From atomic clocks to coordinate times

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Proper time - Coordinate time

- Proper time τ : defined locally and usable only in the vicinity of the instrument. “ what is indicated by the small hand of my watch ” (Einstein)

- Coordinate time T: globally defined and usable.

- Order of magnitude of “relativistic effects”: U/c^2, v^2/c^2

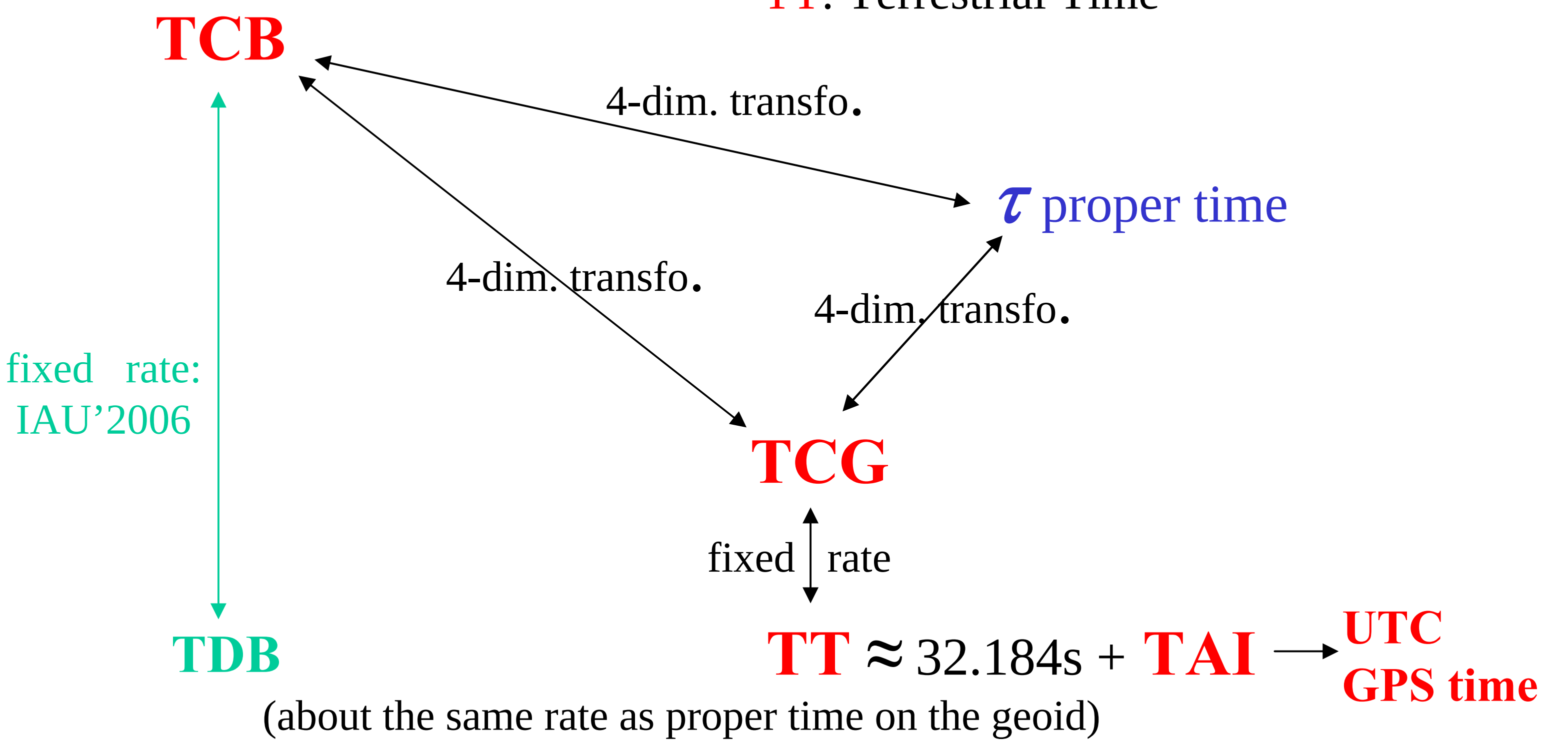
  U: gravitational potential v: velocity c: velocity of light

- $d\tau/dT = 1 - 1/c^2\,[U(X,T) + v(X,T)^2/2\,] + O(c^{-2})$

  Transformation depends on time and space position

Time scales in the solar system: definitions

TCB: Barycentric coordinate time
TDB: Barycentric dynamical time
TCG: Geocentric coordinate time
TT: Terrestrial Time



Barycentric system **Geocentric system**

Transformation from proper time to TCB/TDB

- **Following IAU2000 B1.3-5 notations**

$d\tau/dTCB = 1 - 1/c^2 (w_0 + w_L + v^2/2) + 1/c^4 [4 w_i v^i + (w_0^2/2 - 3w_0 v^2/2 - v^4/8) + \Delta]$

  – The vector potential w^i and the function Δ are given (**Note 2**) by

$$w_A^i(t,\mathbf{x}) = G\left[\frac{-(\mathbf{r}_A \times \mathbf{S}_A)^i}{2r_A^3} + \frac{M_A v_v^i}{r_A}\right], \qquad \Delta_A(t,\mathbf{x}) = \frac{GM_A}{r_A}\left[-2v_a^2 + \sum_{B\neq A}\frac{GM_B}{r_{BA}} + \frac{1}{2}\left(\frac{(r_A^k v_A^k)^2}{r_A^2} + r_A^k a_A^k\right)\right] + \frac{2Gv_A^k(\mathbf{r}_A \times \mathbf{s}_A)^k}{r_A^3},$$

where $\mathbf{S}_A$ is the total angular momentum of body A, v_A and a_A are the coordinate velocity and acceleration of body A.

- Orders of magnitude:
  – Validity = a few 10^{-18}
  – Terms in $1/c^4$: a few 10^{-16}

- IAU'2006: $d\tau/dTDB = d\tau/dTCB / (1 - L_B)$

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Transformation from proper time to TCG/TT

General considerations based on formulas to $1/c^2$ (valid for 10^{-18})

- **For a clock in space**

  $\mathrm{d}\tau/\mathrm{d}TCG = 1 - 1/c^2\,[U_E\,(X) + v^2/2 + U\,(x_E + X) - U\,(x_E) - U_{,k}\,(x_E)\,X^k\,]$

$\Rightarrow$ $\mathrm{d}\tau/\mathrm{d}TT = \mathrm{d}\tau/\mathrm{d}TCG\;/\;(1 - L_G\,)$

Make consistent computation of geopotential model (U_E) and of tidal (last 3) terms (where U is the potential due to external bodies)

But no problem linked to the geoid

- **For a clock on Earth**

  $\mathrm{d}\tau/\mathrm{d}TCG = 1 - 1/c^2\,[U_g - \int g\,\mathrm{d}h + U\,(x_E + X) - U\,(x_E) - U_{,k}\,(x_E)\,X^k\,]$

$\Rightarrow$ $\mathrm{d}\tau/\mathrm{d}TT = (\mathrm{d}\tau/\mathrm{d}TCG\,)/\,(1 - L_G\,)$

Still questions linked to the geoid: how to realize it?

- In practice: $\mathrm{d}\tau/\mathrm{d}TT = 1 + 1/c^2 \int g\,\mathrm{d}h$

$\Rightarrow$ $\mathrm{d}\tau/\mathrm{d}TCG = (\mathrm{d}\tau/\mathrm{d}TT\,)\times(1 - L_G\,)$

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Time scales in the solar system: Realization (2006)

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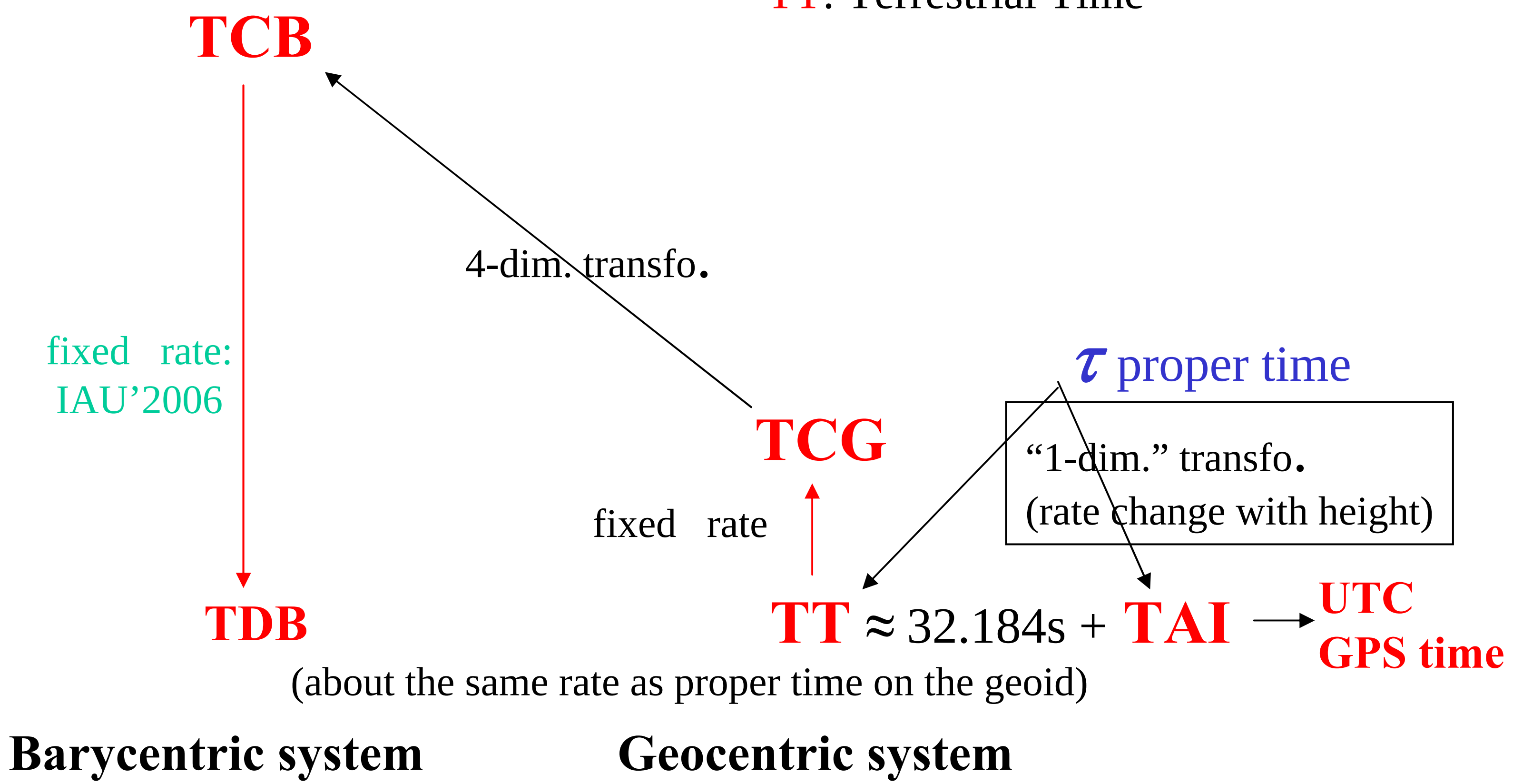
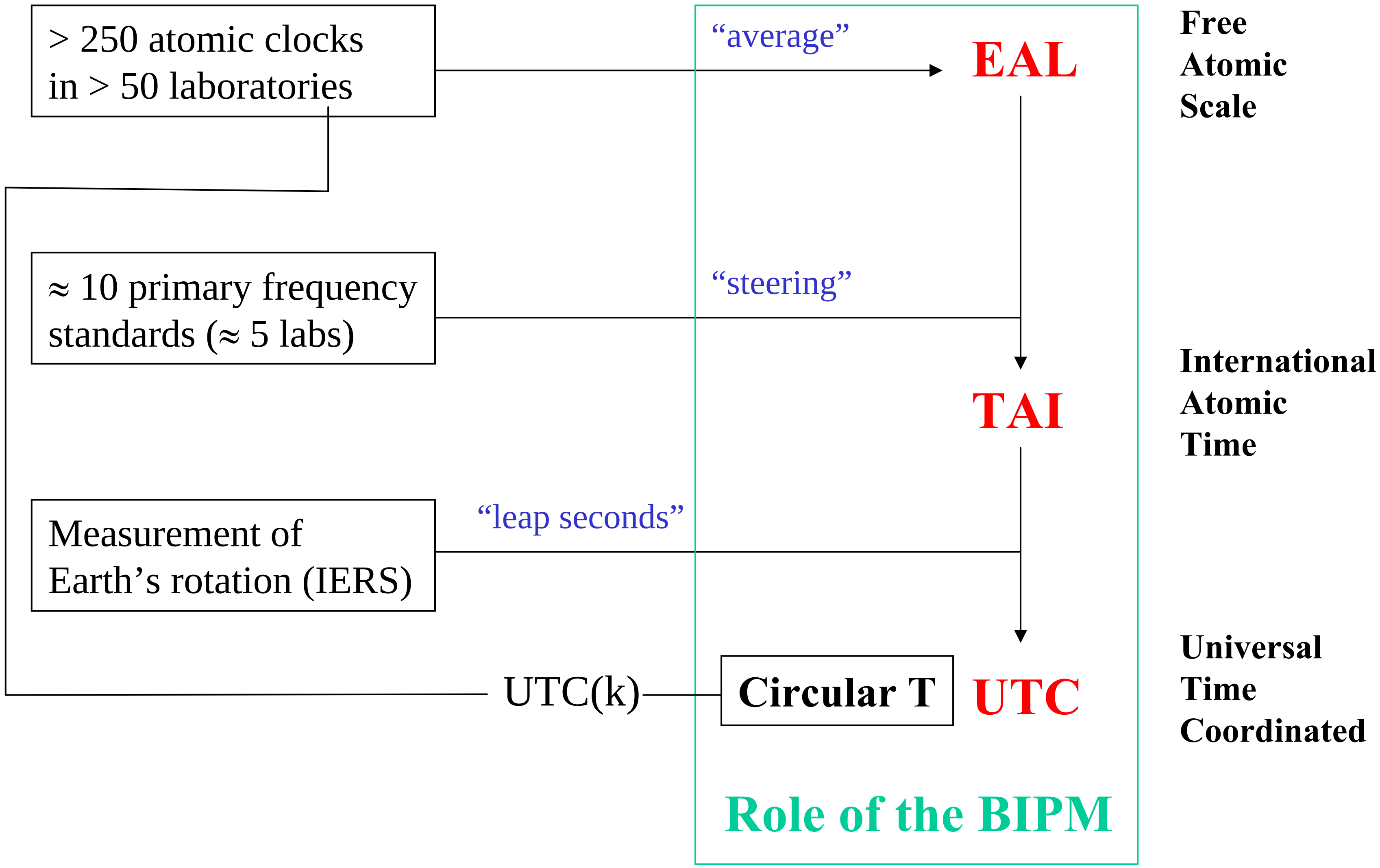
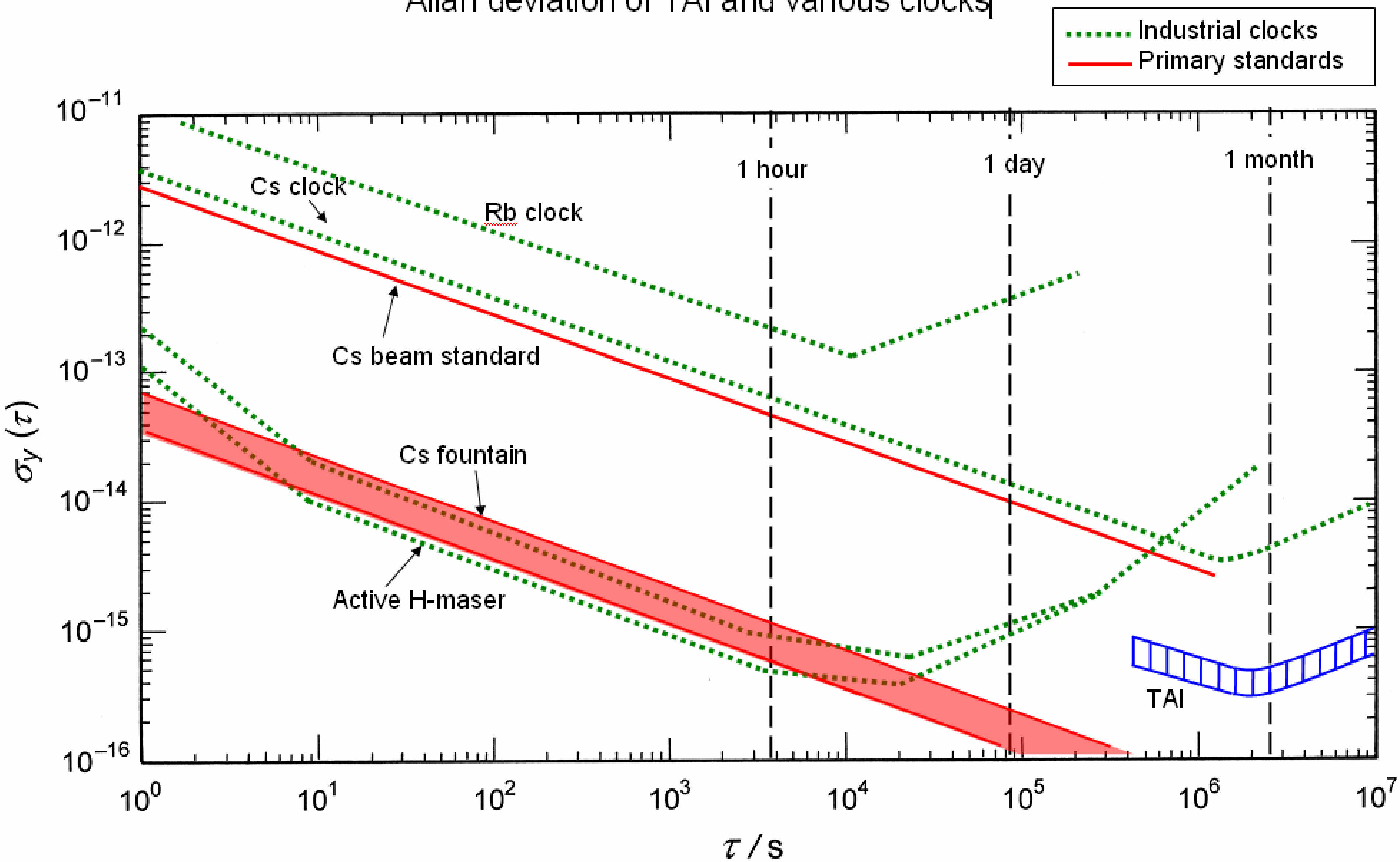


Elaboration of TAI and UTC



Allan deviation of TAI and various clocks

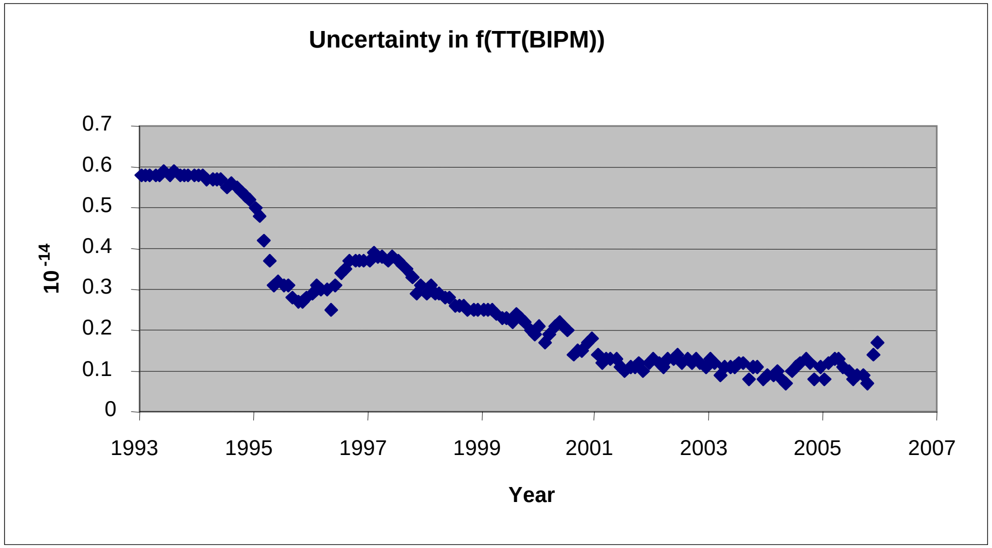


Terrestrial Time and its realizations

- TT is ‘ideal’ and should be distinguished from its realizations (really available time scales).
- Two realizations of TT:
  - TT = TAI + 32.184 s available in “real time”
  - TT(BIPMxx) post-processed
    - Post-processed after the end of year 20xx using all available PFS data
    - Each month, PFS evaluations (within +/- 1 year) are used to estimate f(EAL). In this process, the instability of EAL adds some noise.
    - Monthly estimations of f(EAL) are smoothed and integrated to obtain [EAL-TT(BIPMxx)](t).

The frequency accuracy of TT(BIPMxx)

- Frequency accuracy: decreases from 6×10^{-15} in 1993-1994 to about 1×10^{-15} since 2001.



- Limitations:
  - Sparse data from Cs fountains
  - Noise from frequency transfer techniques over the duration of fountain operation.
- $< 5 \times 10^{-16}$ should be possible now.
- Reaching 1×10^{-16} will require several improvements

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Frequency accuracy: future prospects

- Present definition based on Cs transition may be limited to about $1\text{x}10^{-16}$ because of uncertainty in collisional shift + other.
- Other transitions promise better reproducibility => candidate for new definition of the second (or secondary representations)
  - Hyperfine transition in Rb
  - Optical transitions in atoms Yb, Sr, Ca, …
  - Optical transitions in trapped ions Yb^+, Hg^+, Sr^+, Al^+, …
  - e.g. Hg^+ to Al^+ comparison at NIST < $1\text{x}10^{-16}$ uncertainty

- Problems to transfer this accuracy to a time scale:
  - Stability of a continuous time scale (now 10^{-16}).
  - Uncertainty is time/frequency transfer techniques (now 10^{-16})
  - Uncertainty in transformation from proper to coordinate time (now 10^{-17} on Earth)

- Reaching 10^{-17} will be a challenge

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Time scales in the solar system: Future realization?

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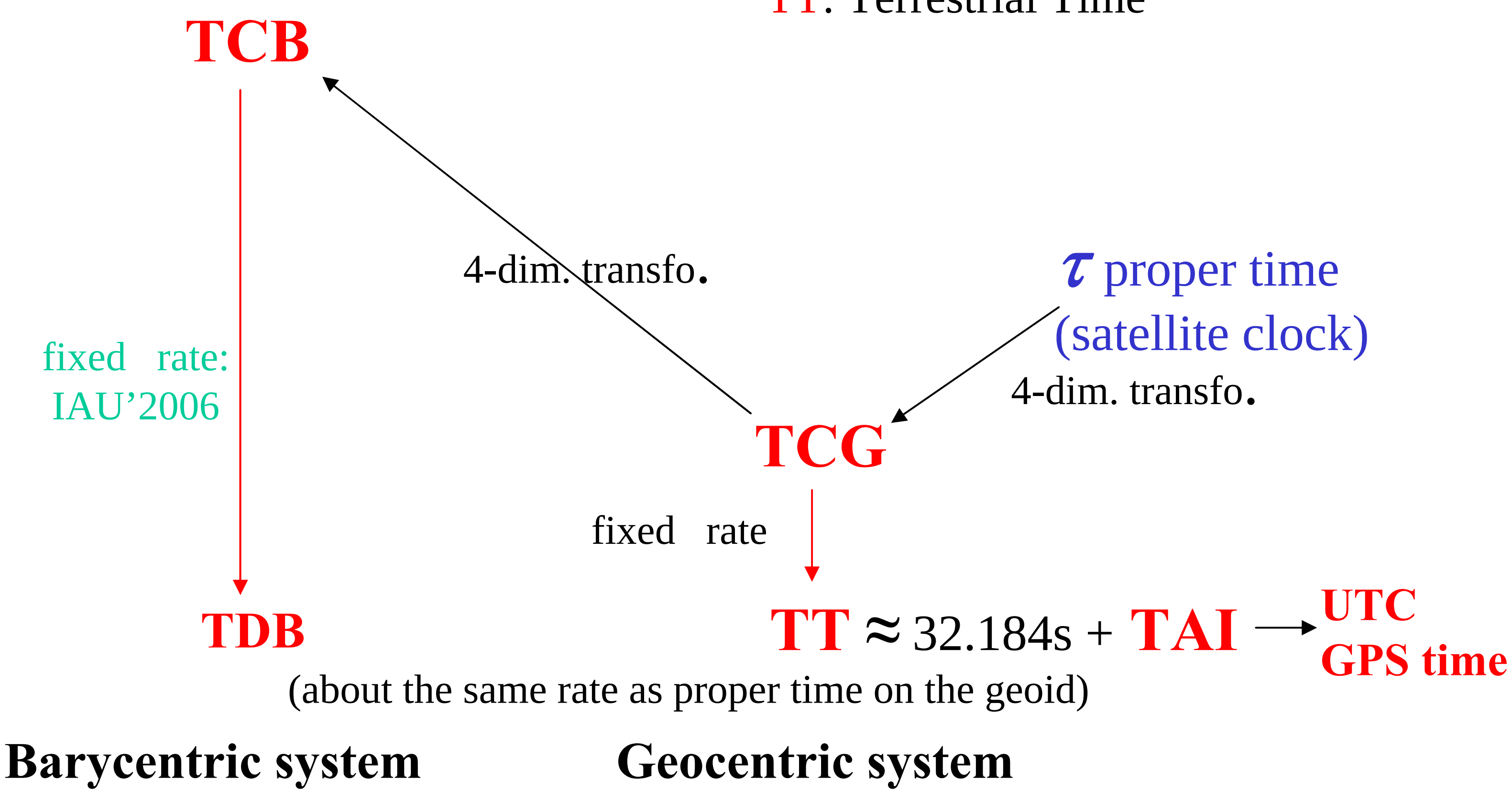

Conclusions

- Time scales in the solar system: $1x10^{-15}$ achieved, 10^{-16} level under exploration.

- Going to $1x10^{-16}$ should be possible without major changes.

- Going to $1x10^{-17}$ (and beyond) will mean
  - New clock transitions
  - Space clocks
  - New comparison techniques

- Beyond $1x10^{-17}$, limitations in the theoretical framework will need to be reviewed.

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