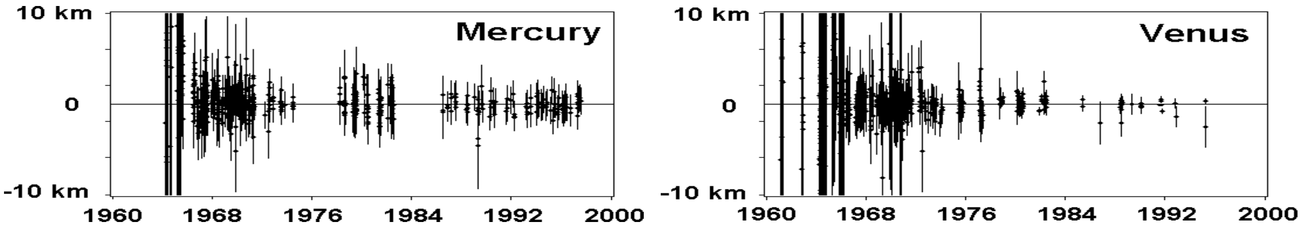
Orientation of EPM onto ICRF

different catalogus → FK4 (Sveshnikov, 1974; 2000)
FK4 → FK5 (Standish, 1995)
FK5 → ICRF (Mignard and Froeschle, 2000)
EPM → ICRF (VLBI measurements of spacecraft):

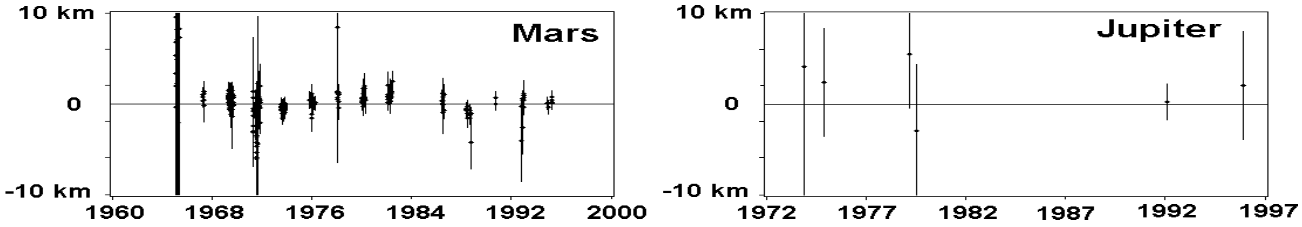
$$\varepsilon_x = 1.9 \pm 0.1, \varepsilon_y = -0.5 \pm 0.2, \varepsilon_z = -1.5 \pm 0.1. \ (mas)$$

The more 230 parameters have been determined from 437883 position observations (1913-2005) of different types:

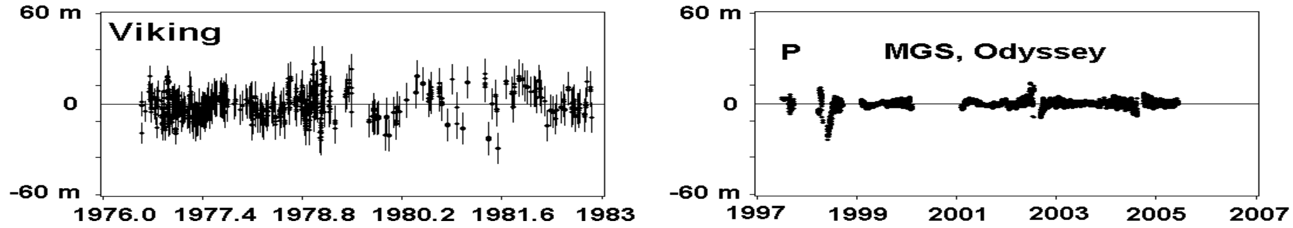
- **the orbital elements of the all planets and 14 satellites of the outer planets the observations of which have been used to improve the orbits of these planets;**
- **the value of Astronomical Unit in kilometers;**
- **the three orientation angles of the ephemerides relative to the International Celestial Reference Frame (ICRF) by the including the ICRF-base VLBI measurements of spacecraft into the solution;**
- **the 13 parameters of Mars's rotation and the coordinates of the three landers on the martian surface;**
- **masses of Jupiter and the seven asteroids that perturb Mars most strongly, mean densities for the three taxonomic classes of asteroids (C, S, M), the mass and radius of the asteroid ring;**
- **the solar quadrupole moment (J_2) and 15 parameters of the solar corona for different conjunctions with the Sun;**
- **the eight coefficients of Mercury's topography and the corrections to the level surfaces of Venus and Mars;**
- **the five coefficients of the phase effect correction for the outer planets;**
- **constant bias for data of Viking-1, Viking-2, Pathfinder, MGS and Odyssey and other series of observations that were interpreted as calibration errors of the instruments or as systematic errors of an unkhown origin;**
- **the relativistic parameters (β, γ, $\dot{G}/G$, the secular trend of the planet perihelia).**



Ranging residuals for Mercury and Venus.

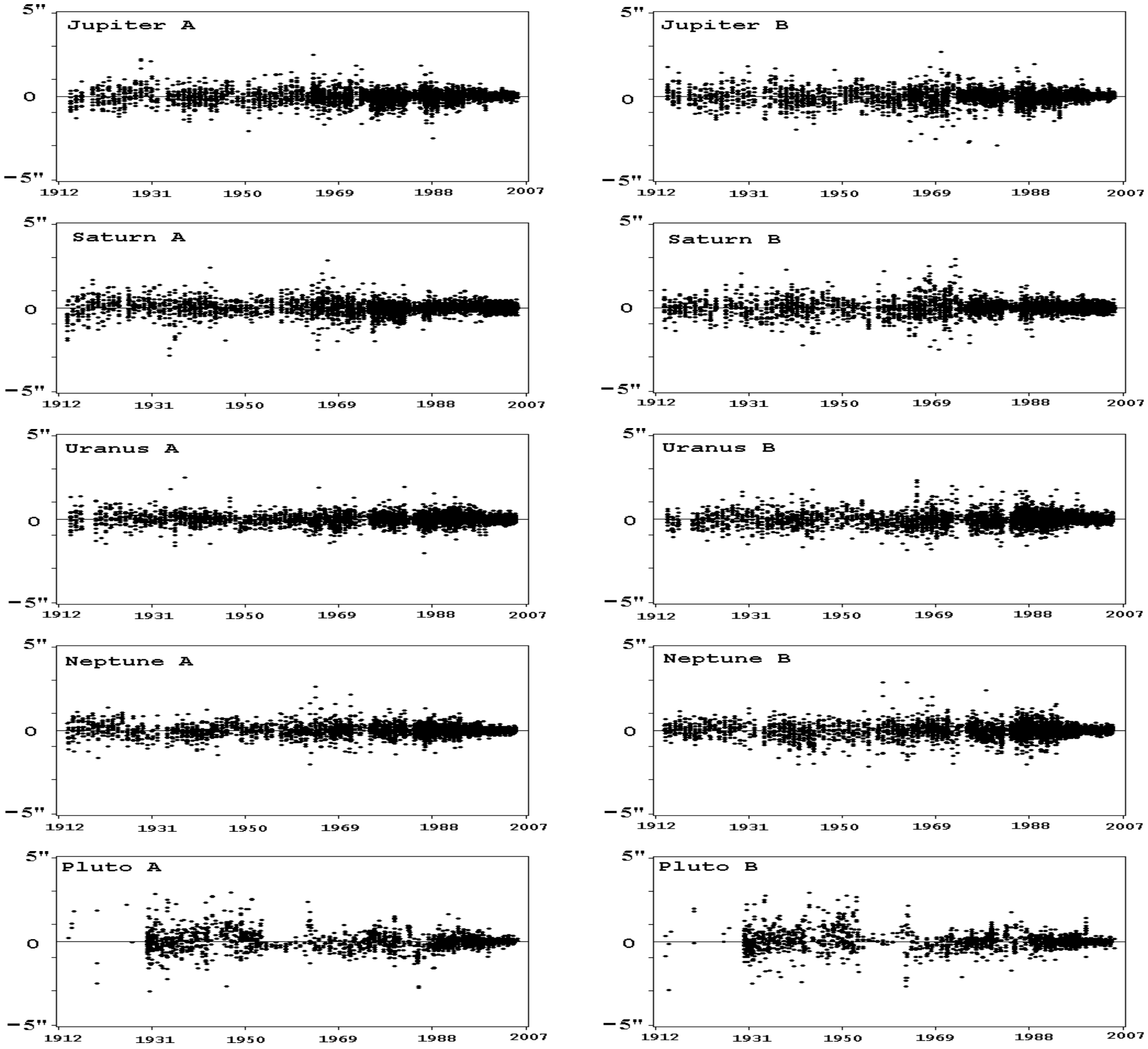


Ranging residuals for Mars and Jupiter.



Ranging residuals of Viking,
Pathfinder-P(1997), MGS(1998–2005), Odyssey(2002–2005).

The rms residuals of ranging for the Mercury are 1.4 km, for Venus and Mars are 0.7 km, for Viking are 8.8 m, for Pathfinder 2.8 m, for MGS and Odyssey 1.2 m.



Residuals of the outer planets 1913–2005 in $\alpha \cos \delta$ (A) and in δ (B), the scale $\pm 5''$.

Table 4. The formal standard deviations of elements of the planet orbits

a - the semi-major axis, i - the inclination of the orbit, Ω - the ascending node, e - the eccentricity, π - the longitude of perihelion, λ - the mean longitude

Planet	a	$\sin i \cos \Omega$	$\sin i \sin \Omega$	$e \cos \pi$	$e \sin \pi$	λ
	[м]	[mas]	[mas]	[mas]	[mas]	[mas]
Mercury	0.333	1.392	1.347	0.105	0.084	0.343
Venus	0.219	0.056	0.030	0.004	0.003	0.031
Earth	0.138	—	—	0.001	0.001	—
Mars	0.267	0.003	0.002	0.001	0.001	0.002
Jupiter	839	4.895	4.219	0.399	0.415	2.021
+ s 1 — 4	616	2.419	2.166	0.313	0.362	1.467
Saturn	10518	14.873	14.676	7.677	7.334	16.658
+ s 3 — 9	4256	3.061	4.117	3.900	2.959	3.501
Uranus	42030	4.455	6.454	5.349	3.665	7.982
+ s 3,4	40294	4.440	6.276	5.057	3.635	7.509
Neptune	489930	4.564	8.958	14.015	19.644	26.123
+ s 1	463307	4.411	8.520	13.115	18.740	24.484
Pluto	3412734	6.790	15.662	80.870	38.847	43.554

The value of the Astronomical Unit and its the formal deviation:

$$\mathbf{AU} = 149597870695.3 \pm 0.1 \ \mathbf{m}.$$

Table 5. Large trans–Neptunian objects

Name	a [AU]	e	i [^{o}]	D [km]	$GM_i/GM_\odot$ [10^{-10}]
Pluto+Charon	39.48	0.25	17.14	2390	73.96
2003 UB_{313}	67.67	0.44	44.18	2755	99.09
Sedna (90377)	523.45	0.85	11.93	1700	19.41
2003 EL_{61}	43.34	0.19	28.19	1617	21.15
2005 FY_9	45.74	0.15	29.00	1500	15.99
Quaoar (50000)	43.09	0.04	8.00	1260	9.48
2002 TC_{302} (84522)	55.49	0.30	35.05	1200	8.19
Varuna (20000)	43.16	0.06	17.15	1040	2.96
Orcus (90482)	39.39	0.22	20.52	900	3.45
Ixion (28978)	39.31	0.25	19.70	890	3.34
1996 TO_{66} (19308)	43.60	0.12	27.33	850	2.91
2002 UX_{25} (55637)	42.85	0.14	19.41	840	2.80
2002 TX_{300} (55636)	43.60	0.13	25.83	810	2.51
2002 AW_{197} (55565)	47.51	0.13	24.31	790	2.34
Chaos (19521)	46.13	0.11	12.00	740	1.92
1999 TC_{36} (47171)	39.63	0.23	8.42	740	1.92
2003 AZ_{84}	39.71	0.17	13.54	730	1.84
2002 MS_4	41.65	0.15	17.69	730	1.84
1995 SM_{55} (24835)	42.15	0.11	26.96	700	1.63
2004 GV_9 (90568)	41.90	0.08	22.03	700	1.63

The total shift of the barycenter of the solar system due to the 19 largest trans-Neptunian objects is 6140 m within the lifetime of GAIA (2011-2020).

For the time interval 1913–2020 the differences of positions of planets for two ephemerides (standart and with the addition of trans–Neptunian objects) are the order of magnitude less than the formal standard deviations of elements of the planet.

It wasn't found the difference of residuals for these ephemerides after adjusting them to the present set of observational data.

TDB and TCB time-base ephemerides

The the version of the EPM ephemerides was constructed in TCB time scale, however the conversion to TCB time scale could not and did not allow greater accuracy of ephemerides and adjusted parameters.

There are different versions of conversion to TCB ephemerides proposed by Brumberg and Standish.
The first version involves the same numerical values in terms of TCB and TDB for the unit of length (AU) in km and for any velocities including the speed of light.
The second version retains the same numerical value in SI units for the heliocentric constant $GM_\odot$ in terms of TDB and TCB.

This situation is rather confusing, especially for different users. In any case, some official recommendation should be adopted before recommending the TCB time ephemerides for public.

The EPM ephemerides has been the base for the Russian "Astronomical Yearbook" since 2006.

the EPM ephemerides are available to outside users:
ftp://quasar.ipa.nw.ru/incoming/EPM2004