The re-definition of the astronomical unit of length: reasons and consequences

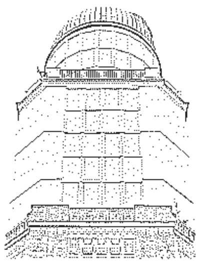
N. Capitaine[(1)], S. Klioner[(2)] , D.D. McCarthy[(3)]

[(1)]: Observatoire de Paris / SYRTE, France

[(2)]: Lohrmann Observatory, Dresden Technical University, Germany

[(3)]: US Naval Observatory, USA

Systèmes de Référence Temps-Espace



What is the astronomical unit?

Traditionally it has been considered to be a measure of the mean distance of the Earth from the Sun.

Originally it was regarded as the length of the semi-major axis of the Earth's orbit.

The practice of using the value of the Gaussian constant, *k* (Gauss, 1809) as a fixed constant which served to define the au has been in use, unofficially since the 19 th century and officially since 1938.

The "old" definition

- The astronomical unit of length is such that the Newtonian gravitational constant G is equal to the square of the Gaussian constant k

$$G = k^2 = 0.000\,295\,912\,208\,285\,591\,102\,5$$

provided that the mass of the Sun and the day (86400 SI seconds) are taken as the units of mass and time, respectively.

The Gaussian constant $k = 0.017\,202\,098\,95$ is a defining constant.

- The need for a specific scale unit in planetary orbit analyses was due to the lack of precise distance measurements in the solar system.

- The concept is similar to other cases when a unit is defined indirectly by fixed values of some natural constants (e.g. $G = c = \hbar = 1$ for the geometrized units)

IAU 1976 definition of the astronomical unit

«The astronomical unit of length is that length (A) for which the Gaussian gravitational constant (k) takes the value of 0.01720209895 when the units of measurements are the astronomical unit of length, mass and time.

The dimensions of k^2 are those of the constant of gravitation (G), i.e., $L^3M^{-1}T^{-2}$. The term "unit distance" is also for the length A. »

Equivalently (cf. The SI brochure): « The radius of an unperturbed circular Newtonian orbit about the Sun of a particle having infinitesimal mass, moving with an angular frequency of 0.017 202 098 95 radians per day. »

Consequences of the current definition of the astronomical unit

- The scale distance in the solar system is provided by the value, A, of the astronomical unit in metres that is fitted to a planetary ephemeris.

- The length of astronomical unit in Système International (SI) meters depends on the theory of motion and observations being used.

- The value in SI units of GM_S is calculated by the formula: $GM_S = A^3k^2/D^2$

The IAU 1976 System of astronomical constants

Defining constants:		
1.	Gaussian gravitational constant	$k = 0{\cdot}017\ 202\ 098\ 95$
2.	Speed of light	$c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$
Primary constants:		
3.	Light-time for unit distance	$\tau_A = 499{\cdot}004\ 782\ \mathrm{s}$ [499·004 7838 ...]
4.	Equatorial radius for Earth	$a_e = 6378\ 140\ \mathrm{m}$ [6378 137]
5.	Dynamical form-factor for Earth	$J_2 = 0{\cdot}001\ 082\ 63$ [0·001 082 626]
6.	Geocentric gravitational constant	$GE = 3{\cdot}986\ 005 \times 10^{14}\ \mathrm{m^3\ s^{-2}}$ [$3{\cdot}986\ 004\ 33 \ldots \times 10^{14}$]
7.	Constant of gravitation	$G = 6{\cdot}672 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$
8.	Ratio of mass of Moon to that of Earth	$\mu = 0{\cdot}012\ 300\ 02$ [0·012 300 038]
9.	General precession in longitude, per Julian century, at standard epoch 2000	$\rho = 5029''0966$
10.	Obliquity of the ecliptic, at standard epoch 2000	$\epsilon = 23^\circ\ 26'\ 21''448$
Derived constants:		
11.	Constant of nutation, at standard epoch 2000	$N = 9''2025$
12.	Unit distance	$c\tau_A = A = 1{\cdot}495\ 978\ 70 \times 10^{11}\ \mathrm{m}$ [$1{\cdot}495\ 978\ 706\ 91 \times 10^{11}$]
13.	Solar parallax	$\arcsin(a_e/A) = \pi_\odot = 8''794\ 148$ [8''794 144]
14.	Constant of aberration, for standard epoch 2000	$\kappa = 20''49\ 552$
15.	Flattening factor for the Earth	$f = 0{\cdot}003\ 352\ 81 = 1/298{\cdot}257$
16.	Heliocentric gravitational constant	$A^3k^2/D^2 = GS = 1{\cdot}327\ 124\ 38 \times 10^{20}\ \mathrm{m^3\ s^{-2}}$ [$1{\cdot}327\ 124\ 40 \ldots \times 10^{20}$]
17.	Ratio of mass of Sun to that of the Earth	$(GS)/(GE) = S/E = 332\ 946{\cdot}0$ [332 946·050 895 ...]
18.	Ratio of mass of Sun to that of Earth + Moon	$(S/E)/(1+\mu) = 328\ 900{\cdot}5$ [328 900·561 400]
19.	Mass of the Sun	$(GS)/G = S = 1{\cdot}9891 \times 10^{30}\ \mathrm{kg}$

The IAU 2009 System of astronomical constants - Table 1

		Auxiliary Defining Constants	
k[a]	Gaussian gravitational constant	$1.720209895 \times 10^{-2}$	[18, 15]
L_G	1−d(TT)/d(TCG)	$6.969290134 \times 10^{-10}$	[19, 32]
L_B	1−d(TDB)/d(TCB)	$1.550519768 \times 10^{-8}$	[20]
TDB_0[b]	TDB−TCB at T_0	-6.55×10^{-5} s	[20]
θ_0[c]	Earth rotation angle at J2000.0	0.7790572732640 revolutions	[19, 5]
$d\theta/dUT1$[c]	Rate of advance of Earth rotation angle	1.00273781191135448 revolutions $UT1\text{-day}^{-1}$	[19, 5]

Constant	Description	Value	Uncertainty	Reference
		Natural Measurable Constants		
G	Constant of gravitation	6.67428×10^{-11} $m^3 kg^{-1} s^{-2}$	6.7×10^{-15} $m^3 kg^{-1} s^{-2}$	CODATA, 2006
		Other Constants		
au[d]	Astronomical unit	$1.49597870700 \times 10^{11}$ m	3 m	Pitjeva & Standish 2009
L_C	Average value of 1−d(TCG)/d(TCB)	$1.48082686741 \times 10^{-8}$	2×10^{-17}	
		Body Constants[e]		
GM_S	Heliocentric gravitational constant	$1.32712442099 \times 10^{20}$ $m^3 s^{-2}$ (TCB-compatible) $1.32712440041 \times 10^{20}$ $m^3 s^{-2}$ (TDB-compatible)	1.0×10^{10} $m^3 s^{-2}$ (TCB-compatible) 1.0×10^{10} $m^3 s^{-2}$ (TDB-compatible)	Folkner et al. 2008, from the ua fitted to DE 421

The IAU 2009 System of astronomical constants - definitions and values

- The IAU 2009 System of astronomical constants (IAU 2009 Resolution B2) retains the IAU 1976 definition of the astronomical unit.

- The TDB-compatible value of the astronomical unit (Table 1) is an average (Pitjeva and Standish 2009) of recent estimates for the astronomical unit defined by *k.*

- The TDB-compatible value for GM_S (Table 1) is consistent with the value of Table 1 for the astronomical unit to within the errors of the estimate.

Modern context

- There is a need for a self-consistent set of units and numerical standards for use in modern dynamical astronomy in the framework of General Relativity,
- the accuracy of modern range measurements makes the use of relative distances unnecessary: BCRS distances between solar system bodies are now known very well so that there is no need to decouple angular positions and distances,
- modern planetary ephemerides (INPOP08, DE423, EPM2008) can provide GM_S directly in SI units and even trace the expected time-dependence of this quantity,
- extending the current definition of the astronomical unit to the relativistic framework would require several additional conventions making the definition even more complicated.

Defining the astronomical unit as a fixed number of SI metres

1. Proposal

- The astronomical unit be re-defined to be a conventional unit of length,
- astronomical unit = L_A metres exactly, L_A being a defining number,
- The defining number should be, for continuity reason, the value for the current best estimate of the ua in m as adopted by IAU 2009 Resolution B2

The CCU (of the CIPM) declared its support to move to a fixed relationship to the SI metre through a defining number

Defining the astronomical unit as a fixed number of SI metres

2. Consequences

– *k* will not have a role any more; it should be deleted from the IAU System of astronomical constants,

– the experimental determination of the ua in SI unit, will be abandoned,

– GM_{Sun} will be determined experimentally,

– the ua would limit its role to that of a unit of length of "convenient" size for some applications.

The IAU 2012 Resolution proposal

- the astronomical unit be re-defined to be a conventional unit of length equal to 149 597 870 700 m exactly, as adopted in IAU 2009 Resolution B2
- this definition be used with all time scales (such as TCB, TDB, TCG, TT, etc.),
- the Gaussian gravitational constant *k* be deleted from the system of astronomical constants,
- the value of the heliocentric gravitation constant, GM_S, be determined observationally in SI units,
- the unique symbol au be used for the astronomical unit.

Defining the astronomical unit as a fixed number of SI metres

3. The advantages with respect to the old definition

- ✓ Eliminates deviation from SI
- ✓ Eliminates dependence of the unit on theories of motion on theories of motion
- ✓ Provides a self-consistent set of units in the relativistic framework
- ✓ Avoids time-dependent units if time variation of solar mass is considered
- ✓ Permits direct determination of time variation in solar mass parameter GM_{Sun} in SI units

Defining the astronomical unit as a fixed number of SI metres

4. Impacts of the change

- Mainly concerns those in the field of high-accuracy solar system dynamics.

- Although astronomical unit defines parsec and thus the whole astronomical distance ladder, the relative difference between the old and the new definitions will not exceed 10^{-10} , so no significant effect considering relative errors of cosmic distances outside solar system.